MIS Quarterly

RESEARCH ARTICLE

QUALITY COMPETITION AND MARKET SEGMENTATION IN THE SECURITY SOFTWARE MARKET

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Appendix A

Proofs

Proof of Lemma 1

Setting $u_{n+1} = 1$, from Figure 2, we can find the market coverage of θ_j, $j = 1, 2, \ldots, n$, as $x_j = u_{j+1} - u_j$, which can be summed over j to obtain $u_i = 1 - \Sigma_{j=i}^{n} x_j$. Substituting this into (2) for $i = 1$ and noting that $p_0 = \theta_0 = 0$, we find

$$p_1 = G\theta_1 u_1 = G\theta_1\left(1-\sum_{j=1}^{n} x_j\right) \tag{A1}$$

We will now prove the lemma by induction. It is clear that (3) reduces to (A1) for $i = 1$. Let (3) hold for $i = k$, implying

$$p_k = G\left(\theta_k\left(1-\sum_{j=k}^{n} x_j\right)-\sum_{j=1}^{k-1}\theta_j x_j\right)$$

We substitute this into (2) for $i = k+1$ to obtain

$$\begin{aligned}
p_{k+1} &= G(\theta_{k+1}-\theta_k)u_{k+1} + p_k \\
&= G(\theta_{k+1}-\theta_k)\left(1-\sum_{j=k+1}^{n} x_j\right) + G\left(\theta_k\left(1-\sum_{j=k}^{n} x_j\right)-\sum_{j=1}^{k-1}\theta_j x_j\right) \\
&= G(\theta_{k+1}-\theta_k)\left(1-\sum_{j=k+1}^{n} x_j\right) + G\left(\theta_k\left(1-\sum_{j=k+1}^{n} x_j\right)-\theta_k x_k-\sum_{j=1}^{k}\theta_j x_j+\theta_k x_k\right) \\
&= G\left(\theta_{k+1}\left(1-\sum_{j=k+1}^{n} x_j\right)-\sum_{j=1}^{k}\theta_j x_j\right)
\end{aligned}$$

In other words, if (3) holds for $i = k$, then it also holds for $i = k + 1$. Since (3) holds for $i = 1$, the proof is now complete. ■

Proof of Proposition 1

(i) Since $R_i = x_i G H_i - c(\theta_i)$ and $x_i < 1$, we can use the first order condition with respect to x_i to obtain

$$\frac{\partial R_i}{\partial x_i} = GH_i - G\theta_i x_i - gH_i\theta_i x_i = 0 \Leftrightarrow \theta_i x_i = \frac{GH_i}{G+gH_i}$$

Therefore, we get

$$\theta_i x_i - \theta_k x_k = \frac{GH_i}{G+gH_i} - \frac{GH_k}{G+gH_k} = \frac{G^2(H_i - H_k)}{(G+gH_i)(G+gH_k)}$$

Now, from definition of H_l, $l = 1, 2, \ldots, n$, we get

$$\begin{aligned} H_l - H_{l-1} &= \left(\theta_l\left(1-\sum_{j=l}^{n} x_j\right) - \sum_{j=1}^{l-1}\theta_j x_j\right) - \left(\theta_{l-1}\left(1-\sum_{j=l-1}^{n} x_j\right) - \sum_{j=1}^{l-2}\theta_j x_j\right) \\ &= (\theta_l - \theta_{l-1})\left(1-\sum_{j=l}^{n} x_j\right) \end{aligned}$$

Since $\theta_i > \theta_k$ implies that $i > k$, summing the above over l, we get

$$H_i - H_k = \sum_{l=k+1}^{i}(H_l - H_{l-1}) = \sum_{l=k+1}^{i}(\theta_l - \theta_{l-1})\left(1-\sum_{j=l}^{n} x_j\right) \qquad \text{(A2)}$$

Now, since $\theta_i > \theta_k$, there must exist some l, $k < l \leq i$, such that $\theta_l - \theta_{l-1} > 0$, implying that the right hand side of (A2) is strictly greater than zero. Thus, $H_i - H_k > 0$ and, hence, $\theta_i x_i - \theta_k x_k > 0$, which completes the proof.

(ii) First, we note that, for all $i = 1, 2, \ldots, n$,

$$Y - \theta_i x_i = \frac{G-1}{g} - \frac{GH_i}{G+gH_i} = \frac{G(G-1) - gH_i}{g(G+gH_i)} = \frac{Y - H_i + gY^2}{G+gH_i}$$

which, of course, means that $Y \geq H_i \Rightarrow Y \geq \theta_i x_i$. Next, we also know that

$$\begin{aligned} Y - H_i &= \left(1-\sum_{j=1}^{n}\theta_j x_j\right) - \theta_i\left(1-\sum_{j=i}^{n} x_j\right) + \sum_{j=1}^{i-1}\theta_j x_j = \left(1-\sum_{j=i}^{n}\theta_j x_j\right) - \theta_i\left(1-\sum_{j=i}^{n} x_j\right) \\ &\geq \left(1-\sum_{j=i}^{n} x_j\right) - \theta_i\left(1-\sum_{j=i}^{n} x_j\right) = (1-\theta_i)\left(1-\sum_{j=i}^{n} x_j\right) \geq 0 \end{aligned}$$

Therefore, $Y \geq H_i$, and hence $Y \geq \theta_i x_i$, for all $i = 1, 2, \ldots, n$. Summing both sides over i, we get

$$nY \geq \sum_{i=1}^{n}\theta_i x_i = 1 - Y \Leftrightarrow Y \geq \tfrac{1}{n+1} \Leftrightarrow \sum_{i=1}^{n}\theta_i x_i = 1 - Y \leq \tfrac{n}{n+1}$$

(iii) If $\theta_i = \theta_k$, then for every l, $k < l \leq i$, $\theta_l - \theta_{l-1} = 0$. Therefore, from (A2), $H_i - H_k = 0$ implying $\theta_i x_i - \theta_k x_k = 0$ or $x_i = x_k$.

(iv) From the proof of part (ii), we know that $Y \geq \theta_i x_i$. Therefore,

$$\frac{\partial^2 R_i}{\partial g \partial \theta_i} = x_i\left(\left(1-\sum_{j=i}^{n} x_j\right)(Y - \theta_i x_i) + x_i\sum_{j=1}^{i-1}\theta_j x_j\right) \geq 0$$

On the other hand, from the proof of part (i), we know that $\theta_i x_i = \frac{GH_i}{G+gH_i}$; we can then show that

$$\frac{\partial^2 R_i}{\partial g \partial x_i} = YH_i - \theta_i x_i (H_i + Y) = -\frac{H_i^2}{G + gH_i} < 0$$

In other words, when g increases, the first order response by a vendor to this change is to increase quality and decrease market share. However, when $\theta_i = 1$, the vendor cannot increase quality any further and its only first order response would be to decrease its market share. Since such a response complements other vendors' actions, in equilibrium, x_i must decrease.

(v) We prove this part by contradiction. Let there be an equilibrium with $\theta_i = \theta_k < 1$, for some $k < i$, with $1 \leq i, k \leq n$. We know that vendor i solves the following maximization problem:

$$\max_{\theta_i, x_i} R_i = x_i G\left(\theta_i\left(1 - \sum_{j=i}^{n} x_j\right) - \sum_{j=1}^{i-1} \theta_j x_j\right) - c(\theta_i)$$

Since $\theta_i < 1$ (by assumption), the first order condition with respect to θ_i must be satisfied:

$$\frac{\partial R_i}{\partial \theta_i} = x_i\left(G\left(1 - \sum_{j=i}^{n} x_j\right) - gx_i\left(\theta_i\left(1 - \sum_{j=i}^{n} x_j\right) - \sum_{j=1}^{i-1} \theta_j x_j\right)\right) - c'(\theta_i) = 0$$

Furthermore, since $\theta_i = \theta_j$, for all j, $k \leq j < i$, we know from above that the market shares of these vendors would be equal. We set $\theta_k = \ldots = \theta_i = \theta$ and $x_k = \ldots = x_i = x$ to get

$$\begin{aligned} c'(\theta) &= Gx\left(1 - \sum_{j=i}^{n} x_j\right) - gx^2\theta\left(1 - \sum_{j=i}^{n} x_j\right) + gx^2 \sum_{j=1}^{i-1} \theta_j x_j \\ &= Gx\left(1 - \sum_{j=k}^{n} x_j + \sum_{j=k}^{i-1} x_j\right) - gx^2\theta\left(1 - \sum_{j=k}^{n} x_j + \sum_{j=k}^{i-1} x_j\right) + gx^2\left(\sum_{j=1}^{k-1} \theta_j x_j + \sum_{j=k}^{i-1} \theta_j x_j\right) \\ &= Gx\left(1 - \sum_{j=k}^{n} x_j + (i-k)x\right) - gx^2\theta\left(1 - \sum_{j=k}^{n} x_j + (i-k)x\right) + gx^2\left(\sum_{j=1}^{k-1} \theta_j x_j + (i-k)\theta x\right) \\ &= Gx\left(1 - \sum_{j=k}^{n} x_j + (i-k)x\right) - gx^2\theta\left(1 - \sum_{j=k}^{n} x_j\right) + gx^2 \sum_{j=1}^{k-1} \theta_j x_j \end{aligned} \quad \text{(A3)}$$

We now consider how the revenue of the k^{th} vendor changes with the quality of its own product:

$$\frac{\partial R_k}{\partial \theta_k} = Gx\left(1 - \sum_{j=k}^{n} x_j\right) - gx^2\theta\left(1 - \sum_{j=k}^{n} x_j\right) + gx^2 \sum_{j=1}^{k-1} \theta_j x_j - c'(\theta)$$

Substituting (A3) into the above expression, we get $\frac{\partial R_k}{\partial \theta_k} = -Gx^2(i-k) < 0$, which is a violation of the first order condition for an interior solution. ■

Proof of Theorem 1

We first show that there exists a g beyond which all vendors offer a quality level of one. To see this, consider vendor 1. Its profit is given by $R_1 = x_1 GH_1 - c(\theta_1)$. Therefore,

$$\frac{\partial R_1}{\partial \theta_1} = x_1\left(1 - \sum_{j=1}^{n} x_j\right)(G - g\theta_1 x_1) - c'(\theta_1)$$

From (4), $Y = 1 - \Sigma_{j=1}^{n}\theta_j x_j$ and $G = 1 + gY$. Furthermore, from the proof of Proposition 1(i), we know that $\theta_1 x_1 = \frac{GH_1}{G+gH_1}$ in equilibrium. Therefore, we have

$$G - g\theta_1 x_1 \geq \frac{(1+gY)^2}{1+g(H_1+Y)} \geq \frac{(1+gY)^2}{1+2gY}$$

The last inequality results from the fact that $H_1 \leq Y$; see the proof of Proposition 1(ii). Now, from that proof, we also know that $Y \geq \frac{1}{n+1}$, so $gY \geq \frac{g}{n+1}$. Furthermore, $\frac{(1+gY)^2}{1+2gY}$ is an increasing function of gY. Hence, we can write

$$G - g\theta_1 x_1 \geq \frac{(1+gY)^2}{1+2gY} \geq \frac{\left(1+\frac{g}{n+1}\right)^2}{1+\frac{2g}{n+1}} = \frac{(n+1+g)^2}{(n+1)(n+1+2g)}$$

which is clearly an increasing function of g. Since $c'(1)$ is bounded, for a sufficiently large g, we will have

$$\left.\frac{\partial R_1}{\partial \theta_1}\right|_{\theta_1=1} \geq x_1\left(1-\sum_{j=1}^{n} x_j\right)\frac{(n+1+g)^2}{(n+1)(n+1+2g)} - c'(1) > 0$$

Since $c(\cdot)$ is an increasing convex function, the above means that, in equilibrium, an interior solution is not possible and $\theta_1 = 1$. This, in turn, implies that $\theta_i = 1$, for all $i = 2, \ldots, n$. In other words, there must exist a threshold for g—we characterize this threshold as $\gamma_n^{-1}(c)$ in Theorem 2—beyond which vertical differentiation would disappear.

We now consider what happens when g starts decreasing below this threshold. Of course, if the development cost is negligible, trivially, all vendors would continue to offer a quality level of one, irrespective of the value of g. However, if the development cost is significant, some vendors would have to drop their quality level below one, but we will show that they can do so only one vendor at a time. To prove this last claim, suppose that two vendors drop the quality level to below one at the same time. At the value of g where this occurs, these vendors must be barely at the same interior solution. However, from the proof of Proposition 1(v), it is clear that no two vendors can have the same interior solution. Therefore, when g decreases, vendors would not only drop their quality levels from one, but would also do so only one at a time, while maintaining the order of their quality levels. Equivalently, as g increases, their qualities would reach one at different values of g. It is also clear from the proof of Proposition 1(iv) that, once a quality level reaches one, it cannot drop when g increases further. Taken together, it is clear that, as g increases, the segmentation level in the market gradually decreases. ■

Proof of Theorem 2

To prove the existence of the inverse function, it is sufficient to show that $\gamma_n(g)$ is a strictly monotonic function. It turns out that $\frac{\partial \gamma_n(g)}{\partial g} > 0$. To see this, we observe that

$$\frac{\partial \gamma_n(g)}{\partial g} = \frac{A-B}{2g^3 n^2 (n+2)^3 \sqrt{4g(1+g)+(n+1)^2}}$$

where

$$A = 2+8g+8g^2+6n+13gn+10g^2n+6n^2+8gn^2+2g^3n^2+4g^4n^2+2n^3+3gn^3,$$
$$B = C\sqrt{4g(1+g)+(n+1)^2}, \quad \text{and}$$
$$C = 2+4g+4n+5gn+2n^2+3gn^2-2g^3n^2$$

Now $A^2 - B^2 = 4g^3(1+g)^2 n^2 (n+2)^3 D$, where $D = 4g+2n-2-gn$; hence, $A^2 > B^2$, or $A > B$, as long as $D > 0$. If $g \leq 2$, D is always positive. Therefore, we only consider the case where $g > 2$. In that case, $D > 0$ if and only if $n < \frac{2(2g-1)}{g-2}$. Suppose not. Then, there is a $\delta \geq 0$ such that $n = \delta + \frac{2(2g-1)}{g-2}$. Substituting this n into C leads to

$$C = \frac{32(1-g)^3(1+g)^2}{(g-2)^2} + \frac{\delta\left(8(1-2g)g^3 + 29g^2 - 2g - 16\right)}{g-2} + \left(2 + 3g - 2g^3\right)\delta^2$$

The first and the third terms are clearly negative since $g > 2$. Furthermore, since $\delta \geq 0$, when $g > 2$, it can be shown, after some algebra, that the second term cannot be positive. Therefore, $C < 0$, implying $B < 0$. Since $A > 0$ always, this, in turn, implies that $A > B$, which completes the proof of the first part.

For the second part, we note that the oligopoly equilibrium can be in only one of $(n + 1)$ regions. Let Region I denote the range of g values with the first market configuration, where all vendors offer the quality level of one. Similarly, let Region II be the range for the second one, where only the lowest quality vendor, namely vendor 1, offers a quality level below one ($\theta_1 < 1$). Vertical differentiation will be observed as soon as the equilibrium outcome moves out of Region I. Therefore, we only need to examine the boundary between Regions I and II. In both the regions, $\theta_2 = \theta_3 = \ldots = \theta_n = 1$, and it follows from Proposition 1(iii) that $x_2 = x_3 = \ldots = x_n$. Let x_h denote this common market share. The optimization problem of vendor 1 can, therefore, be simplified to

$$\begin{aligned} &\underset{\theta_1, x_1}{\text{Max}}\ R_1 = x_1\theta_1\left(1-(n-1)x_h - x_1\right)\left(1 + g\left(1-(n-1)x_h - \theta_1 x_1\right)\right) - c(\theta_1); \\ &\text{s.t.}\quad \theta_1 > 0,\quad (n-1)x_h + x_1 \leq 1 \end{aligned}$$

The following first order condition must be satisfied by the solution of the unconstrained problem:

$$\frac{\partial R_1}{\partial \theta_1} = x_1\left(1-(n-1)x_h - x_1\right)\left(1 + g\left(1-(n-1)x_h - \theta_1 x_1\right)\right) - g x_1^2 \theta_1\left(1-(n-1)x_h - x_1\right) - c'(\theta_1) = 0$$

Since, at the boundary of Regions I and II, $\theta_1 = 1$, we substitute it above to obtain

$$c'(1) = x_1\left(1-(n-1)x_h - x_1\right)\left(1 + g\left(1-(n-1)x_h - 2x_1\right)\right) = \gamma_n \tag{A4}$$

Now, when θ's are all one, first order conditions with respect to x_i, $i = 1, 2, \ldots, n$, result in

$$x_h = x_1 = x = \frac{(1+2g)(n+1) - \sqrt{4g(1+g) + (n+1)^2}}{2gn(n+2)}$$

which can be substituted into (A4) to obtain

$$\gamma_n(g) = \frac{\mu_n(g) + v_n(g)}{2g^2n^2(n+2)^3}$$

where

$$\mu_n(g) = 2g^3n^2 + (n+1)^2 + g^2\left(n(n+1)(n+6) + 4\right) + g\left(n(3n+5) + 4\right)$$

and

$$v_n(g) = \left(g^2n^2 - g(3n+2) - (n+1)\right)\sqrt{4g(1+g) + (n+1)^2}$$

Therefore, the condition $c'(1) > \gamma_n(g)$—which is equivalent to $g < \gamma_n^{-1}(c'(1))$ —ensures that the outcome is not in Region I. ■

Proof of Proposition 2

Since $\gamma_n(g) = \frac{\mu_n(g) + v_n(g)}{2g^2n^2(n+2)^3}$, using l'Hospital's rule twice, we get

$$\gamma_n(0) = \lim_{g \to 0} \gamma_n(g) = \frac{\mu_n''(0) + v_n''(0)}{4n^2(n+2)^3}$$

The result follows directly from the above. ■

Proof of Theorem 3

Recall that

$$R_v = GH_i x_i + GH_k x_k + G\sum_{t=1}^{m} H_{l_t} x_{l_t} - c(\theta_i)$$

where, as before

$$G = 1 + gY, \quad Y = 1 - \sum_{j=1}^{n} \theta_j x_j, \quad \text{and} \quad H_i = \theta_i\left(1 - \sum_{j=i}^{n} x_j\right) - \sum_{j=1}^{i-1} \theta_j x_j$$

Therefore, we have

$$\frac{\partial G}{\partial x_j} = -g\theta_j \quad \text{and} \frac{\partial \mathrm{H_i}}{\partial x_j} \quad = \begin{cases} -\theta_i & \text{if } j \geq i \\ -\theta_j & \text{otherwise} \end{cases}$$

Combining the above, we can write

$$\begin{aligned} \frac{\partial R_v}{\partial x_i} &= GH_i + x_i\left(G\frac{\partial H_i}{\partial x_i} + H_i\frac{\partial G}{\partial x_i}\right) + x_k\left(G\frac{\partial H_k}{\partial x_i} + H_k\frac{\partial G}{\partial x_i}\right) + \sum_{t=1}^{m} x_{l_t}\left(G\frac{\partial H_{l_t}}{\partial x_i} + H_{l_t}\frac{\partial G}{\partial x_i}\right) \\ &= GH_i - G\theta_i x_i - g\theta_i x_i H_i - G\theta_k x_k - g\theta_i x_k H_k - \sum_{t=1}^{m} x_{l_t}\left(G\theta_{l_t} + g\theta_i H_{l_t}\right) \end{aligned}$$

and

$$\begin{aligned} \frac{\partial R_v}{\partial x_k} &= x_i\left(G\frac{\partial H_i}{\partial x_k} + H_i\frac{\partial G}{\partial x_k}\right) + GH_k + x_k\left(G\frac{\partial H_k}{\partial x_k} + H_k\frac{\partial G}{\partial x_k}\right) + \sum_{t=1}^{m} x_{l_t}\left(G\frac{\partial H_{l_t}}{\partial x_k} + H_{l_t}\frac{\partial G}{\partial x_k}\right) \\ &= GH_k - G\theta_k x_i - g\theta_k x_i H_i - G\theta_k x_k - g\theta_k x_k H_k - \sum_{t=1}^{m} x_{l_t}\left(G\theta_{l_t} + g\theta_k H_{l_t}\right) \end{aligned}$$

The above expressions lead to

$$\begin{aligned} \frac{1}{\theta_i}\frac{\partial R_v}{\partial x_i} - \frac{1}{\theta_k}\frac{\partial R_v}{\partial x_k} &= \frac{1}{\theta_i\theta_k}\left[\theta_k\frac{\partial R_v}{\partial x_i} - \theta_i\frac{\partial R_v}{\partial x_k}\right] \\ &= \frac{G}{\theta_i\theta_k}\left[\left(\theta_k H_i - \theta_i H_k\right) + \left(\theta_i - \theta_k\right)\left(\theta_k x_k + \sum_{t=1}^{m} \theta_{l_t} x_{l_t}\right)\right] \end{aligned} \tag{A5}$$

We now observe

$$\begin{aligned} \theta_k H_i - \theta_i H_k &= \theta_i\theta_k\left(1 - \sum_{j=i}^{n} x_j\right) - \theta_k\sum_{j=1}^{i-1}\theta_j x_j - \theta_i\theta_k\left(1 - \sum_{j=k}^{n} x_j\right) + \theta_i\sum_{j=1}^{k-1}\theta_j x_j \\ &= \theta_i\theta_k\sum_{j=k}^{i-1} x_j + \left(\theta_i - \theta_k\right)\sum_{j=1}^{k-1}\theta_j x_j - \theta_k\sum_{j=k}^{i-1}\theta_j x_j \\ &= \left(\theta_i - \theta_k\right)\sum_{j=1}^{k-1}\theta_j x_j + \theta_k\sum_{j=k}^{i-1}\theta_i x_j - \theta_k\sum_{j=k}^{i-1}\theta_j x_j \\ &= \left(\theta_i - \theta_k\right)\sum_{j=1}^{k-1}\theta_j x_j + \theta_k\sum_{j=k}^{i-1}\left(\theta_i - \theta_j\right)x_j \end{aligned}$$

Substituting this into (A5) leads to

$$\frac{1}{\theta_i}\frac{\partial R_v}{\partial x_i} - \frac{1}{\theta_k}\frac{\partial R_v}{\partial x_k} = \frac{G}{\theta_i \theta_k}\left[\theta_k \sum_{j=k}^{i-1}\left(\theta_i - \theta_j\right)x_j + \left(\theta_i - \theta_k\right)\left(\sum_{j=1}^{k}\theta_j x_j + \sum_{t=1}^{m}\theta_{l_t} x_{l_t}\right)\right]$$

Since θ_i is the largest among all the versions provided, it is easy to see that the right hand side of the above expression is positive, which is a violation of the condition in (7). ■

Proof of Lemma 2

This proof is similar to that of Lemma 1. We can show that (9) implies

$$p_i = \left(1 + g\left(1 - \sum_{j=0}^{n}\theta_j x_j\right)\right)\left(\theta_i\left(1 - \sum_{j=i}^{n} x_j\right) - \sum_{j=0}^{i-1}\theta_j x_j\right)$$

Substituting $x_0 = 1 - \Sigma_{j=1}^{n} x_j$ and $\theta_0 = \phi$ and rearranging terms, we get (10). ■

Proof of Theorem 4

It is similar to the proofs of Theorems 1 and 2, with $g' = g(1 - \phi)$. ■