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RESEARCH ARTICLE

ADVERTISING VERSUS BROKERAGE MODEL FOR ONLINE TRADING PLATFORMS

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Appendix

Monotonicity in Buyers' and Sellers' Participation Decisions

Lemma 1. *Let $c < c'$ and $k < k'$. Under the brokerage model, if a buyer with cost c' participates in the platform, the buyer with cost c also participates in the platform. If a seller with cost k' participates in the platform, the seller with cost k also participates in the platform.*

Proof. The buyer with c' participates if her payoff, specified in Equation (3), is positive (i.e., if $nps - c' \geq 0$). Because $c < c'$, we have $nps - c \geq 0$, which indicates that the buyer with c also has an incentive to participate. Similar reasoning applies to the sellers' participation decisions.

Lemma 2. *Let $c < c'$ and $k < k'$. Under the advertising model, if a buyer with cost c' participates in the platform, the buyer with cost c also participates in the platform. If a seller with cost k' participates in the platform, the seller with cost k also participates in the platform. Moreover, if the seller with k' participates in the advertising service, the seller with k also participates in the advertising service.*

Proof. The proof of buyers' and sellers' decisions of participating in the platform is the same as that of Lemma 1. We next show sellers' decisions about participating in the advertising service. The seller with cost k' participates in the advertising service if her payoff with advertising is greater than her payoff without advertising; that is, if

$$m\min\{p_2, 1\}(\pi - k') - \theta > mp_1(\pi - k')$$

by Equation (6). Because $k < k'$, if the above inequality is true, $m\min\{p_2, 1\}(\pi - k) - \theta > mp_1(\pi - k)$ must be true because $\min\{p_2, 1\} > p_1$, which indicates that the seller with c also has incentive to participate in the advertising service.

Proof of Proposition 1

Proof. Notice that the objective function $p^2 s\tau(\pi - \tau)^2$ crosses zero at $\tau = 0$ and $\tau = \pi$, and it is positive over $[0, \pi]$. Its first-order derivative, $p^2 s[(\pi - \tau)^2 - 2\tau(\pi - \tau)] = p^2 s(\pi - \tau)(\pi - 3\tau)$, is positive over $(0, \pi/3)$ and is negative over $(\pi/3, \pi)$. Therefore, the objective function reaches the maximum at $\tau^* = \pi/3$. Substituting τ^* into the objective function results in the maximum revenue.

Proof of Proposition 2

Proof. Notice that the objective function, $\delta s[\pi(\pi - k'_A) - \delta(\pi - k'_A)^2]k'_A$ crosses zero three times at $k'_A = \pi - \pi/\delta$, $k'_A = 0$, and $k'_A = \pi$. We can verify that the objective function is positive over $[0, \pi]$.

By letting the first-order derivative of the objective function be zero, we have (after removing the constant term δs)

$$[\pi(\pi - k'_A) - \delta(\pi - k'_A)^2] + [-\pi + 2\delta(\pi - k'_A)]k'_A = 0$$

which can be reorganized as

$$-3\delta k'^2_A - 2(1 - 2\delta)\pi k'_A + (1 - \delta)\pi^2 = 0$$

Because $\frac{(1-\delta)\pi^2}{-3\delta} < 0$, one root of the above equation is negative and the other is positive. The positive one is

$$k'^+_A = \frac{2(1-2\delta)\pi - \sqrt{4(1-2\delta)^2\pi^2 + 12\delta(1-\delta)\pi^2}}{-6\delta} = \frac{(2\delta - 1) + \sqrt{1-\delta+\delta^2}}{3\delta}\pi$$

which can be verified to be less than π because $\sqrt{1-\delta+\delta^2} < 1+\delta$. Therefore, its first-order derivative is positive over $(0, k'^+_A)$ and is negative over (k'^+_A, π), which indicates that the objective function is increasing over $(0, k'^+_A)$ and decreasing over (k'^+_A, π).

Notice that the constraint in Inequality (18) is equivalent to $k'_A \le ap\pi/\delta$. Therefore, if $ap\pi/\delta \ge k'^+_A$, the objective function reaches the maximum at $k'^*_A = k'^+_A$; otherwise, it reaches the maximum at $k'^*_A = ap\pi/\delta$ (when the constraint binds). The condition $ap\pi/\delta \ge k'^+_A$ can be rewritten as

$$\frac{ap\pi}{\delta} > \frac{(2\delta - 1) + \sqrt{1-\delta+\delta^2}}{3\delta}\pi$$

which is equivalent to $(\delta - 2 + 3p) > \sqrt{1-\delta+\delta^2}$. By substituting in $\delta = 1 - (1 - a)p$, the above condition can be simplified to $p > \frac{1+a}{1+2a} = \hat{p}(a)$.

Therefore, if $p < \hat{p}(a)$, substituting $k'^*_A = ap\pi/\delta$ into θ^*, we have

$$\theta^* = \delta\pi s(\pi - k'_A) - \delta^2 s(\pi - k'_A)^2 = s\pi^2\left[(\delta - ap) - (\delta - ap)^2\right] = p(1-p)s\pi^2$$

and thus $\Pi^*_A = \theta^* k'^*_A = \frac{a}{\delta}p^2(1-p)s\pi^3$.

If $p > \hat{p}(a)$, substituting $k'^*_A = k'^+_A$ into θ^*, we have

$$\theta^* = s\pi^2\left[\left(\delta - \frac{-(1-2\delta)+\sqrt{1-\delta+\delta^2}}{3}\right) - \left(\delta - \frac{-(1-2\delta)+\sqrt{1-\delta+\delta^2}}{3}\right)^2\right] = s\pi^2\left[\frac{1+2\delta-2\delta^2+(2\delta-1)\sqrt{1-\delta+\delta^2}}{9}\right]$$

and thus

$$\Pi^*_A = \theta^* k'^*_A = s\pi^2\left[\frac{1+2\delta-2\delta^2+(2\delta-1)\sqrt{1-\delta+\delta^2}}{9}\right]\frac{(2\delta-1)+\sqrt{1-\delta+\delta^2}}{3\delta}\pi$$

which can be simplified to the value in the proposition.

Proof of Corollary 2

Proof. From the proof of Proposition 2, if $p > \hat{p}(a)$, the constraint in Inequality (18) does not bind, and therefore $p_2^* > 1$; otherwise, the constraint binds and $p_2^* = 1$.

Proof of Proposition 3

Proof. We define $\Delta \equiv \Pi_B^* - \Pi_A^*$. When $p < \hat{p}(a)$ (in which $\hat{p}(a)$ is as defined in Proposition 2),

$$\Delta = \tfrac{4p^2 s\pi^3}{27} - \tfrac{(1-p)a}{\delta} p^2 s\pi^3 = p^2 s\pi^3\left(\tfrac{4}{27} - \tfrac{(1-p)a}{\delta}\right) = \tfrac{p^2 s\pi^3}{27\delta}\left[p(31a-4) - (27a-4)\right] \tag{47}$$

For any $a \in [0, 4/27]$, $\Delta > 0$ because when $a < 4/31$, $4 - 27a > 4 - 31a$, and when $a > 4/31$ in that range, $31a - 4 > 0$ and $27a - 4 < 0$. When $a \in [4/27, 1]$, $\Delta > 0$ defines a curve, $p(a) = (27a - 4)/(31a - 4)$, on the (a, p) space, which intersects with $\hat{p}(a)$ at $a^* = 8/23$. When $a < a^*$, we can verify $(27a - 4)/(31a - 4) < \hat{p}(a)$ and, therefore, if $p > (27a - 4)/(31a - 4)$, $\Delta > 0$; otherwise, $\Delta < 0$. When $a > a^*$, $(27a - 4)/(31a - 4) > \hat{p}(a)$ and therefore within $p < \hat{p}(a)$, $\Delta < 0$.

When $p > \hat{p}(a)$,

$$\Delta = \frac{s\pi^3}{27\delta}\left[4p^2\delta - \left(-2 + 3\delta + 3\delta^2 - 2\delta^3 + 2\left(1 - \delta + \delta^2\right)^{\frac{3}{2}}\right)\right]$$

Notice that $\Delta = 0$ defines a curve $p^*(a)$ on the (a, p) space, which intersects with $\hat{p}(a)$ at $a^* = 8/23$. When $a < a^*$, we can verify $p^*(a) < \hat{p}(a)$, and, therefore, within $p > \hat{p}(a)$, $\Delta > 0$. When $a > a^*$, $p^*(a) < \hat{p}(a)$, and, therefore, if $p > \hat{p}(a)$, $\Delta > 0$; otherwise, $\Delta < 0$.

To summarize, for $a \in [0, 4/27]$, we have $\Delta > 0$. For $a \in [4/27, 8/23]$, if and only if $p > (27a - 4)/(31a - 4)$, $\Delta > 0$. For $a \in [8/23, 1]$, if and only if $p > p^*(a)$, $\Delta > 0$. Then $\bar{p}(a)$ in the proposition follows.

Proof of Proposition 4

Proof. (a) The payoffs of these sellers under the advertising model, by Equation (25), are positive because the marginal seller with k_A^* derives zero payoff, and these sellers have lower costs than that marginal seller. These sellers do not participate under the brokerage model and derive zero payoff. Therefore, they are all better off under the advertising model.

(b) For sellers in $[k_A'^*, k_B^*]$, when $p > \hat{p}(a)$ (in which $\hat{p}(a)$ is defined as in Proposition 2), sellers are better off under the advertising model if

$$\frac{2 - \delta + \sqrt{1 - \delta + \delta^2}}{3}\pi s(1-a)p(\pi - k) > \tfrac{2}{3} p^2 s\pi\left(\tfrac{2}{3}\pi - k\right) \tag{48}$$

where the term in the left-hand side is the sellers' payoffs under the advertising model by substituting m_A^* from Equation (22) into Equation (25), and the term in the right-hand side is the sellers' payoff under the brokerage model from Equation (24). We can verify that Inequality (48) is true for all $p > \hat{p}(a)$ and that these sellers are better off under the advertising model.

Similarly, when $p < \hat{p}(a)$, sellers are better off under the advertising model if

$$ps\pi(1-a)p(\pi - k) > \frac{2}{3} p^2 s\pi\left(\frac{2}{3}\pi - k\right)$$

The condition can be simplified to $k(a - 1/3) > (a - 5/9)\pi$. For $a \in [0, 1/3]$, the condition is satisfied because $k \leq 2\pi/3 = k_B^*$. For $a \in [1/3, 5/9]$, the condition is satisfied because the right-hand side is non-positive. When $a > 5/9$, the inequality condition reduces to

$$k > (9a - 5)\pi/(9a - 3) \tag{49}$$

We next examine the condition for $(9a-5)\pi/(9a-3) > k'^*_A = ap\pi/\delta$. Substituting δ in, by simple algebra, the condition is equivalent to $p < (9a-5)/(11a-5)$. Therefore, if $p > (9a-5)/(11a-5)$, $(9a-5)\pi/(9a-3) < k'^*_A$, and all $k \in \left[k'^*_A, k'^*_B\right]$ satisfy Equation (49) and sellers are better off; otherwise, sellers with $k > (9a-5)\pi/(9a-3)$ are better off under the advertising model and other sellers are worse off.

For sellers with costs in $[0, k'^*_A]$, when $p > \hat{p}(a)$, sellers are better off under the advertising model if

$$\frac{2-\delta+\sqrt{1-\delta+\delta^2}}{3}\pi s(\pi-k)-s\pi^2\left[\frac{1+2\delta-2\delta^2+(2\delta-1)\sqrt{1-\delta+\delta^2}}{9}\right]>\frac{2}{3}p^2 s\pi\left(\frac{2}{3}\pi-k\right) \tag{50}$$

where the term in the left-hand side is sellers' payoffs under the advertising model by substituting m^*_A from Equation (22) and θ^* from Equation (19) into Equation (26), and the term in the right-hand side is sellers' payoff under the brokerage model from Equation (24). We can verify that Inequality (50) is true for all $p > \hat{p}(a)$ and these sellers are better off under the advertising model.

Similarly, when $p < \hat{p}(a)$, sellers are better off under the advertising model if

$$ps\pi(\pi-k)-p(1-p)s\pi^2>\frac{2}{3}p^2 s\pi\left(\frac{2}{3}\pi-k\right)$$

By simple algebra, the condition can be reduced to $\left(1-\frac{2}{3}p\right)k<\frac{5}{9}p\pi$. Therefore, any $k<\frac{5p\pi}{3(3-2p)}$ satisfies the condition. We next check the condition for $\frac{5p\pi}{3(3-2p)}<k'^*_A=\frac{ap\pi}{\delta}$. Substituting δ in, by simple algebra, the condition is equivalent to $p < (9a-5)/(11a-5)$. Therefore, if $p > (9a-5)/(11a-5)$, $\frac{5p\pi}{3(3-2p)}>k'^*_A$ and all $k \in [0, k'^*_A]$ satisfy $k<\frac{5p\pi}{3(3-2p)}$ and sellers are better off; otherwise, sellers with $k<\frac{5p\pi}{3(3-2p)}$ are better off under the advertising model and other sellers are worse off.

All together, we can summarize the results using function $\tilde{p}(a)$ as specified in the proposition.

Proof of Proposition 5

Proof. When $p < \hat{p}(a)$, the result follows because $p\pi s-c>\frac{2}{3}p\pi s-c$ by Equations (28) and (29). When $p > \hat{p}(a)$, the result can be established if

$$\frac{2-\delta+\sqrt{1-\delta+\delta^2}}{3}\pi s-c>\frac{2}{3}p\pi s-c$$

or, equivalently, if $2-\delta+\sqrt{1-\delta+\delta^2}>2p$. Furthermore, the condition is equivalent to $1-\delta+\delta^2 > (2p-2+\delta)^2$, which reduces to $3+a-4ap > 0$ by substituting in δ. Because a and p are in [0,1], the inequality $3+a-4ap > 0$ is true.

Proof of Proposition 6

Proof. When $p \le \hat{p}(a)$, the average matching probability under the advertising model is the same as under the brokerage model; that is, $[(n^*_A - n'^*_A)p_1 + n'^*_A] = pn^*_A$ (which can also be seen from Equation (29)). Notice that $m^*_B < m^*_A$ and $n^*_B < n^*_A$. We check the social welfare under the brokerage model generated by the low-cost buyers with a mass m^*_B (among m^*_A) and the low-cost sellers with a mass n^*_B (among n^*_A). These segments of buyers and sellers under the advertising model generate the same total value $m^*_B n^*_B p(s+\pi)$ as under the brokerage model. The total fixed cost on the seller side is lower than the fixed cost under the brokerage model because, among m^*_B, the lower-cost sellers participate in the advertising and sell their products more often. The total opportunity costs on the buyer side are the same under the two models. Therefore, these segments of buyers and sellers under the advertising model generate more social welfare than that under the brokerage model. Other participating buyers and sellers generate additional social welfare because their decisions to participate means that the expected benefits are greater than their costs. When $p > \hat{p}(a)$, we can also verify that the advertising model generates more social welfare than the brokerage model.