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AN ECONOMIC ANALYSIS OF ONLINE ADVERTISING USING BEHAVIORAL TARGETING

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Appendix

A.1 Summary of Notations

Notation	Definition and Comments
i	Index for advertisers
j	Index for users
n	Number of advertisers
p_i	Probability that advertiser i's most targeted user clicks on his advertisement
x_{ij}	Distance between advertiser i and user j along the circle
γ	Decay factor, which also measures the heterogeneity of users' preferences
q_{ij}	Decay in the probability that user j clicks on advertiser i
v_i	Unit value that advertiser i derives from each click
z_i	Advertiser i's reference value, defined as $z_i = v_i p_i$
$z_{(i)}$	The i^{th} highest value among all z_i
π_T	The publisher's revenue under traditional advertising
π_B	The publisher's revenue under behavioral targeting
x_i	Marginal user for allocation under behavioral targeting who has the same value to advertisers i and $i + 1$
y_i	Marginal user for payment under behavioral targeting who has the same value to advertisers $i - 1$ and $i + 1$
Δ_i	Cross-border effect under behavioral targeting
H_i	Advertiser i's expected payment for the users won under behavioral targeting
A_i	Advertiser i's payoff under behavioral targeting

A.2 Proof of Lemma 1

Proof. We consider any advertiser i's bidding strategy b_i. We suppose that among the rest of advertisers, advertiser l has the highest proposed expected payment $b_l \cdot p_l \mathrm{E}[q]$. If $b_i \cdot p_i \mathrm{E}[q] > b_l \cdot p_l \mathrm{E}[q]$, advertiser i wins the auction with total expected payment $b_l p_l \mathrm{E}[q]$, and his payoff is

$$v_i p_i \mathrm{E}[q] - b_l p_l \mathrm{E}[q] \tag{16}$$

Otherwise, the advertiser loses the auction and his payoff is zero.

We first consider bidding $b_i < v_i$. If $b_i \cdot p_i \mathrm{E}[q] > b_l \cdot p_l \mathrm{E}[q]$, the advertiser wins the auction and derives the payoff in Equation (16), just as if bidding v_i. If $b_l \cdot p_l \mathrm{E}[q] > v_i \cdot p_i \mathrm{E}[q]$, the advertiser loses the auction and derives zero payoff, just as if bidding v_i. If $b_i \cdot p_i \mathrm{E}[q] < b_l \cdot p_l \mathrm{E}[q] < v_i \cdot p_i \mathrm{E}[q]$, bidding b_i causes the advertiser to lose the auction and receive zero payoff, whereas if he had bid v_i, the advertiser would have won the auction and derived positive payoff as in Equation (16). Therefore, bidding $b_i < v_i$ is (weakly) dominated by bidding v_i.

We then consider bidding $b_i > v_i$. If $v_i \cdot p_i \mathrm{E}[q] > b_l \cdot p_l \mathrm{E}[q]$, the advertiser wins the auction and derives the payoff in Equation (16), just as if bidding v_i. If $b_l \cdot p_l \mathrm{E}[q] > b_i \cdot p_i \mathrm{E}[q]$, the advertiser loses the auction and derives zero payoff, just as if bidding v_i. If $v_i \cdot p_i \mathrm{E}[q] < b_l \cdot p_l \mathrm{E}[q] < b_i \cdot p_i \mathrm{E}[q]$, bidding b_i causes the advertiser to win the auction with negative payoff, whereas bidding v_i leads to zero payoff. Therefore, bidding $b_i > v_i$ is (weakly) dominated by bidding v_i. All together, bidding true value v_i is advertiser i's (weakly) dominant strategy.

Under behavioral targeting, the same argument applies, simply replacing $p_i \mathrm{E}[q]$ with $p_i q_{ij}$ for user j.

A.3 Proof of Lemma 2

Proof. For any advertiser k different from advertisers i and $i + 1$, we first consider the case where the shortest path from user j (located between advertiser i and $i + 1$) to advertiser k passes advertiser i. If $z_i \geq z_k$, then $z_k(1 - \gamma x_{kj}) < z_i(1 - \gamma x_{ij})$ because $x_{ij} < x_{kj}$. If $z_i < z_k$, we have (assuming $k < i$)

$$z_k(1-\gamma x_{kj}) = z_k\left[1-\gamma\left(\frac{i-k}{n}+x_{ij}\right)\right] < z_i - z_k \gamma x_{ij} < z_i(1-\gamma x_{ij})$$

where the first inequality follows from the comparable value assumption and the second inequality is because $z_i < z_k$. Therefore, in this case, advertiser i derives higher value than advertiser k from user j. Similarly, when the shortest path from user j to advertiser k passes advertiser $i + 1$, advertiser $i + 1$ derives higher value than advertiser k from user j. All together, either advertiser i or advertiser $i + 1$ derives higher value than any other advertiser k from user j, and thus user j must be assigned to either advertiser i or advertiser $i + 1$ in equilibrium.

A.4 Proof of Lemma 3

Proof. We denote $V_{ij} \equiv z_i q_{ij}$, the value that the advertiser i derives from user j, and denote $V_{(1)j}$ and $V_{(2)j}$ as the highest and second highest values among all i's. When z_i is increased to $z_i^{'}$ (i.e., $z_i^{'} > z_i$), $V_{ij}^{'} > V_{ij}$ and thus $V_{(2)j}^{'} \geq V_{(2)j}$. In addition, for the additional users that advertiser i wins under $z_i^{'}$, the highest values under z_i become the second highest ones under $z_i^{'}$, and thus $V_{(2)j}^{'} = V_{(1)j} > V_{(2)j}$. Because the publisher's revenue is the sum of the second highest value from each user, $\pi_B^{'} > \pi_B$.

A.5 Proof of Proposition 1

Proof. (a) By Lemma 3, $\partial\pi_B/\partial z_i > 0$. Given $z_{(2)}$ and the order constraint $z_{(i)} \geq z_{(i+1)}$, to maximize π_B, $z_{(3)} = z_{(4)} = \ldots = z_{(n)} = z_{(2)}$. For $z_{(1)}$, we have $\partial\pi_B/\partial z_{(1)} > 0$ subject to $z_{(n)} \geq z_{(1)}\left(1-\frac{\gamma}{n}\right)$ (i.e., the comparable value assumption). Noticing $z_{(n)} = z_{(2)}$, we conclude the $z_{(1)}$ that maximizes π_B is $z_{(1)} = z_{(2)} \Big/ \left(1-\frac{\gamma}{n}\right)$ (when the condition binds).

(b) Similarly, because $\partial\pi_B/\partial z_i > 0$, to minimize π_B subject to $z_{(2)}$ and $z_{(i)} \geq z_{(i+1)}$, we have $z_{(1)} = z_{(2)}$ and $z_{(3)} = z_{(4)} = \ldots = z_{(n)}$. Because of the comparable value assumption $z_{(n)} \geq z_{(1)}\left(1-\frac{\gamma}{n}\right)$, the $z_{(n)}$ that minimizes π_B is $z_{(n)} = \left(1-\frac{\gamma}{n}\right)z_{(1)} = \left(1-\frac{\gamma}{n}\right)z_{(2)}$.

We next determine how the relative location of the two advertisers with $z_{(2)}$ affects the revenue. When $n = 2$ or 3, there is a unique relative distribution of these advertisers' values. We next focus on the case with $n \geq 4$.

We first examine the possible revenue for a user segment $i \mid (i+1)$. For a user segment with $z_i = z_{(2)}$ and $z_{i+1} = z_{(2)}$, or for *user segment* $z_{(2)} \mid z_{(2)}$, the revenue is from a base payment (in Table A1) by Equation (7) with $x_i = \frac{1}{2n}$. For user segment $z_{(2)} \mid z_{(n)}$, if $z_{(2)}$'s other neighbor is of $z_{(n)}$, the revenue is from a base payment (in Table A1) by Equation (7) with $x_i = 1/n$; if $z_{(2)}$'s other neighbor is of $z_{(2)}$, the revenue is the base payment plus the cross-border effect. By Equation (10), the cross-border effect is

$$\frac{\left(1-\frac{\gamma}{n}\right)^2\left(z_{(2)}-z_{(n)}\right)^2}{2\gamma\left(z_{(2)}+z_{(n)}\right)} = \frac{\left(1-\frac{\gamma}{n}\right)^2\left(\frac{\gamma}{n}\right)^2}{2\gamma\left(2-\frac{\gamma}{n}\right)}z_{(2)} = \frac{\gamma(n-\gamma)^2}{2n^3(2n-\gamma)}z_{(2)} \tag{17}$$

For user segment $z_{(n)} \mid z_{(n)}$, if the neighbors are of $z_{(n)}$, the revenue is from a base payment (in Table A1) by Equation (7) with $x_i = 1/(2n)$; if one neighbor is of $z_{(2)}$ and the other is of $z_{(n)}$, the revenue is the base payment plus one piece of the cross-border effect defined in (17). If both neighbors are of $z_{(2)}$, the revenue is the base payment plus two pieces of the cross-border effect (with one piece at each border).

Table A1. The Base Payments for Possible User Segments

User Segment	$z_{(2)} \mid z_{(2)}$	$z_{(2)} \mid z_{(n)}$	$z_{(n)} \mid z_{(n)}$
Base Payment	$\frac{1}{n}\left(1-\frac{3\gamma}{4n}\right)z_{(2)}$	$\frac{1}{n}\left(1-\frac{\gamma}{n}\right)\left(1-\frac{\gamma}{2n}\right)z_{(2)}$	$\frac{1}{n}\left(1-\frac{\gamma}{n}\right)\left(1-\frac{3\gamma}{4n}\right)z_{(2)}$

Notice the base payment from each user segment $i \mid (i+1)$ is solely determined by the values z_i and z_{i+1} (while the cross-border effect depends on the neighbors' values). Therefore, the revenue from base payments (excluding the cross-border effect) can differ only when the two highest-value advertisers are adjacent and when they are not. In the former, the revenue consists revenue from one $z_{(2)} \mid z_{(2)}$ segment, from two $z_{(2)} \mid z_{(n)}$ segments, and from $(n-3)$ $z_{(n)} \mid z_{(n)}$ segments; in the latter (the non-adjacent case), the revenue consists of revenue from four $z_{(2)} \mid z_{(n)}$ segments and from $(n-4)$ $z_{(n)} \mid z_{(n)}$ segments. The difference in these revenues is the revenue from one $z_{(2)} \mid z_{(2)}$ and from one $z_{(n)} \mid z_{(n)}$ minus the revenue from two $z_{(2)} \mid z_{(n)}$; that is,

$$\frac{1}{n}\left(1-\frac{3\gamma}{4n}\right)z_{(2)} + \frac{1}{n}\left(1-\frac{\gamma}{n}\right)\left(1-\frac{3\gamma}{4n}\right)z_{(2)} - \frac{2}{n}\left(1-\frac{\gamma}{n}\right)\left(1-\frac{\gamma}{2n}\right)z_{(2)} = \frac{1}{n}\left(1-\frac{3\gamma}{4n}\right)z_{(2)} - \frac{1}{n}\left(1-\frac{\gamma}{n}\right)\left(1-\frac{\gamma}{4n}\right)z_{(2)} = \frac{1}{n}\frac{\gamma}{2n}\left(1-\frac{\gamma}{2n}\right)z_{(2)} > 0$$

which indicates that the revenue from base payments is greater when the highest-value advertisers are adjacent.

If the two highest-value advertisers are adjacent, the total revenue includes four pieces of cross-border effects, in addition to the revenue from the base payment. When $n = 4$, the four pieces consist of two from the two $z_{(2)} \mid z_{(n)}$ segments (because in each segment $z_{(2)}$ neighbors the other $z_{(2)}$) and two from the $z_{(n)} \mid z_{(n)}$ segment (because both $z_{(n)}$'s neighbor $z_{(2)}$). When $n \geq 5$, the four pieces consist of two from the two $z_{(2)} \mid z_{(n)}$ segments and two from the two $z_{(n)} \mid z_{(n)}$ segments with one $z_{(n)}$ in each segment neighbored by $z_{(2)}$.

If the two highest-value advertisers are not adjacent, the total revenue contains no cross-border effect when $n = 4$. Therefore, the total revenue (including the revenue from the base payment and the cross-border effect) is lower when the two highest-value advertisers are not adjacent than when they are. When $n \geq 5$, if the two highest-value advertisers are $2/n$ distant to each other (i.e., there is one lowest-value advertiser in between), the total revenue contains two pieces of cross-border effect: either from the two $z_{(n)} \mid z_{(n)}$ segments with one $z_{(n)}$ neighbored by $z_{(2)}$ (if $n > 5$) or from the $z_{(n)} \mid z_{(n)}$ segment with both $z_{(n)}$'s neighbored by $z_{(2)}$ (if $n = 5$). If at least two lowest-value advertisers are located between the two highest-value advertisers, the total revenue contains four pieces of cross-border effect. The reason is that, around each arc connecting the two $z_{(2)}$'s, we should have either two $z_{(n)} \mid z_{(n)}$ segments with one $z_{(n)}$ neighbored by $z_{(2)}$ in each segment or one $z_{(n)} \mid z_{(n)}$ segment with both $z_{(n)}$'s neighbored by $z_{(2)}$. Either case leads to two pieces of cross-border effect with one arc, and we have two different arcs. Therefore, when $n \geq 5$, the structure with the two highest-value advertisers being $2/n$ distant from each other generates the least total revenue (with the lowest revenue from the base payment and the least number of cross-border effects).

A.6 Proof of Proposition 2

Proof. (a) When $n = 2$, by Proposition 1, given $z_{(2)}$, the structure with $z_{(1)} = z_{(2)}/(1 - \gamma/2)$ generates the highest π_B, in which the dominant advertiser wins all users (except the other advertiser's most targeted user, from which both advertisers derive the same value). In this case, by Equation (7) with $x_i = 1/2$, $\pi_B = \left(1-\frac{\gamma}{4}\right)z_{(2)}$, which is the same as π_T by Equation (3). Therefore, $\pi_B \leq \pi_T$, and the equality occurs only if $z_{(1)} = z_{(2)}/(1 - \gamma/2)$.

When $n \geq 3$, $\pi_B > \pi_T$ is always possible. For example, when $z_{(1)} = z_{(2)} = \ldots z_{(n)}$, π_B consists of the base payments from the n $z_{(2)} \mid z_{(2)}$ segments. According to Table A1, $\pi_B = \left(1-\frac{3\gamma}{4n}\right)z_{(2)}$. Noticing $\pi_T = \left(1-\frac{\gamma}{4}\right)z_{(2)}$, we can conclude $\pi_B \geq \pi_T$ when $n \geq 3$. Furthermore, a value structure with $z_{(1)} > z_{(2)}$ results in a higher revenue under behavioral targeting (by Lemma 3) and in the same revenue under traditional advertising, which leads to $\pi_B > \pi_T$.

(b) For $2 < n < 6$, traditional advertising might generate higher revenue than behavioral targeting as well. For $n = 5$, see the proof in part (c). When $n = 4$, the least revenue under behavioral advertising is $\left(1-\frac{\gamma}{4}\right)\left(1-\frac{\gamma}{8}\right)z_{(2)}$ (from four $z_{(2)} \mid z_{(n)}$ segments), which is less than $\pi_T = \left(1-\frac{\gamma}{4}\right)z_{(2)}$ regardless of γ. When $n = 3$, the least revenue consists of the base payment (from two $z_{(2)} \mid z_{(n)}$ segments and one $z_{(2)} \mid z_{(2)}$ segment) and two pieces of cross-border effect:

$$\frac{1}{3}\left(1-\frac{\gamma}{4}\right)z_{(2)} + \frac{2}{3}\left(1-\frac{\gamma}{3}\right)\left(1-\frac{\gamma}{6}\right)z_{(2)} + \frac{\gamma(3-\gamma)^2}{27(6-\gamma)}z_{(2)} = \left(1-\frac{\gamma}{4}\right)z_{(2)} + \left[-\frac{1}{6}+\frac{\gamma}{27}+\frac{(1-\gamma/3)^2}{3(6-\gamma)}\right]\gamma z_{(2)}$$

which is less than π_T regardless of γ because the term in the square bracket is negative.

(c) When $n \geq 5$, according to the proof of Proposition 1, the least revenue consists of the base payment (from four $z_{(2)} \mid z_{(n)}$ segments and $(n-4)$ $z_{(n)} \mid z_{(n)}$ segments) and two pieces of cross-border effect:

$$\pi_B = \frac{4}{n}\left(1-\frac{\gamma}{n}\right)\left(1-\frac{\gamma}{2n}\right)z_{(2)} + \frac{n-4}{n}\left(1-\frac{\gamma}{n}\right)\left(1-\frac{3\gamma}{4n}\right)z_{(2)} + \frac{\gamma(n-\gamma)^2}{n^3(2n-\gamma)}z_{(2)}$$

The difference between this π_B and π_T in Equation (3) is

$$\left[\left(1-\frac{\gamma}{n}\right)\left(1-\frac{3n-4}{4n^2}\gamma\right)+\frac{\gamma(n-\gamma)^2}{n^3(2n-\gamma)}-\left(1-\frac{\gamma}{4}\right)\right]z_{(2)} = \left[n^2(n-4)-(3n-4)(n-\gamma)+\frac{4(n-\gamma)^2}{2n-\gamma}\right]\frac{\gamma z_{(2)}}{4n^3}$$

The term in the second square bracket is increasing in γ by noticing the first-order derivative

$$3n-4-\frac{8(n-\gamma)}{2n-\gamma}+\frac{4(n-\gamma)^2}{(2n-\gamma)^2} > 3n-4-\frac{8(n-\gamma)}{2n-\gamma} = 3n-8+\frac{4\gamma}{2n-\gamma} > 0$$

and thus is greater than its value at zero $n^2(n-4) - (3n-4)n + 2n = n(n-6)(n-1)$ (which is nonnegative if $n \geq 6$). Therefore, the difference is positive, and behavioral advertising generates higher revenue if $n \geq 6$. If $n = 5$, the difference is negative for any γ by noting that the value of the term in the square bracket at $\gamma = 2$ is -3.5. All together, we can conclude that if $n < 6$, the revenue under behavioral targeting may be less than the revenue under traditional advertising. If $n \geq 6$, the least amount of revenue under behavioral targeting is still greater than the revenue under traditional advertising. Therefore, if and only if $n \geq 6$, the publisher is better off using behavioral targeting.

A.7 Proof of Corollary 1

Proof. Under behavioral targeting, $x_i = \frac{1}{2n}$ by Equation (6). According to Equation (13), $\pi_B = \left(1-\frac{3\gamma}{4n}\right)$ by substituting in x_i and noting $\Delta_i = 0$. According to Equation (3), $\pi_T = \left(1-\frac{\gamma}{4}\right)$. Therefore, when $n = 3$, $\pi_B = \pi_T$, and when $n > 3$, $\pi_B > \pi_T$.

A.8 Proof of Proposition 3

Proof. (a) When $n = 2$, according to Proposition 2, the publisher is (weakly) better off under traditional advertising. Meanwhile, when the lower-value advertiser gets no market share under behavioral targeting, two advertising strategies could lead to the same revenue for the publisher. Therefore, if $n = 2$, the maximum gain is zero.

When $n > 2$, without loss of generality, we let $z_1 = z_{(1)}$ and normalize $z_{(2)} = 1$. π_B is maximized when $z_{(2)} = z_{(3)} = \ldots = z_{(n)} = 1$ and $z_{(1)} = 1\Big/\left(1-\frac{\gamma}{n}\right)$ by Proposition 1. By substituting $z_{(1)}$ and $x_1 = 1/n$ into (14), we can obtain the maximum π_B and thus calculate the maximum gain as

$$\frac{\pi_B}{\pi_T}-1=\frac{\frac{n-2}{n}\left(1-\frac{3\gamma}{4n}\right)+\frac{2}{n}\left(1-\frac{\gamma}{2n}\right)+\left(\frac{n-\gamma}{(2n-\gamma)n}\right)^2\gamma}{1-\frac{\gamma}{4}}-1=\frac{\frac{\gamma}{4}-\frac{\gamma}{n}\left(\frac{n-2}{n}\frac{3}{4}+\frac{1}{n}\right)+\left(\frac{n-\gamma}{(2n-\gamma)n}\right)^2\gamma}{1-\frac{\gamma}{4}}$$

$$=\frac{\left(1-\frac{3n-2}{n^2}\right)\gamma+\left(\frac{2(n-\gamma)}{n(2n-\gamma)}\right)^2\gamma}{4-\gamma}=\frac{\gamma}{4-\gamma}\left[\frac{(n-2)(n-1)}{n^2}+\left(\frac{2(n-\gamma)}{n(2n-\gamma)}\right)^2\right] \tag{18}$$

(b) We notice the first-order derivative of the term in the square bracket on the right-hand side of (18) with respect to n,

$$\frac{3n-4}{n^3}-2\frac{2(n-\gamma)}{n(2n-\gamma)}\frac{2(2n^2-4n\gamma+\gamma^2)}{(2n-\gamma)^2n^2}>\frac{3n-4}{n^3}-\frac{2(2n^2-4n\gamma+\gamma^2)}{(2n-\gamma)^2n^2}=\frac{(2n-4)(2n-\gamma)^2+(4n^2\gamma-n\gamma^2)}{(2n-\gamma)^2n^3}>0$$

which indicates the maximum gain is increasing in n.

We notice the first-order derivative of the right-hand side of (18) with respect to γ,

$$\frac{4}{(4-\gamma)^2}\left[\frac{(n-2)(n-1)}{n^2}+\left(\frac{2(n-\gamma)}{n(2n-\gamma)}\right)^2\right]-\frac{2\gamma}{(4-\gamma)}\frac{2(n-\gamma)}{(2n-\gamma)n}\frac{2}{(2n-\gamma)^2}=\frac{8(n-\gamma)}{(4-\gamma)^2(2n-\gamma)^3n^2}\left[\frac{(n-2)(n-1)(2n-\gamma)^3}{2(n-\gamma)}+2(n-\gamma)(2n-\gamma)-(4-\gamma)n\gamma\right]$$

Note that the term in the above square bracket is greater than $(2n-\gamma)^2+2(n-\gamma)(2n-\gamma)-(4-\gamma)n\gamma = 8n^2-14n\gamma+3\gamma^2+n\gamma^2$, which is positive. Therefore, the above first-order derivative is positive, and the maximum gain is increasing in γ.

A.9 Proof of Proposition 4

Proof. Part (a) can be found to be true from the discussion in the body.

(b) We first derive advertiser i's payoff under behavioral targeting:

$$\int_0^{x_i}\left[z_i(1-\gamma x)-z_{i+1}\left(1-\frac{\gamma}{n}+\gamma x\right)\right]dx+\int_0^{\frac{1}{n}-x_{i-1}}\left[z_i(1-\gamma x)-z_{i-1}\left(1-\frac{\gamma}{n}+\gamma x\right)\right]dx-\Delta_i$$

$$=\left[z_i-z_{i+1}\left(1-\frac{\gamma}{n}\right)\right]x_i-\frac{\gamma}{2}(z_i+z_{i+1})x_i^2+\left[z_i-z_{i-1}\left(1-\frac{\gamma}{n}\right)\right]\left(\frac{1}{n}-x_{i-1}\right)-\frac{\gamma}{2}(z_i+z_{i-1})\left(\frac{1}{n}-x_{i-1}\right)^2-\Delta_i \tag{19}$$

$$=\frac{1}{2}x_i\left[z_i-z_{i+1}\left(1-\frac{\gamma}{n}\right)\right]+\frac{1}{2}\left(\frac{1}{n}-x_{i-1}\right)\left[z_i-z_{i-1}\left(1-\frac{\gamma}{n}\right)\right]-\Delta_i$$

where the first integral is the expected value net the base payment for the users in segment $i \mid (i+1)$, the second integral is the expected value net the base payment for the users in segment $(i-1) \mid i$, Δ_i is the cross-border effect, and the last equality is because

$$\gamma(z_i + z_{i+1})x_i = z_i - z_{i+1}\left(1-\frac{\gamma}{n}\right)$$

$$\gamma(z_i + z_{i-1})\left(\frac{1}{n} - x_{i-1}\right) = z_i - z_{i-1}\left(1-\frac{\gamma}{n}\right)$$

from Equation (6).

(b.1) When $n = 2$, the difference in the payoff under behavioral targeting and under traditional advertising is

$$\frac{\left[z_1 - z_2\left(1-\frac{\gamma}{2}\right)\right]^2}{\gamma(z_1+z_2)} - (z_1 - z_2)\left(1-\frac{\gamma}{4}\right) = \frac{1}{\gamma(z_1+z_2)}\left[\left(1-\frac{\gamma}{2}\right)^2 z_1^2 - 2\left(1-\frac{\gamma}{2}\right)z_1 z_2 + z_2^2\right] = \frac{1}{\gamma(z_1+z_2)}\left[\left(1-\frac{\gamma}{2}\right)z_1 - z_2\right]^2 \geq 0$$

where the first term (the advertiser's payoff under behavioral targeting) is from Equation (19) (noticing $\Delta_1 = 0$).

(b.2) Advertiser 1's payoff under behavioral targeting can be formulated by letting $i = 1$ in Equation (19), and the payoff under traditional advertising is $(z_1 - z_{(2)})\left(1-\frac{\gamma}{4}\right)$ by Equation (4). The condition for advertiser 1 to be better off under behavioral targeting specified in the proposition then follows.

A.10 Proof of Corollary 4

Proof. The difference in the payoff under behavioral targeting and under traditional advertising is

$$\Delta(z_1) = \frac{\left[z_1 - \left(1-\frac{\gamma}{n}\right)\right]^2}{\gamma(z_1+1)} - (z_1 - 1)\left(1-\frac{\gamma}{4}\right) = \frac{1}{\gamma(z_1+1)}\left[\left(1-\frac{\gamma}{2}\right)^2 z_1^2 - 2\left(1-\frac{\gamma}{n}\right)z_1 + \left(1-\frac{\gamma}{n}\right)^2 + \gamma\left(1-\frac{\gamma}{4}\right)\right]$$

$\Delta(z_1) = 0$ has two roots:

$$z_1^{'} = \frac{\left(1-\frac{\gamma}{n}\right) - \sqrt{\gamma\left(1-\frac{\gamma}{4}\right)\left[\left(1-\frac{\gamma}{n}\right)^2 - \left(1-\frac{\gamma}{2}\right)^2\right]}}{\left(1-\frac{\gamma}{2}\right)^2} \text{ and } z_1^{''} = \frac{\left(1-\frac{\gamma}{n}\right) + \sqrt{\gamma\left(1-\frac{\gamma}{4}\right)\left[\left(1-\frac{\gamma}{n}\right)^2 - \left(1-\frac{\gamma}{2}\right)^2\right]}}{\left(1-\frac{\gamma}{2}\right)^2}$$

with $z_1^{'} < z_1^{''}$. Notice that $\Delta(1) > 0$ because when $z_1 = 1$ the advertiser's payoff under behavioral targeting is positive and his payoff under traditional advertising is zero. Furthermore, because $z_1^{'} + z_1^{''} = 2\left(1-\frac{\gamma}{n}\right)\Big/\left(1-\frac{\gamma}{2}\right)^2 > 2$, we have $z_1^{''} > 1$. Under the comparable value assumption, $z_1 \leq 1\Big/\left(1-\frac{\gamma}{n}\right)$. Because $1\Big/\left(1-\frac{\gamma}{n}\right) < \left(1-\frac{\gamma}{n}\right)\Big/\left(1-\frac{\gamma}{2}\right)^2 < z_1^{''}$, it follows that $z_1 < z_1^{''}$. Therefore, if $z_1 < z_1^{'}$, $\Delta(z_1) > 0$ and thus the advertiser is better off under behavioral targeting; otherwise, it is worse off. Notice that $z_1 \in \left[1, 1\Big/\left(1-\frac{\gamma}{n}\right)\right]$. $z_1 < z_1^{'}$ can occur because $z_1^{'} > 1$. $z_1 > z_1^{'}$ can occur because

$$\frac{1}{1-\frac{\gamma}{n}} - z_1^{'} = \frac{1}{\left(1-\frac{\gamma}{n}\right)\left(1-\frac{\gamma}{2}\right)^2}\left[\left(1-\frac{\gamma}{2}\right)^2 - \left(1-\frac{\gamma}{n}\right)^2 + \left(1-\frac{\gamma}{n}\right)\sqrt{\gamma\left(1-\frac{\gamma}{4}\right)\left[\left(1-\frac{\gamma}{n}\right)^2 - \left(1-\frac{\gamma}{2}\right)^2\right]}\right]$$

$$= \frac{\sqrt{\left(1-\frac{\gamma}{n}\right)^2 - \left(1-\frac{\gamma}{2}\right)^2}}{\left(1-\frac{\gamma}{n}\right)\left(1-\frac{\gamma}{2}\right)^2}\left[\left(1-\frac{\gamma}{n}\right)\sqrt{\gamma\left(1-\frac{\gamma}{4}\right)} - \sqrt{\left(1-\frac{\gamma}{n}\right)^2 - \left(1-\frac{\gamma}{2}\right)^2}\right] = \frac{\sqrt{\left(1-\frac{\gamma}{n}\right)^2 - \left(1-\frac{\gamma}{2}\right)^2}}{\left(1-\frac{\gamma}{n}\right)\left(1-\frac{\gamma}{2}\right)^2}\frac{\left(1-\frac{\gamma}{2}\right)^2\left[1-\left(1-\frac{\gamma}{n}\right)^2\right]}{\left(1-\frac{\gamma}{n}\right)\sqrt{\gamma\left(1-\frac{\gamma}{4}\right)} + \sqrt{\left(1-\frac{\gamma}{n}\right)^2 - \left(1-\frac{\gamma}{2}\right)^2}} > 0$$

A.11 Proof of Proposition 5

Proof. We assume that advertiser 1 is the dominant advertiser. The joint payoff of the publisher and the advertisers under traditional advertising can be formulated as $2z_1\int_0^{\frac{1}{2}}(1-\gamma x)dx$. The joint payoff under behavioral targeting is $\sum_{i=1}^{n}\int_0^{x_i} z_i(1-\gamma x)dx+\int_0^{\frac{1}{n}-x_{i-1}} z_i(1-\gamma x)dx$. For any user j in segment $i \mid (i+1)$, if $x_{ij} \in [0, x_i]$, by Lemma 2 and the definition of x_i, advertiser i wins the user and derives the highest value among all advertisers, which implies

$$z_i(1-\gamma x_{ij}) > z_1(1-\gamma x_{1j})$$

The left-hand side of the inequality is also user j's contribution to the joint payoff under behavioral targeting, and the right-hand side is her contribution to the joint payoff under traditional advertising. Therefore, the joint payoff derived from user j is higher under behavioral targeting than under traditional advertising. The same argument applies if $x_{ij} \in [x_i, 1/n]$ (such that advertiser $i+1$ wins the user and derives the highest value). Because the joint payoff is the sum of the joint payoff from each user, the joint payoff under behavioral targeting is greater than the joint payoff under traditional advertising.

A.12 Maximum Gain in the General Case

Proof. Without loss of generality, we let $z_1 = z_{(1)}$ and normalize $z_{(2)} = 1$. By Lemma 3, π_B is maximized when $z_{(2)} = z_{(3)} = \ldots = z_{(n)}$ and when $z_{(1)}$ is large enough to win all of the users. For each of the two segments with one end at $z_{(1)}$ (i.e., segments $n \mid 1$ and $1 \mid 2$), the expected revenue is the lower-value advertiser's expected value of all the users in the segment, which is $\frac{1}{n}\left(1-\frac{\gamma}{2n}\right)$. For each of the other $n-2$ segments (e.g., $i \mid i+1$), the expected revenue is advertiser i's expected value of the half of the users who are closer to him than to advertiser $i+1$ and advertiser $i+1$'s expected value of the other half, which is $\frac{1}{n}\left(1-\frac{\gamma}{4n}\right)$. We can thus obtain the maximum and calculate the maximum π_B gain as

$$\frac{\pi_B}{\pi_T}-1=\frac{\frac{n-2}{n}\left(1-\frac{\gamma}{4n}\right)+\frac{2}{n}\left(1-\frac{\gamma}{2n}\right)}{1-\frac{\gamma}{4}}-1=\frac{\frac{\gamma}{4}-\frac{\gamma}{n}\left(\frac{n-2}{n}\frac{1}{4}+\frac{1}{n}\right)}{1-\frac{\gamma}{4}}=\frac{\gamma\left(1-\frac{n+2}{n^2}\right)}{4-\gamma}=\frac{\gamma}{4-\gamma}\frac{(n-2)(n+1)}{n^2}$$

The maximum gain is clearly increasing in γ. The gain is also increasing in n because its first-order derivative with respect to n is positive; that is, $\frac{\gamma}{4-\gamma}\left(\frac{1}{n^2}+\frac{4}{n^3}\right)>0$.