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IS VOLUNTARY PROFILING WELFARE ENHANCING?

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Appendix A

Detailed Mathematical Derivation and Proofs for the Theorems

The Conditions for Assumption 2

The mathematical conditions for Assumption 2 under each of the high-privacy-cost and low-privacy-cost beliefs are

(a) For the high-privacy-cost belief,

$$Max\left\{\frac{\sqrt{X^2+4\lambda\left[v_L-[1-\alpha]c\right]\left[v_H-\lambda v_L-[1-\alpha-\lambda]c\right]}-X}{2\lambda\left[v_L-[1-\alpha]c\right]},\frac{v_H-[2+\lambda]v_L+[1-\alpha+\lambda]c}{\lambda\left[v_L-[1-\alpha]c\right]}\right\}<\beta\leq 1$$

(b) For the low-privacy-cost belief,

$$Max\left\{\frac{\left[v_H-[1-\alpha]c\right]\left[Y+\lambda^2 r+\sqrt{\left[2Y+\lambda^2 r\right]\lambda^2 r}\right]}{[1-\lambda]\left[2v_H-v_L-[1-\alpha]c\right]^2},1-\frac{2\left[v_L-[1-\alpha]c\right]\lambda^2 r}{\left[v_H-[2-\lambda]v_L+[1-\lambda][1-\alpha]c\right]^2}\right\}<\beta\leq 1$$

where $X = 2v_H - [1+3\lambda]v_L - [1-[1-\lambda]\alpha - 3\lambda]c$ and $Y = [1-\lambda][2v_H - v_L - [1-\alpha]c$. We assume in our analysis that the lower bound for β is an interior value between 0 and 1.

The Derivation of Optimal Solutions for Problems in Stages 1 and 2

Stage 2: The Price for a Participating Consumer'

C:\in\The seller maximizes profit from a participating consumer in the high-privacy-cost belief given by

$$\max_{p_1^\nu} \pi_1^\nu(p_1^\nu) = \begin{cases} p_1^\nu[1-\lambda]\left[\beta + \dfrac{[1-\beta][\bar{v}_{NS} - p_1^\nu - [1-\alpha]c]}{\bar{v}_{NS} - v_L}\right] & \text{if } p_1^\nu \le \hat{v} - [1-\alpha]c \\ \dfrac{p_1^\nu[1-\lambda][1-\beta][\bar{v}_{NS} - p_1^\nu - [1-\alpha]c]}{\bar{v}_{NS} - v_L} & \text{if } p_1^\nu > \hat{v} - [1-\alpha]c \end{cases}$$

It yields the following first order condition

$$\frac{\partial \pi_1^\nu(p_1^\nu)}{\partial p_1^\nu} = \begin{cases} [1-\lambda]\left[\beta + \dfrac{[1-\beta][\bar{v}_{NS} - 2p_1^\nu - [1-\alpha]c]}{\bar{v}_{NS} - v_L}\right] = 0 & \text{for } p_1^\nu \le \hat{v} - [1-\alpha]c \\ \dfrac{[1-\lambda][1-\beta][\bar{v}_{NS} - 2p_1^\nu - [1-\alpha]c]}{\bar{v}_{NS} - v_L} = 0 & \text{for } p_1^\nu > \hat{v} - [1-\alpha]c \end{cases}$$

If the following conditions hold: $\beta - \frac{[1-\beta][\bar{v}_{NS} - [1-\alpha]c]}{\bar{v}_{NS} - v_L} > 0$ and $\bar{v}_{NS} - 2v_L + [1-\alpha]c < 0$, then we have

$$p_1^{\nu^*} = \hat{v} - [1-\alpha]c$$

Under Assumption 2, substituting $\bar{v}_{NS}$ given in (6), we verify that the above conditions are satisfied.

The seller maximizes profit from a participating consumer in the low-privacy-cost belief given by

If $\hat{v} \in [v_L, \bar{v}_S]$,

$$\pi_1^\nu(p_1^\nu) = \begin{cases} p_1^\nu\left[\beta + \dfrac{[1-\beta]\left[\bar{v}_S' - \bar{v}_S + [1-\lambda][\bar{v}_S - p_1^\nu - [1-\alpha]c + \bar{v}_{NS} - \bar{v}_S']\right]}{\lambda[\bar{v}_S' - \bar{v}_S] + [1-\lambda][\bar{v}_{NS} - v_L]}\right] \\ \qquad \text{if } p_1^\nu \le \hat{v} - [1-\alpha]c, \\ \dfrac{p_1^\nu[1-\beta]\left[\bar{v}_S' - \bar{v}_S + [1-\lambda][\bar{v}_S - p_1^\nu - [1-\alpha]c + \bar{v}_{NS} - \bar{v}_S']\right]}{\lambda[\bar{v}_S' - \bar{v}_S] + [1-\lambda][\bar{v}_{NS} - v_L]} \\ \qquad \text{if } \hat{v} - [1-\alpha]c < p_1^\nu \le \bar{v}_S - [1-\alpha]c \\ \dfrac{p_1^\nu[1-\beta]\left[\bar{v}_S' - p_1^\nu - [1-\alpha]c + [1-\lambda][\bar{v}_{NS} - \bar{v}_S']\right]}{\lambda[\bar{v}_S' - \bar{v}_S] + [1-\lambda][\bar{v}_{NS} - v_L]} \\ \qquad \text{if } \bar{v}_S - [1-\alpha]c < p_1^\nu \le \bar{v}_S' - [1-\alpha]c \\ \dfrac{p_1^\nu[1-\beta][1-\lambda][\bar{v}_{NS} - p_1^\nu - [1-\alpha]c]}{\lambda[\bar{v}_S' - \bar{v}_S] + [1-\lambda][\bar{v}_{NS} - v_L]} \quad \text{if } \bar{v}_S' - [1-\alpha]c < p_1^\nu \le \bar{v}_{NS} - [1-\alpha]c \end{cases}$$

if $\hat{v} \in (\bar{v}_S, \bar{v}_S']$,

$$\pi_1^\nu(p_1^\nu) = \begin{cases} p_1^\nu\left[\beta + \dfrac{[1-\beta]\left[\bar{v}_S' - \bar{v}_S + [1-\lambda][\bar{v}_S - p_1^\nu - [1-\alpha]c + \bar{v}_{NS} - \bar{v}_S']\right]}{\lambda[\bar{v}_S' - \bar{v}_S] + [1-\lambda][\bar{v}_{NS} - v_L]}\right] \\ \qquad \text{if } p_1^\nu \le \bar{v}_S - [1-\alpha]c \\ p_1^\nu\left[\beta + \dfrac{[1-\beta]\left[\bar{v}_S' - p_1^\nu - [1-\alpha]c + [1-\lambda][\bar{v}_{NS} - \bar{v}_S']\right]}{\lambda[\bar{v}_S' - \bar{v}_S] + [1-\lambda][\bar{v}_{NS} - v_L]}\right] \\ \qquad \text{if } \bar{v}_S - [1-\alpha]c < p_1^\nu \le \hat{v} - [1-\alpha]c \\ \dfrac{p_1^\nu[1-\beta]\left[\bar{v}_S' - p_1^\nu - [1-\alpha]c + [1-\lambda][\bar{v}_{NS} - \bar{v}_S']\right]}{\lambda[\bar{v}_S' - \bar{v}_S] + [1-\lambda][\bar{v}_{NS} - v_L]} \\ \qquad \text{if } \hat{v} - [1-\alpha]c < p_1^\nu \le \bar{v}_S' - [1-\alpha]c \\ \dfrac{p_1^\nu[1-\beta][1-\lambda][\bar{v}_{NS} - p_1^\nu - [1-\alpha]c]}{\lambda[\bar{v}_S' - \bar{v}_S] + [1-\lambda][\bar{v}_{NS} - v_L]} \quad \text{if } \bar{v}_S' - [1-\alpha]c < p_1^\nu \le \bar{v}_{NS} - [1-\alpha]c \end{cases}$$

and if $\hat{v} \in (\bar{v}_S', \bar{v}_{NS}]$,

$$\pi_1^{\gamma}(p_1^{\gamma}) = \begin{cases} p_1^{\gamma}\left[\beta + \dfrac{[1-\beta]\left[\bar{v}_S' - \bar{v}_S + [1-\lambda][\bar{v}_S - p_1^{\gamma} - [1-\alpha]c + \bar{v}_{NS} - \bar{v}_S']\right]}{\lambda[\bar{v}_S' - \bar{v}_S] + [1-\lambda][\bar{v}_{NS} - v_L]}\right] & \text{if } p_1^{\gamma} \le \bar{v}_S - [1-\alpha]c \\ p_1^{\gamma}\left[\beta + \dfrac{[1-\beta]\left[\bar{v}_S' - p_1^{\gamma} - [1-\alpha]c + [1-\lambda][\bar{v}_{NS} - \bar{v}_S']\right]}{\lambda[\bar{v}_S' - \bar{v}_S] + [1-\lambda][\bar{v}_{NS} - v_L]}\right] & \text{if } \bar{v}_S - [1-\alpha]c < p_1^{\gamma} \le \bar{v}_S' - [1-\alpha]c \\ p_1^{\gamma}\left[\beta + \dfrac{[1-\beta][1-\lambda][\bar{v}_{NS} - p_1^{\gamma} - [1-\alpha]c]}{\lambda[\bar{v}_S' - \bar{v}_S] + [1-\lambda][\bar{v}_{NS} - v_L]}\right] & \text{if } \bar{v}_S' - [1-\alpha]c < p_1^{\gamma} \le \hat{v} - [1-\alpha]c \\ \dfrac{p_1^{\gamma}[1-\beta][1-\lambda][\bar{v}_{NS} - p_1^{\gamma} - [1-\alpha]c]}{\lambda[\bar{v}_S' - \bar{v}_S] + [1-\lambda][\bar{v}_{NS} - v_L]} & \text{if } \hat{v} - [1-\alpha]c < p_1^{\gamma} \le \bar{v}_{NS} - [1-\alpha]c \end{cases}$$

It yields the following first order condition:

If $\hat{v} \in [v_L, \bar{v}_S]$,

$$\frac{\partial \pi_1^{\gamma}(p_1^{\gamma})}{\partial p_1^{\gamma}} = \begin{cases} \left[\beta + \dfrac{[1-\beta]\left[\bar{v}_S' - \bar{v}_S + [1-\lambda][\bar{v}_S - 2p_1^{\gamma} - [1-\alpha]c + \bar{v}_{NS} - \bar{v}_S']\right]}{\lambda[\bar{v}_S' - \bar{v}_S] + [1-\lambda][\bar{v}_{NS} - v_L]}\right] = 0 & \text{for } p_1^{\gamma} \le \hat{v} - [1-\alpha]c \\ \dfrac{[1-\beta]\left[\bar{v}_S' - \bar{v}_S + [1-\lambda][\bar{v}_S - 2p_1^{\gamma} - [1-\alpha]c + \bar{v}_{NS} - \bar{v}_S']\right]}{\lambda[\bar{v}_S' - \bar{v}_S] + [1-\lambda][\bar{v}_{NS} - v_L]} = 0 & \text{for } \hat{v} - [1-\alpha]c < p_1^{\gamma} \le \bar{v}_S - [1-\alpha]c \\ \dfrac{[1-\beta]\left[\bar{v}_S' - 2p_1^{\gamma} - [1-\alpha]c + [1-\lambda][\bar{v}_{NS} - \bar{v}_S']\right]}{\lambda[\bar{v}_S' - \bar{v}_S] + [1-\lambda][\bar{v}_{NS} - v_L]} = 0 & \text{for } \bar{v}_S - [1-\alpha]c < p_1^{\gamma} \le \bar{v}_S' - [1-\alpha]c \\ \dfrac{[1-\beta][1-\lambda][\bar{v}_{NS} - 2p_1^{\gamma} - [1-\alpha]c]}{\lambda[\bar{v}_S' - \bar{v}_S] + [1-\lambda][\bar{v}_{NS} - v_L]} = 0 & \text{for } \bar{v}_S' - [1-\alpha]c < p_1^{\gamma} \le \bar{v}_{NS} - [1-\alpha]c \end{cases}$$

if $\hat{v} \in (\bar{v}_S, \bar{v}_S']$,

$$\frac{\partial \pi_1^{\gamma}(p_1^{\gamma})}{\partial p_1^{\gamma}} = \begin{cases} \left[\beta + \dfrac{[1-\beta]\left[\bar{v}_S' - \bar{v}_S + [1-\lambda][\bar{v}_S - 2p_1^{\gamma} - [1-\alpha]c + \bar{v}_{NS} - \bar{v}_S']\right]}{\lambda[\bar{v}_S' - \bar{v}_S] + [1-\lambda][\bar{v}_{NS} - v_L]}\right] = 0 & \text{for } p_1^{\gamma} \le \bar{v}_S - [1-\alpha]c \\ \left[\beta + \dfrac{[1-\beta]\left[\bar{v}_S' - 2p_1^{\gamma} - [1-\alpha]c + [1-\lambda][\bar{v}_{NS} - \bar{v}_S']\right]}{\lambda[\bar{v}_S' - \bar{v}_S] + [1-\lambda][\bar{v}_{NS} - v_L]}\right] = 0 & \text{for } \bar{v}_S - [1-\alpha]c < p_1^{\gamma} \le \hat{v} - [1-\alpha]c \\ \dfrac{[1-\beta]\left[\bar{v}_S' - 2p_1^{\gamma} - [1-\alpha]c + [1-\lambda][\bar{v}_{NS} - \bar{v}_S']\right]}{\lambda[\bar{v}_S' - \bar{v}_S] + [1-\lambda][\bar{v}_{NS} - v_L]} = 0 & \text{for } \hat{v} - [1-\alpha]c < p_1^{\gamma} \le \bar{v}_S' - [1-\alpha]c \\ \dfrac{[1-\beta][1-\lambda][\bar{v}_{NS} - 2p_1^{\gamma} - [1-\alpha]c]}{\lambda[\bar{v}_S' - \bar{v}_S] + [1-\lambda][\bar{v}_{NS} - v_L]} = 0 & \text{for } \bar{v}_S' - [1-\alpha]c < p_1^{\gamma} \le \bar{v}_{NS} - [1-\alpha]c \end{cases}$$

and if $\hat{v} \in (\bar{v}_S', \bar{v}_{NS}]$,

$$\frac{\partial \pi_1^{\gamma}(p_1^{\gamma})}{\partial p_1^{\gamma}} = \begin{cases} \left[\beta + \dfrac{[1-\beta]\left[\bar{v}_S' - \bar{v}_S + [1-\lambda][\bar{v}_S - 2p_1^{\gamma} - [1-\alpha]c + \bar{v}_{NS} - \bar{v}_S']\right]}{\lambda[\bar{v}_S' - \bar{v}_S] + [1-\lambda][\bar{v}_{NS} - v_L]}\right] = 0 & \text{for } p_1^{\gamma} \le \bar{v}_S - [1-\alpha]c \\ \left[\beta + \dfrac{[1-\beta]\left[\bar{v}_S' - 2p_1^{\gamma} - [1-\alpha]c + [1-\lambda][\bar{v}_{NS} - \bar{v}_S']\right]}{\lambda[\bar{v}_S' - \bar{v}_S] + [1-\lambda][\bar{v}_{NS} - v_L]}\right] = 0 & \text{for } \bar{v}_S - [1-\alpha]c < p_1^{\gamma} \le \bar{v}_S' - [1-\alpha]c \\ \left[\beta + \dfrac{[1-\beta][1-\lambda][\bar{v}_{NS} - 2p_1^{\gamma} - [1-\alpha]c]}{\lambda[\bar{v}_S' - \bar{v}_S] + [1-\lambda][\bar{v}_{NS} - v_L]}\right] = 0 & \text{for } \bar{v}_S' - [1-\alpha]c < p_1^{\gamma} \le \hat{v} - [1-\alpha]c \\ \dfrac{[1-\beta][1-\lambda][\bar{v}_{NS} - 2p_1^{\gamma} - [1-\alpha]c]}{\lambda[\bar{v}_S' - \bar{v}_S] + [1-\lambda][\bar{v}_{NS} - v_L]} = 0 & \text{for } \hat{v} - [1-\alpha]c < p_1^{\gamma} \le \bar{v}_{NS} - [1-\alpha]c \end{cases}$$

If the following conditions hold: $\beta - \frac{[1-\beta][v_H-[1-\alpha]c]}{[v_H-v_L]-\lambda[\bar{v}_S-v_L]} > 0$ and $\lambda[v_H - \bar{v}_S] + [1-\lambda][v_H - 2v_L + [1-\alpha]c] < 0$, then we have

$$p_1^{\nu^*} = \hat{v} - [1-\alpha]c$$

Under Assumption 2, substituting $\bar{v}_S$, $\bar{v}_S'$, and $\bar{v}_{NS}$ given in (8) and (9), we verify that the above conditions are satisfied.

Stage 1: The Price for a Nonparticipating Consumer and the Valuation of Consumer Indifferent Between Participating and Not

Under the high-privacy-cost belief the seller maximizes profit from a nonparticipating consumer given by

$$\max_{p_0^\nu} \pi_0^\nu(p_0^\nu) = p_0^\nu \left[\lambda \int_{p_0^\nu+c}^{v_H} f(v)\, dv + [1-\lambda] \int_{\bar{v}_{NS}}^{v_H} f(v)\, dv\right]$$

The first order condition is

$$\frac{\partial \pi_0^\nu(p_0^\nu)}{\partial p_0^\nu} = \frac{v_H - [1-\lambda]\bar{v}_{NS} - 2\lambda p_0^\nu - \lambda c}{v_H - v_L} = 0$$

Further, using (3) and (4), from $U_0^\nu(v = \bar{v}_{NS}, NS) = U_1^\nu(v = \bar{v}_{NS}, NS)$, we have

$$\bar{v}_{NS} - p_0^\nu - c = \int_{v_L}^{\bar{v}_{NS}} q(\hat{v} \neq \bar{v}_{NS} | \bar{v}_{NS})[\bar{v}_{NS} - \hat{v}]\, d\hat{v}$$

Hence, we obtain

$$p_0^{\nu^*} = \frac{[1+\beta]v_H + [1-\lambda][1-\beta]v_L - [2-[1-\beta]\lambda]c}{2[1+\lambda\beta]}$$

and

$$\bar{v}_{NS} = \frac{v_H - \lambda[1-\beta]v_L + \lambda c}{1+\lambda\beta}$$

Under the low-privacy-cost belief the seller maximizes profit from a nonparticipating consumer given by

$$\max_{p_0^\nu} \pi_0^\nu(p_0^\nu) = p_0^\nu \left[\lambda \int_{\bar{v}_S'}^{v_H} f(v)\, dv + [1-\lambda] \int_{\bar{v}_{NS}}^{v_H} f(v)\, dv\right]$$

The first order condition is

$$\frac{\partial \pi_0^\nu(p_0^\nu)}{\partial p_0^\nu} = \frac{v_H - \lambda\bar{v}_S' - [1-\lambda]\bar{v}_{NS}}{v_H - v_L} = 0$$

Further, using (3) and (4), from $U_0^\nu(v = \bar{v}_{NS}, NS) \leq U_1^\nu(v = \bar{v}_{NS}, NS)$, we have

$$\bar{v}_{NS} - p_0^\nu - c \leq \int_{v_L}^{\bar{v}_{NS}} q(\hat{v} \neq \bar{v}_{NS} | \bar{v}_{NS})[\bar{v}_{NS} - \hat{v}]\, d\hat{v}$$

from $U_0^\nu(v = \bar{v}_S', S) \leq U_1^\nu(v = \bar{v}_S', S)$, we have

$$\bar{v}_S' - p_0^\nu - c \leq \int_{v_L}^{\bar{v}_S'} q(\hat{v} \neq \bar{v}_S' | \bar{v}_S')[\bar{v}_S' - \hat{v}]\, d\hat{v} - r$$

and from $U_1^\nu(v = \bar{v}_S, S) = U_0^\nu(v = \bar{v}_S, S) = 0$, we have

$$\int_{v_L}^{\bar{v}_S} q(\hat{v} \neq \bar{v}_S | \bar{v}_S)[\bar{v}_S - \hat{v}]\, d\hat{v} - r = 0$$

Hence, we obtain

$$p_0^{v^*} > v_H - c$$
$$\bar{v}_S = \frac{[1-\lambda][1-\beta]v_L - \lambda r + \sqrt{2r[1-\lambda][1-\beta][v_H - v_L] + \lambda^2 r^2}}{[1-\lambda][1-\beta]}$$

and

$$\bar{v}'_S = \bar{v}_{NS} = v_H$$

Proof of Theorem 1

Using $\bar{v}_{NS}$ given in (6) and $\bar{v}_S$ given in (8), we show

$$\frac{\partial \bar{v}_{NS}}{\partial \beta} = -\frac{\lambda\big[[v_H - v_L] - \lambda[v_L - c]\big]}{[1+\lambda\beta]^2} \leq 0, \qquad \frac{\partial \bar{v}_{NS}}{\partial \lambda} = -\frac{\beta[v_H - v_L] + [v_L - c]}{[1+\lambda\beta]^2} \leq 0,$$
$$\frac{\partial \bar{v}_S}{\partial r} = \frac{[1-\lambda][1-\beta][v_H - v_L] + \lambda^2 r^2 - \lambda\sqrt{2r[1-\lambda][1-\beta][v_H - v_L] + \lambda^2 r^2}}{[1-\lambda][1-\beta]\sqrt{2r[1-\lambda][1-\beta][v_H - v_L] + \lambda^2 r^2}} \geq 0,$$
$$\frac{\partial \bar{v}_S}{\partial \beta} = \frac{\Big[[1-\lambda][1-\beta][v_H - v_L] + \lambda^2 r - \lambda\sqrt{2r[1-\lambda][1-\beta][v_H - v_L] + \lambda^2 r^2}\Big] r}{[1-\lambda][1-\beta]^2\sqrt{2r[1-\lambda][1-\beta][v_H - v_L] + \lambda^2 r^2}} \geq 0, \text{ and}$$
$$\frac{\partial \bar{v}_S}{\partial \lambda} = \frac{\Big[[1-\lambda][1-\beta][v_H - v_L] + \lambda r - \lambda\sqrt{2r[1-\lambda][1-\beta][v_H - v_L] + \lambda^2 r^2}\Big] r}{[1-\lambda]^2[1-\beta]\sqrt{2r[1-\lambda][1-\beta][v_H - v_L] + \lambda^2 r^2}} \geq 0. \ \blacksquare$$

Proof of Theorem 2

(i) In the high-privacy-cost equilibrium, comparing the price under no profiling (given in (1)) and the price for a nonparticipating consumer under voluntary profiling (given in (5)), we show

$$p_0^{v^*} - p^{b^*} = \frac{[1-\lambda][\beta v_H + [1-\beta]v_L - c]}{2[1+\lambda\beta]} \geq 0$$

In the low-privacy-cost equilibrium, there is no price at which a nonparticipating consumer purchases (i.e., $p_0^{v^*} > v_H - c$).

(ii) Comparing the price under no profiling (given in (1)) and the expected price paid by a participating consumer under voluntary profiling (given in (11)), we show

$$E_v\Big(E_{\hat{v}}\big(I(\hat{v}, v)\, p_1^v(\hat{v}|v)\big)\Big) - p^{b^*} \begin{cases} > 0 & \text{if } c > c^* \\ \leq 0 & \text{otherwise} \end{cases}$$

where, $c^* = \frac{[2[1-\beta]+3\lambda\beta]v_H - [2+\beta[1+\lambda] - [1-3\beta^2]\lambda]v_L}{[1+2\beta]\lambda - 3\beta[1+\lambda\beta] + 3\alpha[1+\beta][1+\lambda\beta]}$ for the high-privacy-cost equilibrium, and $\frac{3v_H - [2+\beta]v_L}{3[\alpha[1+\beta]-\beta]} - \frac{[1+2\beta][v_H - \lambda\bar{v}_S]}{3[1-\lambda][\alpha[1+\beta]-\beta]} + \frac{\lambda[v_H - \bar{v}_S]^2\big[[1+2\beta][v_H - \lambda\bar{v}_S] - [1-\lambda]\big[[1-\beta]\bar{v}_S + 3\beta v_L\big]\big]}{3[1-\lambda][\alpha[1+\beta]-\beta]\big[[v_H - v_L] - \lambda[\bar{v}_S - v_L]\big]^2}$ for the low-privacy-cost equilibrium. $\blacksquare$

Proof of Theorem 3

(i) From $p_0^{v^*} \geq p^{b^*}$ (Theorem 2(i)), we show $U_0^v(v) \leq U^b(v)$ for all v.

(ii) Using $U_1^v(\bar{v}_{NS}, NS) = U_0^v(\bar{v}_{NS}, NS) \leq U^b(\bar{v}_{NS}, NS)$ and $U_1^v(v, NS) > U_0^v(v, NS)$ for $v \leq \bar{v}_{NS}$, we show: there is a $\tilde{v}_{NS} \leq \bar{v}_{NS}$ such that $U_1^v(\tilde{v}_{NS}, NS) = U^b(\tilde{v}_{NS}, NS)$. Further, from $U_1^v(v, NS) > U_1^v(v, S)$, we show there is a $\tilde{v}_S \leq \tilde{v}_{NS}$ such that $U_1^v(\tilde{v}_S, S) = U^b(\tilde{v}_S, S)$. $\blacksquare$

Illustration of Aggregate Consumer Surplus under Voluntary Profiling Versus No Profiling

In the high-privacy-cost equilibrium (i.e., $r > 2$), for $v_H = 100$, $v_L = 70$, $c = 87$, $\lambda = 0.6$, and $\alpha = 0.9$, we have

$$CS^v = 0.79 > CS^b = 0.70 \text{ when } \beta = 0.80$$

whereas

$$CS^v = 0.63 < CS^b = 0.70 \text{ when } \beta = 0.88$$

Similarly, in the low-privacy-cost equilibrium (i.e., $r = 0.1$), for $v_H = 100$, $v_L = 70$, $c = 93$, $\lambda = 0.6$, and $\alpha = 0.9$, we have

$$CS^v = 0.31 > CS^b = 0.20 \text{ when } \beta = 0.90$$

whereas

$$CS^v = 0.16 < CS^b = 0.20 \text{ when } \beta = 0.94. \ \blacksquare$$

Illustration of Social Welfare under Voluntary Profiling Versus No Profiling

In the high-privacy-cost equilibrium (i.e., $r > 2$), for $v_H = 100$, $v_L = 70$, $c = 44$, $\lambda = 0.1$, and $\alpha = 0.1$, we have

$$SW^v = 38.54 < SW^b = 39.20 \text{ when } \beta = 0.80$$

whereas

$$SW^v = 40.47 > SW^b = 39.20 \text{ when } \beta = 0.95$$

Similarly, in the low-privacy-cost equilibrium (i.e., $r = 0.4$), for $v_H = 100$, $v_L = 70$, $c = 41.2$, $\lambda = 0.1$, and $\alpha = 0.1$, we have

$$SW^v = 43.07 < SW^b = 43.22 \text{ when } \beta = 0.87$$

whereas

$$SW^v = 43.81 > SW^b = 43.22 \text{ when } \beta = 0.92. \ \blacksquare$$

Proof of Theorem 4

If $\beta = 1$, no privacy-sensitive consumer participates in profiling and privacy-nonsensitive consumers whose valuations are not greater than $p_0^{v^*} + c$ participate in profiling. Hence, from $p_0^{v^*} + c = \bar{v}_{NS}$, we have: $p_0^{v^*} = \frac{v_H - c}{1+\lambda}$, and from: $\pi^v = [1-\lambda]\int_{v \le p_0^v + c} f(v)[v - [1-\alpha]c]\, dv + \int_{v > p_0^v + c} f(v) p_0^v \, dv$, and $CS^v = \int_{v > p_0^v + c} f(v)[v - p_0^v - c]\ dv$, we show

$$\frac{\partial \pi^v}{\partial \lambda} = -\frac{\left[p_0^{v^*} - v_L + c\right]\left[p_0^{v^*} + v_L - [1-2\alpha]c\right] - 2\left[v_H - [1+\lambda]p_0^{v^*} - [1-[1-\lambda]\alpha]c\right]\frac{\partial p_0^{v^*}}{\partial \lambda}}{2[v_H - v_L]} \le 0$$

$$\frac{\partial CS^v}{\partial \lambda} = -\frac{\partial p_0^{v^*}}{\partial \lambda} \cdot \frac{v_H - p_0^{v^*} - c}{v_H - v_L} \ge 0$$

and

$$SW^v(\lambda = 0) - SW^v(\lambda = 1) = \int_{v \le \frac{v_H + c}{2}} f(v)[v - c]\, dv + \int_{v > \frac{v_H + c}{2}} f(v)\alpha c \, dv \ge 0. \ \blacksquare$$

Proof of Theorem 5

(i) The seller charges a uniform price p^{v^*} for all consumers under voluntary profiling. Hence, from

$$U_1^v - U_0^v = \alpha c \geq 0$$

all privacy-nonsensitive consumers participate in profiling. Further, from

$$U_1^v - U_0^v = \alpha c - r \begin{cases} \geq 0 & \text{if } r \leq \alpha c \\ < 0 & \text{if } r > \alpha c \end{cases}$$

privacy-sensitive consumers participate in profiling if $r \leq \alpha c$.

The seller's profit is given by

$$\pi^v = \begin{cases} \dfrac{p^v[v_H - p^v - [1 - [1-\lambda]\alpha]c]}{v_H - v_L} & \text{if } r > \alpha c \\ \dfrac{p^v[v_H - p^v - [1-\alpha]c - \lambda r]}{v_H - v_L} & \text{if } r \leq \alpha c \end{cases}$$

and maximizing it yields

$$p^{v*} = \begin{cases} \dfrac{1}{2}[v_H - [1 - [1-\lambda]\alpha]c] & \text{if } r > \alpha c \\ \dfrac{1}{2}[v_H - [1-\alpha]c - \lambda r] & \text{if } r \leq \alpha c \end{cases}$$

Using P^{v*} obtained above, we show

$$U^v(v, NS) - U^b(v, NS) = \begin{cases} \dfrac{1}{2}[1+\lambda]\alpha c \geq 0 & \text{if } r > \alpha c \\ \dfrac{1}{2}[\alpha c + \lambda r] \geq 0 & \text{if } r \leq \alpha c \end{cases}$$

$$U^v(v, S) - U^b(v, S) = \begin{cases} -\dfrac{1}{2}[1-\lambda]\alpha c \leq 0 & \text{if } r > \alpha c \\ \dfrac{1}{2}[\alpha c - [2-\lambda]r] \begin{cases} \leq 0 & \text{if } \dfrac{\alpha c}{2-\lambda} \leq r \leq \alpha c \\ > 0 & \text{if } r < \dfrac{\alpha c}{2-\lambda} \end{cases} \end{cases}$$

(ii) Aggregate consumer surplus is given by

$$CS^v = \begin{cases} \dfrac{\lambda[v_H - p^v - c]^2 - [1-\lambda][v_H - p^v - [1-\alpha]c]^2}{2[v_H - v_L]} & \text{if } r > \alpha c \\ \dfrac{[v_H - p^v - [1-\alpha]c]^2 - \lambda r^2}{2[v_H - v_L]} & \text{if } r \leq \alpha c \end{cases}$$

Hence, we show

$$CS^v - CS^b = \begin{cases} \dfrac{[1-\lambda][2[v_H - c] + [3\lambda + c]\alpha]\alpha c}{8[v_H - v_L]} \geq 0 & \text{if } r > \alpha c \\ \dfrac{[\alpha c + \lambda r]^2 + 2[v_H - c][\alpha c + \lambda r] - 4\lambda r^2}{8[v_H - v_L]} \geq 0 & \text{if } r \leq \alpha c \end{cases}$$

(iii) As the seller can always choose to ignore the profile information, we have $\pi^v \geq \pi^b$. Hence, together with (ii) we have $SW^v \geq SW^b$. ■

Proof of Corollary 1

As is shown in Proof of Theorem 5(i), $U^v(v, NS) \geq U^b(v, NS)$ for all v, and $U^v(v, S) > U^b(v, S)$ for all v if $r < \frac{\alpha c}{2-\lambda}$. ■

Proof of Theorem 6

From $U_1^v(v,NS) = v - p_1^v - [1-\alpha]c \geq U_0^v(v,NS) = v - p_0^v - c$, privacy-nonsensitive consumers participate in profiling if $\alpha c - [p_1^v - p_0^v] \geq 0$. Similarly, privacy-sensitive consumers participate in profiling if $\alpha c - r - [p_1^v - p_0^v] \geq 0$.

(a) If $r \leq \alpha c - [p_1^v - p_0^v]$, then both privacy-sensitive and privacy-nonsensitive consumers (whose surplus are positive) participate in profiling. Hence, there is only one price (for participating consumers) and results are identical to those for price discrimination-free voluntary profiling.

(b) If $r > \alpha c - [p_1^v - p_0^v]$, then privacy-sensitive consumers do not participate in profiling. Suppose privacy-nonsensitive consumers participate in profiling. Then, maximizing the seller's profit from participating consumers given by

$$\max_{p_1^v} \pi_1^v(p_1^v) = p_1^v[1-\lambda]\int_{p_1^v+[1-\alpha]c}^{v_H} f(v)\,dv$$

we have

$$p_1^{v^*} = \frac{1}{2}[v_H - [1-\alpha]c]$$

and maximizing the seller's profit from nonparticipating consumers given by

$$\max_{p_0^v} \pi_0^v(p_0^v) = p_0^v\lambda\int_{p_0^v+c}^{v_H} f(v)\,dv$$

we have

$$p_0^{v^*} = \frac{1}{2}[v_H - c]$$

Using $p_1^{v^*}$ and $p_0^{v^*}$, from $\alpha c - \left[p_1^{v^*} - p_0^{v^*}\right] = \frac{\alpha c}{2} \geq 0$, we confirm that privacy-nonsensitive consumers participate. Further, we have (i) $U_1^v - U^b = \frac{\alpha c}{2} \geq 0$, and from: $p_0^{v^*} = p^{b^*}$, we have: (ii) $U_0^v = U^b$. Therefore, we have: (iii) $CS^v \geq CS^b$ and (iv) $SW^v \geq SW^b$. ■

Proof of Theorem 7

Given that a consumer can purchase a product elsewhere at p^o, the seller's price under no profiling (given as (1) in the monopoly model) is now

$$p^b = \min\left\{\frac{v_H - c}{2}, p^o\right\}$$

and the seller's price for a participating and a nonparticipating consumer (given as (2) and (5) in the monopoly model) under voluntary profiling is

$$p_1^v = \begin{cases} \hat{v} - [1-\alpha]c & \text{if } v_L \leq \hat{v} \leq p^o + [1-\alpha]c \\ p^o & \text{if } p^o + [1-\alpha]c < \hat{v} \leq \bar{v}_{NS} \end{cases}$$

$$p_0^v = \min\left\{\frac{[1+\beta]v_H + [1-\lambda][1-\beta]v_L - [2-[1-\beta]\lambda]c}{2[1+\lambda\beta]}, p^o\right\}$$

We identify three cases depending on p^o as follows:

Case 1: $p^o \geq \bar{v}_{NS} - [1-\alpha]c$

The seller's and consumers' decisions are not affected by p^o. Hence, all results are identical to those for a monopoly seller.

Case 2: $\frac{[1+\beta]v_H + [1-\lambda][1-\beta]v_L - [2-[1-\beta]\lambda]c}{2[1+\lambda\beta]} \leq p^o < \bar{v}_{NS} - [1-\alpha]c$

Consider a privacy nonsensitive consumer with valuation v. If this consumer participates in profiling, their surplus is

$$U_1^v(v,NS) = \begin{cases} q(\hat{v}=v|v)\cdot 0 + \int_{v_L}^{v} q(\hat{v}\neq v|v)[v-\hat{v}]\,d\hat{v} & \text{if } v_L \le v \le p^o + [1-\alpha]c \\ q(\hat{v}=v|v)[v-p^o-[1-\alpha]c] + \int_{v_L}^{p^o+[1-\alpha]c} q(\hat{v}\neq v|v)[v-\hat{v}]\,d\hat{v} & \\ \quad + \int_{p^o+[1-\alpha]c}^{\bar{v}_{NS}} q(\hat{v}\neq v|v)[v-p^o-[1-\alpha]c]\,d\hat{v} & \text{if } p^o+[1-\alpha]c < v \le \bar{v}_{NS} \end{cases}$$

and if this consumer does not participate in profiling, their surplus is

$$U_0^v(v,NS) = \begin{cases} 0 & \text{if } v < p_0^v + c \\ v - p_0^v - c & \text{if } v \ge p_0^v + c \end{cases}$$

Hence, solving $\frac{\partial \pi_0^v(p_0^v)}{\partial p_0^v} = \frac{v_H - [1-\lambda]\bar{v}_{NS} - 2\lambda p_0^v - \lambda c}{v_H - v_L} = 0$ and $U_1^v(v=\bar{v}_{NS},NS) = U_0^v(v=\bar{v}_{NS},NS)$ together we have

$$p_0^{v*} = \frac{1}{2}\left[\frac{v_H - [1-\lambda]\bar{v}_{NS}}{\lambda} - c\right]$$

and

$$\bar{v}_{NS} = \frac{\psi - \sqrt{\psi^2 + 4[1-\lambda]\big[[1-\beta]\lambda[p^o+[1-\alpha]c-v_L]^2 - [v_H - [2p^o+[1-2\alpha]c]\lambda]v_L\big]}}{2[1-\lambda]}$$

where $\psi = v_H + [1-\lambda]v_L - [2p^o + [1-2\alpha]c]\lambda$. Therefore, we show

$$U_1^v(v,NS) - U_0^v(v,NS) \begin{cases} \ge 0 & \text{if } v_L \le v \le \bar{v}_{NS} \\ < 0 & \text{if } \bar{v}_{NS} < v \le v_H \end{cases}$$

We have $p_0^{v*} \ge p^{b*}$. Further, given that there is a $\bar{v}_{NS} \le v_H$, all nonparticipating consumers as well as some participating consumers are worse off, and aggregate consumer surplus and social welfare under voluntary profiling can be higher or lower compared to no profiling. Proofs are analogous to those for a monopoly seller.

Case 3: $p^o < \frac{[1+\beta]v_H + [1-\lambda][1-\beta]v_L - [2-[1-\beta]\lambda]c}{2[1+\lambda\beta]}$

As $p_1^v \le p_0^v$ regardless of the signal, we have $U_1^v(v,NS) \ge U_0^v(v,NS)$ for all v. Thus, all privacy-nonsensitive consumers participate (i.e., $\bar{v}_{NS} = v_H$) and we have $p_0^{v*} = p^{b*}$. Therefore, no nonparticipating consumer is worse off and all participating consumers are weakly better off under voluntary profiling compared to no profiling. ■

Appendix B

Mathematical Derivation of Extensions

Network Effects in the Profiler

We restrict our analysis to the high-privacy-cost equilibrium and assume Assumption 2 continues to hold. Suppose α increases in proportion to the number of participating consumers (i.e., $\alpha = a\bar{v}_{NS}$). Then the seller maximizes profit from a participating consumer given by

$$\pi_1^v(p_1^v) = \begin{cases} p_1^v[1-\lambda]\left[\beta + \frac{[1-\beta][\bar{v}_{NS} - p_1^v - [1-a\bar{v}_{NS}]c]}{\bar{v}_{NS} - v_L}\right] & \text{if } p_1^v \le \hat{v} - [1-a\bar{v}_{NS}]c \\ \frac{p_1^v[1-\lambda][1-\beta][\bar{v}_{NS} - p_1^v - [1-a\bar{v}_{NS}]c]}{\bar{v}_{NS} - v_L} & \text{if } p_1^v > \hat{v} - [1-a\bar{v}_{NS}]c \end{cases}$$

It yields the following first order condition

$$\frac{\partial\pi_1^v(p_1^v)}{\partial p_1^v}=\begin{cases}[1-\lambda]\left[\beta+\dfrac{[1-\beta][\bar{v}_{NS}-2p_1^v-[1-a\bar{v}_{NS}]c]}{\bar{v}_{NS}-v_L}\right]=0 & \text{for } p_1^v\le\hat{v}-[1-a\bar{v}_{NS}]c\\ \dfrac{[1-\lambda][1-\beta][\bar{v}_{NS}-2p_1^v-[1-a\bar{v}_{NS}]c]}{\bar{v}_{NS}-v_L}=0 & \text{for } p_1^v>\hat{v}-[1-a\bar{v}_{NS}]c\end{cases}$$

If (i) $\beta-\frac{[1-\beta][\bar{v}_{NS}-[1-a\bar{v}_{NS}]c]}{\bar{v}_{NS}-v_L}>0$ and (ii) $\bar{v}_{NS}-2v_L+[1-a\bar{v}_{NS}]c<0$, then

$$p_1^{v*}=\hat{v}-[1-a\bar{v}_{NS}]c$$

Under Assumption 2, substituting $\bar{v}_{NS}$, we verify that the above conditions are satisfied. Further, solving $\frac{\partial\pi_0^v(p_0^v)}{\partial p_0^v}=\frac{v_H-[1-\lambda]\bar{v}_{NS}-2\lambda p_0^v-\lambda c}{v_H-v_L}=0$ and $U_0^v(v=\bar{v}_{NS},NS)=U_1^v(v=\bar{v}_{NS},NS)$ together we have

$$p_0^{v*}=\frac{[1+\beta]v_H+[1-\lambda][1-\beta]v_L-[2-[1-\beta]\lambda]c}{2[1+\lambda\beta]}$$

and

$$\bar{v}_{NS}=\frac{v_H-\lambda[1-\beta]v_L+\lambda c}{1+\lambda\beta}$$

We show: $p_0^{v*}-p^{b*}=\frac{[1-\lambda][\beta v_H+[1-\beta]v_L-c]}{2[1+\lambda\beta]}\ge 0$. Hence, compared to no profiling, under voluntary profiling, no nonparticipating consumers are better off. Also, given that there is a $\bar{v}_{NS}$ such that $U_1^v(v,NS)\ge U_0^v(v,NS)$ if $v\le\bar{v}_{NS}$ and $U_1^v(v,NS)<U_0^v(v,NS)$ otherwise, some participating consumers are worse off. Therefore, aggregate consumer surplus and social welfare under voluntary profiling can be higher or lower. Proofs are analogous to those for the base model.

Generic Search Support for a Nonparticipating Consumer

We restrict our analysis to the high-privacy-cost equilibrium. Suppose the search cost of nonparticipating consumer is $[1-\alpha_0]c$, where $\alpha_0=\alpha-\epsilon$. Under no profiling the seller maximizes profit given by

$$\max_{p^b}\pi^b(p^b)=p^b\int_{p^b+[1-\alpha_0]c}^{v_H}f(v)\,dv$$

It yields

$$p^{b*}=\frac{1}{2}[v_H-[1-\alpha_0]c]$$

Under voluntary profiling, p_1^{v*} is given in (2). Further, solving: $\frac{\partial\pi_0^v(p_0^v)}{\partial p_0^v}=\frac{v_H-[1-\lambda]\bar{v}_{NS}-2\lambda p_0^v-\lambda[1-\alpha_0]c}{v_H-vL}=0$ and $U_1^v(v=\bar{v}_{NS},NS)=U_0^v(v=\bar{v}_{NS},NS)$ together, we have

$$p_0^v=\frac{[1+\beta]v_H+[1-\beta][1-\lambda]v_L-[2-[1-\beta]\lambda][1-\alpha_0]c}{2[1+\lambda\beta]}$$

and

$$\bar{v}_{NS}=\frac{v_H-\lambda[1-\beta]v_L+\lambda[1-\alpha_0]c}{1+\lambda\beta}$$

We show: $p_0^{v*}-p^{b*}=\frac{[1-\lambda][\beta v_H+[1-\beta]v_L-[1-\alpha_0]c]}{2[1+\lambda\beta]}\ge 0$. Hence, compared to no profiling, under voluntary profiling, no nonparticipating consumers are better off. Also, given that there is a $\bar{v}_{NS}$ such that $U_1^v(v,NS)\ge U_0^v(v,NS)$ if $v\le\bar{v}_{NS}$ and $U_1^v(v,NS)<U_0^v(v,NS)$ otherwise, some participating consumers are worse off. Therefore, aggregate consumer surplus and social welfare under voluntary profiling can be higher or lower. Proofs are analogous to those for the base model.

Appendix C

Analysis of the Alterative Model

Heterogeneity in Search Cost

A consumer derives a fixed value $v \epsilon R^+$ from consuming her ideal product, and without providing profile information to the seller, the consumer incurs search cost $c \epsilon [c_L, c_H]$ where c follows a uniform probability density function $f(c)$. Hence, the *net value* a consumer obtains when purchasing her ideal product $w = v - c$ follows a uniform probability density function $f(w)$ with support $[w_L = v - c_H, w_H = v - c_L]$. The seller receives a signal $\hat{w}$ about the net value of a participating consumer. Other aspects of the model remain the same as those for our original model.

Benchmark: No Profiling

Assumption 1: $w_H > 2w_L$.

The seller maximizes the expected profit given by

$$\max_{p^b} \pi^b(p^b) = p^b \int_{p^b}^{w_H} f(w)\, dw$$

and it yields

$$p^{b^*} = \frac{w_H}{2} = \frac{v - c_L}{2}$$

Voluntary Profiling

The Belief about Consumers' Participation Structure

Definition 1:[1] The belief about consumers' participation structure consists of either (a) high-privacy-cost belief or (b) low-privacy-cost belief. The belief can be formally defined using the probability distribution function of net value of a participating consumer $g(w)$:
(a) In a high-privacy-cost belief,

$$g(w) = \frac{1}{\bar{w}_{NS} - w_L} \text{ for } w_L \le w \le \bar{w}_{NS}$$

(b) In a low-privacy-cost belief,

$$g(w) = \begin{cases} \dfrac{1 - \lambda}{[w_H - w_L] - \lambda[\bar{w}_S - w_L]} & \text{for } w_L \le w \le \bar{w}_S \\ \dfrac{1}{[w_H - w_L] - \lambda[\bar{w}_S - w_L]} & \text{for } \bar{w}_S < w \le w_H \end{cases}$$

and the posterior distribution of net value of a participating consumer given the signal $\hat{w}$ is computed as

$$s(w = y|\hat{w}) = \begin{cases} \beta & \text{if } y = \hat{w} \\ [1 - \beta]g(w) & \text{if } y \ne \hat{w} \end{cases}$$

Assumption 2:[2]

(a) For the high-privacy-cost belief,

$$\text{(i) } \beta - \frac{[1-\beta]\left[\bar{w}_{NS} + \frac{\alpha v}{1-\alpha}\right]}{\bar{w}_{NS} - w_L} > 0 \text{ and (ii) } \bar{w}_{NS} - 2w_L - \frac{\alpha v}{1-\alpha} < 0$$

(b) For the low-privacy-cost belief,

[1] For brevity, we impose $\bar{w}_S' = \bar{w}_{NS} = w_H$ in a low privacy-cost belief. We later show that the consumers' participation strategies do not deviate from this.
[2] As those for the main model, all conditions in Assumption 2 can be written in terms of β after we substitute $\bar{w}_{NS}$ and $\bar{w}_S$.

$$\text{(i) } \beta - \frac{[1-\beta]\left[w_H+\frac{\alpha v}{1-\alpha}\right]}{[w_H-w_L]-\lambda[\overline{w}_S-w_L]} > 0 \text{ and (ii) } \lambda[w_H-\overline{w}_S]+[1-\lambda]\left[w_H-2w_L-\frac{\alpha v}{1-\alpha}\right]<0$$

Stage 3: Consumers' Purchase Decisions

A participating consumer with net value w purchases their ideal product if: $[1-\alpha]w+\alpha v-p_1^v(\widehat{w})\geq 0$, and a nonparticipating consumer with net value w purchases their ideal product if: $w-p_0^v\geq 0$.

Stage 2: The Optimal Price for a Participating Consumer

Given Assumption 2, we have

$$p_1^{v*}=[1-\alpha]\widehat{w}+\alpha v$$

Proof:
The seller maximizes the expected profit from a participating consumer in a high-privacy-cost belief given by

$$\pi_1^v(p_1^v)=\begin{cases}[1-\lambda]p_1^v\left[\beta+[1-\beta]g(w)\left[\overline{w}_{NS}-\dfrac{p_1^v-\alpha v}{1-\alpha}\right]\right] & \text{if } p_1^v\leq[1-\alpha]\widehat{w}+\alpha v\\ [1-\lambda]p_1^v[1-\beta]g(w)\left[\overline{w}_{NS}-\dfrac{p_1^v-\alpha v}{1-\alpha}\right] & \text{otherwise}\end{cases}$$

It yields the following first order conditions

$$\frac{\partial\pi_1^v(p_1^v)}{\partial p_1^v}=\begin{cases}[1-\lambda]\left[\beta+[1-\beta]g(w)\left[\overline{w}_{NS}-\dfrac{2p_1^v-\alpha v}{1-\alpha}\right]\right]=0 & \text{for } p_1^v\leq[1-\alpha]\widehat{w}+\alpha v\\ [1-\lambda][1-\beta]g(w)\left[\overline{w}_{NS}-\dfrac{2p_1^v-\alpha v}{1-\alpha}\right]=0 & \text{for } p_1^v>[1-\alpha]\widehat{w}+\alpha v\end{cases}$$

Given Assumption 2, we have

$$p_1^{v*}=[1-\alpha]\widehat{w}+\alpha v$$

The seller maximizes the expected profit from a participating consumer in a low-privacy-cost belief given by

If $\widehat{w}\in[w_L,\overline{w}_S]$,

$$\pi_1^v(p_1^v)=\begin{cases}p_1^v\left[\beta+\dfrac{[1-\beta]\left[\lambda[w_H-\overline{w}_S]+[1-\lambda]\left[w_H-\frac{p_1^v-\alpha v}{1-\alpha}\right]\right]}{[w_H-w_L]-\lambda[\overline{w}_S-w_L]}\right] & \text{if } p_1^v\leq[1-\alpha]\widehat{w}+\alpha v\\ p_1^v\cdot\dfrac{[1-\beta]\left[\lambda[w_H-\overline{w}_S]+[1-\lambda]\left[w_H-\frac{p_1^v-\alpha v}{1-\alpha}\right]\right]}{[w_H-w_L]-\lambda[\overline{w}_S-w_L]} & \text{if } [1-\alpha]\widehat{w}+\alpha v<p_1^v\leq[1-\alpha]\overline{w}_S+\alpha v\\ p_1^v\cdot\dfrac{[1-\beta]\left[w_H-\frac{p_1^v-\alpha v}{1-\alpha}\right]}{[w_H-w_L]-\lambda[\overline{w}_S-w_L]} & \text{otherwise}\end{cases}$$

and if $\widehat{w}\in(\overline{w}_S,w_H]$,

$$\pi_1^\nu(p_1^\nu) = \begin{cases} p_1^\nu\left[\beta + \dfrac{[1-\beta]\left[\lambda[w_H - \overline{w}_S] + [1-\lambda]\left[w_H - \dfrac{p_1^\nu - \alpha v}{1-\alpha}\right]\right]}{[w_H - w_L] - \lambda[\overline{w}_S - w_L]}\right] & \text{if } p_1^\nu \le [1-\alpha]\overline{w}_S + \alpha v \\ p_1^\nu\left[\beta + \dfrac{[1-\beta]\left[w_H - \dfrac{p_1^\nu - \alpha v}{1-\alpha}\right]}{[w_H - w_L] - \lambda[\overline{w}_S - w_L]}\right] & \text{if } [1-\alpha]\overline{w}_S + \alpha v < p_1^\nu \le [1-\alpha]\widehat{w} + \alpha v \\ p_1^\nu \cdot \dfrac{[1-\beta]\left[w_H - \dfrac{p_1^\nu - \alpha v}{1-\alpha}\right]}{[w_H - w_L] - \lambda[\overline{w}_S - w_L]} & \text{otherwise} \end{cases}$$

It yields the following first order conditions:

If $\widehat{w} \in [w_L, \overline{w}_S]$,

$$\frac{\partial \pi_1^\nu(p_1^\nu)}{\partial p_1^\nu} = \begin{cases} \beta + \dfrac{[1-\beta]\left[\lambda[w_H - \overline{w}_S] + [1-\lambda]\left[w_H - \dfrac{2p_1^\nu - \alpha v}{1-\alpha}\right]\right]}{[w_H - w_L] - \lambda[\overline{w}_S - w_L]} = 0 \ \text{ for } p_1^\nu \le [1-\alpha]\widehat{w} + \alpha v \\ \dfrac{[1-\beta]\left[\lambda[w_H - \overline{w}_S] + [1-\lambda]\left[w_H - \dfrac{2p_1^\nu - \alpha v}{1-\alpha}\right]\right]}{[w_H - w_L] - \lambda[\overline{w}_S - w_L]} = 0 \\ \qquad \text{for } [1-\alpha]\widehat{w} + \alpha v < p_1^\nu \le [1-\alpha]\overline{w}_S + \alpha v \\ \dfrac{[1-\beta]\left[w_H - \dfrac{2p_1^\nu - \alpha v}{1-\alpha}\right]}{[w_H - w_L] - \lambda[\overline{w}_S - w_L]} = 0 \ \text{ for } p_1^\nu > [1-\alpha]\overline{w}_S + \alpha v \end{cases}$$

and if $\widehat{w} \in (\overline{w}_S, w_H]$,

$$\frac{\partial \pi_1^\nu(p_1^\nu)}{\partial p_1^\nu} = \begin{cases} \beta + \dfrac{[1-\beta]\left[\lambda[w_H - \overline{w}_S] + [1-\lambda]\left[w_H - \dfrac{2p_1^\nu - \alpha v}{1-\alpha}\right]\right]}{[w_H - w_L] - \lambda[\overline{w}_S - w_L]} = 0 \ \text{ for } p_1^\nu \le [1-\alpha]\overline{w}_S + \alpha v \\ \beta + \dfrac{[1-\beta]\left[w_H - \dfrac{2p_1^\nu - \alpha v}{1-\alpha}\right]}{[w_H - w_L] - \lambda[\overline{w}_S - w_L]} = 0 \ \text{ for } [1-\alpha]\overline{w}_S + \alpha v < p_1^\nu \le [1-\alpha]\widehat{w} + \alpha v \\ \dfrac{[1-\beta]\left[w_H - \dfrac{2p_1^\nu - \alpha v}{1-\alpha}\right]}{[w_H - w_L] - \lambda[\overline{w}_S - w_L]} = 0 \ \text{ for } p_1^\nu > [1-\alpha]\widehat{w} + \alpha v \end{cases}$$

Given Assumption 2, we have

$$p_1^{\nu *} = [1-\alpha]\widehat{w} + \alpha v$$

Stage 1: The Optimal Price for a Nonparticipating Consumer and Consumers' Participation Decisions

In a high-privacy-cost equilibrium, the seller maximizes the expected profit given by

$$\max_{p_0^\nu} \pi_0^\nu(p_0^\nu) = p_0^\nu\left[\lambda \int_{p_0^\nu}^{w_H} f(w)\, dw + [1-\lambda]\int_{\overline{w}_{NS}}^{w_H} f(w)\, dw\right]$$

and it yields the following first order condition:

$$\frac{\partial \pi_0^\nu(p_0^\nu)}{\partial p_0^\nu} = \frac{w_H - [1-\lambda]\overline{w}_{NS} - 2\lambda p_0^\nu}{w_H - w_L} = 0$$

Further, from $U_0^\nu(w = \overline{w}_{NS}, NS) = U_1^\nu(w = \overline{w}_{NS}, NS)$, we have

$$\bar{w}_{NS} - \frac{w_H - [1-\lambda]\bar{w}_{NS}}{2\lambda} = \int_{w_L}^{\bar{w}_{NS}} q(\hat{w} \neq \bar{w}_{NS}|\bar{w}_{NS})\,[1-\alpha][\bar{w}_{NS} - \hat{w}]\,d\hat{w}$$

Solving simultaneously, we have

$$p_0^{v*} = \frac{[1+\alpha-\alpha\beta+\beta]w_H + [1-\lambda][1-\alpha][1-\beta]w_L}{2\big[1+\lambda[\alpha-\alpha\beta+\beta]\big]}$$

and

$$\bar{w}_{NS} = \frac{w_H - \lambda[1-\beta][1-\alpha]w_L}{1+\lambda[\alpha-\alpha\beta+\beta]}$$

In a low-privacy-cost equilibrium, solving $U_0^v(w = \bar{w}_S, S) = 0$, we have

$$\bar{w}_S = \frac{[1-\lambda][1-\beta][1-\alpha]w_L - \lambda r + \sqrt{2r[1-\lambda][1-\beta][1-\alpha][w_H - w_L] + \lambda^2 r^2}}{[1-\lambda][1-\beta][1-\alpha]}$$

The Perfect Bayesian Equilibrium

The Condition

If $U_1^v(w,S) < 0$ for $w \le p_0^v$ then we have: $U_1^v(w,S) < U_0^v(w,S)$ for all w. Hence, using p_0^{v*} above, we obtain

$$\bar{r} = \frac{[1-\lambda][1-\beta][1-\alpha]\big[w_H - \bar{w}_{NS} + \lambda[\bar{w}_{NS} - 2w_L]\big]^2}{4\lambda^2\big[w_H + [1-\lambda][\bar{w}_{NS} - 2w_L]\big]}$$

If $r > \bar{r}$ then we have the high-privacy-cost equilibrium; otherwise, we have the low-privacy-cost equilibrium.

The Equilibrium

If $r > \bar{r}$ (i.e., a high-privacy-cost equilibrium), then we verify that

$$U_1^v(w,S) < U_0^v(w,S) \;\text{ for all } w$$

and

$$\begin{cases} U_1^v(w,NS) \ge U_0^v(w,NS) & \text{for } w_L \le w \le \bar{w}_{NS} \\ U_1^v(w,NS) < U_0^v(w,NS) & \text{for } \bar{w}_{NS} < w \le w_H \end{cases}$$

If $r \le \bar{r}$ (i.e., a low-privacy-cost equilibrium), then we verify that

$$\begin{cases} U_1^v(w,S) < U_0^v(w,S) & \text{for } w_L \le w < \bar{w}_S \\ U_1^v(w,S) \ge U_0^v(w,S) & \text{for } \bar{w}_S \le w \le w_H \end{cases}$$

and

$$U_1^v(w,NS) \ge U_0^v(w,NS) \;\text{ for all } w$$

Theorem 1. In each equilibrium, the proportion of consumers that choose to participate is decreasing in the (i) privacy cost (r), (ii) valuation accuracy (β), and (iii) fraction of privacy-sensitive consumers (λ).

Proof:

$$\frac{\partial \bar{w}_{NS}}{\partial \beta} = -\frac{\lambda[1-\alpha][w_H - [1+\lambda]w_L]}{\big[1+\lambda[\alpha-\alpha\beta+\beta]\big]^2} \le 0, \qquad \frac{\partial \bar{w}_{NS}}{\partial \lambda} = -\frac{[\alpha-\alpha\beta+\beta][w_H - w_L] + w_L}{\big[1+\lambda[\alpha-\alpha\beta+\beta]\big]^2} \le 0,$$

$$\frac{\partial \bar{w}_S}{\partial r} = \frac{[1-\lambda][1-\beta][1-\alpha][w_H - w_L] + \lambda^2 r - \lambda\sqrt{2r[1-\lambda][1-\beta][1-\alpha][w_H - w_L] + \lambda^2 r^2}}{[1-\lambda][1-\beta][1-\alpha]\sqrt{2r[1-\lambda][1-\beta][1-\alpha][w_H - w_L] + \lambda^2 r^2}} \ge 0,$$

$$\frac{\partial \bar{w}_S}{\partial \beta} = \frac{\Big[[1-\lambda][1-\beta][1-\alpha][w_H - w_L] + \lambda^2 r - \lambda\sqrt{2r[1-\lambda][1-\beta][1-\alpha][w_H - w_L] + \lambda^2 r^2}\Big]\,r}{[1-\lambda][1-\beta]^2[1-\alpha]\sqrt{2r[1-\lambda][1-\beta][1-\alpha][w_H - w_L] + \lambda^2 r^2}} \ge 0, \text{ and}$$

$$\frac{\partial \overline{w}_S}{\partial \lambda} = \frac{\left[[1-\lambda][1-\beta][1-\alpha][w_H - w_L] + \lambda r - \sqrt{2r[1-\lambda][1-\beta][1-\alpha][w_H - w_L] + \lambda^2 r^2}\right] r}{[1-\lambda]^2[1-\beta][1-\alpha]\sqrt{2r[1-\lambda][1-\beta][1-\alpha][w_H - w_L] + \lambda^2 r^2}} \geq 0.$$

Voluntary Profiling Versus No Profiling

Price Paid by Consumers

Theorem 2. Compared to no profiling, under voluntary profiling: (i) the expected price paid by a nonparticipating consumer is higher, and (ii) if the valuation is sufficiently high, then the expected price paid by a participating consumer is higher.

Proof:
(i) In a high-privacy-cost equilibrium, we show

$$p_0^{v^*} - p^{b^*} = \frac{[1-\lambda]\big[[\alpha - \alpha\beta + \beta]w_H - [1-\alpha][1-\beta]w_L\big]}{2\big[1 + \lambda[\alpha - \alpha\beta + \beta]\big]} \geq 0$$

In a low-privacy-cost equilibrium, there is no price at which a nonparticipating consumer purchases.

(ii) The expected average price paid by a participating consumer is computed as

$$E_w\left(E_{\widehat{w}}\big(I(\widehat{w}, w) \cdot p_1^v(\widehat{w}|w)\big)\right) = \int_w g(w)\left[q(\widehat{w} = w|w)\big[[1-\alpha]w + \alpha v\big] + \int_{\widehat{w}<w} q(\widehat{w} \neq w|w)\big[[1-\alpha]\widehat{w} + \alpha v\big]\, d\widehat{w}\right] dw$$

Hence, we show

$$E_w\left(E_{\widehat{w}}\big(I(\widehat{w}, w) \cdot p_1^v(\widehat{w}|w)\big)\right) - p^{b^*} \begin{cases} > 0 & \text{if } v > v^* \\ \leq 0 & \text{otherwise} \end{cases}$$

where $v^* = \frac{3w_H - [1-\alpha]\big[[1+2\beta]\overline{w}_{NS} + [2+\beta]w_L\big]}{3\alpha[1+\beta]}$ for a high-privacy-cost equilibrium and
$\frac{w_H - 2\left[\int_w g(w)\left[q(\widehat{w} = w|w)[1-\alpha]w + \int_{\widehat{w}} q(\widehat{w} \neq w|w)[1-\alpha]\widehat{w}\, d\widehat{w}\right]dw\right]}{\alpha[1+\beta]}$ for a low-privacy-cost equilibrium.

Consumer Surplus

Theorem 3. Compared to no profiling, under voluntary profiling (i) the surplus of a nonparticipating consumer is not larger and (ii) the surplus of a participating consumer whose valuation is greater than $\widetilde{w}_i$ is smaller.

Proof:
(i) From $p_0^{v^*} \geq p^{b^*}$ (Theorem 2(i)), we show $U_0^v(w) \leq U^b(w)$ for all w.
(ii) Using $U_1^v(\overline{w}_{NS}, NS) = U_0^v(\overline{w}_{NS}, NS) \leq U^b(\overline{w}_{NS}, NS)$ and $U_1^v(w, NS) > U_0^v(w, NS)$ for $w \leq \overline{w}_{NS}$, we show there is a $\widetilde{w}_{NS} \leq \overline{w}_{NS}$ such that $U_1^v(\widetilde{w}_{NS}, NS) = U^b(\widetilde{w}_{NS}, NS)$. Further, from $U_1^v(w, NS) > U_1^v(w, S)$, we show there is a $\widetilde{w}_S \leq \widetilde{w}_{NS}$ such that $U_1^v(\widetilde{w}_S, S) = U^b(\widetilde{w}_S, S)$.

Social Policy

The Impact of Privacy Sensitivity and Valuation Accuracy on the Seller, Consumers, and Society

Theorem 4. If valuation accuracy is perfect ($\beta = 1$), then aggregate consumer surplus is increasing and the seller's profit is decreasing in the fraction of privacy-sensitive consumers. Social welfare is non-monotonic in the fraction of privacy-sensitive consumers but is higher when there are no privacy sensitive consumers than when all consumers are privacy sensitive.

Proof:
If $\beta = 1$, we have

$$U_1^v(w, NS) = 0 \text{ for all } w, \text{ and } U_1^v(w, S) = -r < 0 \text{ for all } w$$

Hence, we have

$$\overline{w}_{NS} = p_0^v, \text{ and there is no } \overline{w}_S \text{ that satisfies } U_0^v(w = \overline{w}_S, S) = U_1^v(w = \overline{w}_S, S)$$

Therefore, from $p_0^v = \frac{w_H - [1-\lambda]\bar{w}_{NS}}{2\lambda} = \bar{w}_{NS}$, we have

$$p_0^{v^*} = \bar{w}_{NS} = \frac{w_H}{1+\lambda}$$

and we have

$$\pi^v = [1-\lambda]\int_{w_L}^{p_0^v} f(w)\big[[1-\alpha]w + \alpha v\big]\, dw + \int_{p_0^v}^{w_H} f(w) p_0^v\, dw$$

$$CS^v = \int_{p_0^v}^{w_H} f(w)[w - p_0^v]\, dw \text{, and } SW^v = \pi^v + CS^v$$

We show

$$\frac{\partial \pi^v}{\partial \lambda} = -\frac{[p_0^v - w_L]\big[[1-\alpha][p_0^v + w_L] + 2\alpha v\big] - 2[w_H - [1+\alpha+[1-\alpha]\lambda]p_0^v + [1-\lambda]\alpha v]\frac{\partial p_0^v}{\partial \lambda}}{2[w_H - w_L]} \le 0$$

$$\frac{\partial CS^v}{\partial \lambda} = -\frac{[w_H - p_0^v]\frac{\partial p_0^v}{\partial \lambda}}{w_H - w_L} \ge 0$$

and

$$SW^v(\lambda = 0) - SW^v(\lambda = 1) = \frac{[w_H - 2w_L][w_H + 2w_L]}{8[w_H - w_L]} + \alpha\left[v - \frac{w_H + w_L}{2}\right] \ge 0$$

Price Discrimination and Pareto Optimality

Price Discrimination-Free Voluntary Profiling

Theorem 5. (a) All privacy-nonsensitive consumers participate in profiling and privacy-sensitive consumers whose net value is low (or search cost is high) participate in profiling if the privacy cost is sufficiently low. Otherwise, no privacy-sensitive consumer participates in profiling. (b) Compared to no profiling, under voluntary profiling, (i) the surplus of participating consumers whose net value is low (or search cost is high) is higher, (ii) the surplus of nonparticipating consumers is higher if the fraction of privacy-sensitive consumers is sufficiently high and the privacy cost is moderate; otherwise, the surplus of all nonparticipating consumers is lower, (iii) the aggregate consumer surplus and social welfare can be higher or lower.

Proof:
(a) As under no profiling, under voluntary profiling, the seller charges a uniform price p^v for all consumers; hence, we have

$$U_1^v(w, NS) - U_0^v(w, NS) = \alpha[v - w] \ge 0 \text{ for all } w$$

and we have

$$U_1^v(w, S) - U_0^v(w, S) = \alpha[v - w] - r \begin{cases} \ge 0 & \text{if } w \le v - \frac{r}{\alpha} \\ < 0 & \text{otherwise} \end{cases}$$

where from $v - \frac{r}{\alpha} < w_L$, if $r > \alpha[v - w_L]$, then $U_1^v(w, S) < U_0^v(w, S)$ for all w.

(b) The seller's expected profit is given by

$$\pi^v = \begin{cases} p^v\left[\lambda \int_{\frac{p^v - \alpha v + r}{1-\alpha}}^{w_H} f(w)\, dw + [1-\lambda]\int_{\frac{p^v - \alpha v}{1-\alpha}}^{w_H} f(w)\, dw\right] & \text{if } r \le \alpha[v - w_L] \\ p^v\left[\lambda \int_{p^v}^{w_H} f(w)\, dw + [1-\lambda]\int_{\frac{p^v - \alpha v}{1-\alpha}}^{w_H} f(w)\, dw\right] & \text{otherwise} \end{cases}$$

Maximizing it yields,

$$p^{v*} = \begin{cases} \dfrac{[1-\alpha]w_H + \alpha v - \lambda r}{2} & \text{if } r \le \alpha[v - w_L] \\ \dfrac{[1-\alpha]w_H + [1-\lambda]\alpha v}{2[1-\alpha\lambda]} & \text{otherwise} \end{cases}$$

Consider a privacy-nonsensitive consumer whose net value is w. We show

$$U_1^v(w, NS) - U^b(w, NS) \begin{cases} \ge 0 & \text{if } w \le v - \dfrac{p^v - p^b}{\alpha} \\ < 0 & \text{otherwise} \end{cases}$$

where

$$v - \frac{p^v - p^b}{\alpha} = \begin{cases} \dfrac{\alpha[w_H + v] + \lambda r}{2a} & \text{if } r \le \alpha[v - w_L] \\ \dfrac{[1-\lambda]w_H + [1+\lambda-2\alpha\lambda]v}{2[1-\alpha\lambda]} & \text{otherwise} \end{cases}$$

Consider a privacy-sensitive consumer with net value w who participates in profiling. We show

$$U_1^v(w, S) - U^b(w, S) \begin{cases} \ge 0 & \text{if } w \le v - \dfrac{p^v + r - p^b}{\alpha} \\ < 0 & \text{otherwise} \end{cases}$$

where

$$v - \frac{p^v - p^b + r}{\alpha} = \frac{\alpha[v + w_H] - [2-\lambda]r}{2\alpha}$$

Consider a privacy-sensitive consumer with net value w who does not participate in profiling. We have

$$U_0^v(w, S) - U^b(w, S) = \begin{cases} -\dfrac{\alpha[v - w_H] - \lambda r}{2} \begin{cases} \le 0 & \text{if } r \le \dfrac{\alpha[v - w_H]}{\lambda} \\ > 0 & \text{if } \dfrac{\alpha[v - w_H]}{\lambda} < r \le \alpha[v - w_L] \end{cases} \\ -\dfrac{\alpha[1-\lambda][v - w_H]}{2[1-\alpha\lambda]} \le 0 \quad \text{otherwise} \end{cases}$$

where if $\lambda \le \frac{v-w_H}{v-w_L} = \frac{c_L}{c_H}$, then $U_0^v(w, S) \le U^b(w, S)$ for all w.

Group-Pricing Voluntary Profiling

The seller charges a uniform price p_1^v for all participating consumer and a uniform price p_0^v for all nonparticipating consumer. From

$$U_1^v(w, NS) = [1-\alpha]w + \alpha v - p_1^v \ge U_0^v(w, NS) = w - p_0^v$$

a privacy-nonsensitive consumer whose net value is w participates in profiling if and only if

$$w \le v - \frac{p_1^v - p_0^v}{\alpha}$$

and from

$$U_1^v(w, S) = [1-\alpha]w + \alpha v - p_1^v - r \ge U_0^v(w, S) = w - p_0^v$$

a privacy-sensitive consumer whose net value is w participates in profiling if and only if

$$w \le v - \frac{p_1^v + r - p_0^v}{\alpha}$$

where from $v - \frac{p_1^v + r - p_0^v}{\alpha} < w_L$, if $r > \alpha[v - w_L] - [p_1^v - p_0^v]$, then $U_1^v(w,S) < U_0^v(w,S)$ for all w.

(a) A high-privacy-cost equilibrium (i.e., $r > \alpha[v - w_L] - [p_1^v - p_0^v]$)

Maximizing the seller's profit from a participating consumer given by

$$\max_{p_1^v} \pi_1^v(p_1^v) = p_1^v[1-\lambda] \int_{\frac{p_0^v - \alpha v}{1-\alpha}}^{\overline{w}_{NS}} f(w)dw$$

we have

$$p_1^{v^*} = \frac{[1-\alpha]\overline{w}_{NS} + \alpha v}{2}$$

and maximizing the seller's profit from a nonparticipating consumer given by

$$\max_{p_0^v} \pi_0^v(p_0^v) = p_0^v \left[\lambda \int_{p_0^v}^{w_H} f(w)\,dw + [1-\lambda] \int_{\overline{w}_{NS}}^{w_H} f(w)\,dw\right]$$

we have

$$p_0^{v^*} = \frac{w_H - [1-\lambda]\overline{w}_{NS}}{2\lambda}$$

Solving $U_0^v(w = \overline{w}_{NS}, NS) = U_1^v(w = \overline{w}_{NS}, NS)$, we have

$$\overline{w}_{NS} = \frac{w_H + \alpha\lambda v}{1 + \alpha\lambda}$$

Here,

$$\frac{w_H + \alpha\lambda v}{1 + \alpha\lambda} \geq w_H$$

Therefore, we have

$$\overline{w}_{NS} = w_H, p_1^{v^*} = \frac{[1-\alpha]w_H + \alpha v}{2}, \text{ and } p_0^{v^*} = \frac{w_H}{2}$$

and we show

$$p_0^{v^*} = p^{b^*}, \text{ and}$$

$$U_1^v(w,NS) - U^b(w,NS) = \frac{\alpha[v + w_H - 2w]}{2} \geq 0 \text{ for all } w.$$

(b) A low-privacy cost equilibrium (i.e., $r \leq \alpha[v - w_L] - [p_1^v - p_0^v]$)

Maximizing the seller's profit from a participating consumer given by

$$\max_{p_1^v} \pi_1^v(p_1^v) = p_1^v \left[\lambda \int_{\frac{p_1^v - \alpha v + r}{1-\alpha}}^{\overline{w}_S} f(w)\,dw + [1-\lambda] \int_{\frac{p_1^v - \alpha v}{1-\alpha}}^{\overline{w}_{NS}} f(w)\,dw\right]$$

we have

$$p_1^{v^*} = \frac{\lambda[1-\alpha]\overline{w}_S + [1-\lambda][1-\alpha]\overline{w}_{NS} + \alpha v - \lambda r}{2}$$

and maximizing the seller's profit from a nonparticipating consumer given by

$$\max_{p_0^v} \pi_0^v(p_0^v) = p_0^v \left[\lambda \int_{\overline{w}_S}^{w_H} f(w)\, dw + [1-\lambda] \int_{\overline{w}_{NS}}^{w_H} f(w)\, dw \right]$$

we have

$$p_0^{v^*} > [1-\alpha]w_H + \alpha v$$

Therefore, we have

$$U_0^v(w,i) = 0 \ \text{ for all } w, i \in \{S, NS\}, \qquad \overline{w}_{NS} = \overline{w}_S = w_H,$$
$$p_1^{v^*} = \frac{[1-\alpha]w_H + \alpha v - \lambda r}{2}$$

and we show

$$U_1^v(w,NS) - U^b(w,NS) = \begin{cases} \geq 0 \ \text{ if } w \leq \dfrac{\alpha v - [2-\alpha]w_H + \lambda r}{2\alpha} \\ < 0 \ \text{ otherwise} \end{cases}$$

$$U_1^v(w,S) - U^b(w,S) = \begin{cases} \geq 0 \ \text{ if } w \leq \dfrac{\alpha v - [2-\alpha]w_H - [2-\lambda]r}{2\alpha} \\ < 0 \ \text{ otherwise} \end{cases}$$

Extensions

Presence of an Outside Option

Theorem 7. When consumers have an outside option at p^o, compared to no profiling, under voluntary profiling, there is a threshold value for p^o: (i) if p^o is less than the threshold value then no nonparticipating consumer is worse off and all participating consumers are better; hence, aggregate consumer surplus as well as social welfare are higher. Otherwise (ii) if p^o is greater than or equal to the threshold value, then the results are qualitatively similar to those in the "Model Analysis" and "Voluntary Profiling Versus No Profiling" sections of the paper; all nonparticipating consumers as well as some participating consumers are worse off and aggregate consumer surplus and social welfare can be higher or lower.

Proof:
The seller's price under no profiling is

$$p^{b^*} = min\left\{\frac{w_H}{2}, p^o\right\}$$

and the seller's price for a participating consumer and a nonparticipating consumer under voluntary profiling is

$$p_1^{v^*} = \begin{cases} [1-\alpha]\widehat{w} + \alpha v \ \text{ if } \widehat{w} \leq \dfrac{p^o - \alpha v}{1-\alpha} \\ p^o \ \text{ otherwise} \end{cases}$$

$$p_0^{v^*} = min\left\{\frac{[1+\alpha-\alpha\beta+\beta]w_H + [1-\lambda][1-\alpha][1-\beta]w_L}{2\big[1+\lambda[\alpha-\alpha\beta+\beta]\big]}, p^o\right\}$$

We identify three cases depending on p^o as the following:

Case 1: $p^o \geq [1-\alpha]\overline{w}_{NS} + \alpha v$

The prices are not affected by p^o; hence, all results are identical to those without an outside option scenario.

Case 2: $\frac{[1+\alpha-\alpha\beta+\beta]w_H + [1-\lambda][1-\alpha][1-\beta]w_L}{2[1+\lambda[\alpha-\alpha\beta+\beta]]} \leq p^o < [1-\alpha]\overline{w}_{NS} + \alpha v$

Consider a privacy-nonsensitive consumer whose net value is w. If she participates in profiling, her expected surplus is

$$U_1^v(w,NS)=\begin{cases} q(\widehat{w}=w|w)\cdot 0+\int_{w_L}^{w} q(\widehat{w}\neq w|w)[1-\alpha][w-\widehat{w}]d\widehat{w} \ \text{ if } w_L\leq w\leq \dfrac{p^o-\alpha v}{1-\alpha} \\ q(\widehat{w}=w|w)\big[[1-\alpha]w+\alpha v-p^o\big]+\int_{w_L}^{\frac{p^o-\alpha v}{1-\alpha}} q(\widehat{w}\neq w|w)[1-\alpha][w-\widehat{w}]d\widehat{w} \\ \quad +\int_{\frac{p^o-\alpha v}{1-\alpha}}^{\overline{w}_{NS}} q(\widehat{w}\neq w|w)\big[[1-\alpha]w+\alpha v-p^o\big]d\widehat{w} \ \text{ if } \dfrac{p^o-\alpha v}{1-\alpha}<w\leq \overline{w}_{NS} \end{cases}$$

Solving $\frac{\partial \pi_0^v(p_0^v)}{\partial p_0^v}=\frac{w_H-[1-\lambda]\overline{w}_{NS}-2\lambda p_0^v}{w_H-w_L}=0$ and $U_1^v(w=\overline{w}_{NS},NS)=U_0^v(w=\overline{w}_{NS},NS)$ together, we have

$$p_0^{v*}=\frac{w_H-[1-\lambda]\overline{w}_{NS}}{2\lambda}$$

and

$$\overline{w}_{NS}=\frac{[1-\alpha]\left[p_0^{v*}-p^o+\alpha[v+w_L]\right]+\sqrt{\psi}}{2\alpha[1-\alpha]}$$

where $\psi=[1-\alpha]\Big[[1-\alpha]\left[p_0^{v*}-p^o+\alpha[v+w_L]\right]^2+2\alpha\left[[1-\beta][p^o-\alpha v]^2-2[1-\alpha][p_0^{v*}-\beta p^o+\alpha\beta v]w_L+[1-\alpha]^2[1-\beta]{w_L}^2\right]\Big]$.

We have: $p_0^{v*}-p^{b*}=\frac{[1-\lambda][w_H-\overline{w}_{NS}]}{2\lambda}\geq 0$; hence, compared to no profiling, under voluntary profiling, all nonparticipating consumers are not better off. Also, given that there is a $\overline{w}_{NS}\leq w_H$ such that $U_1^v(w=\overline{w}_{NS},NS)=U_0^v(w=\overline{w}_{NS},NS)$, we show: some participating consumers are worse off; therefore, aggregate consumer surplus and social welfare under voluntary profiling can be higher or lower compared to no profiling.

Case 3: $p^o<\frac{[1+\alpha-\alpha\beta+\beta]w_H+[1-\lambda][1-\alpha][1-\beta]w_L}{2[1+\lambda[\alpha-\alpha\beta+\beta]]}$

Since $p_1^v\leq p_0^v$ regardless of the signal, we have $U_1^v(w,NS)\geq U_0^v(w,NS)$ for all w; that is, all privacy-nonsensitive consumers participate in profiling (i.e., $\overline{w}_{NS}=w_H$). Further, when $\overline{w}_{NS}=w_H$, we have $p^b=p_0^v\geq p_1^v(\widehat{w})$ for all w; hence, no consumer (both participating and nonparticipating) is worse off under voluntary profiling compared to no profiling.

Search Support

Network Effects in the Profiler

We analyze the scenario when α is a function of the number of participating consumers. We restrict our analysis to the high privacy-cost equilibrium and assume (i) $\beta-[1-\beta]g(w)\left[\overline{w}_{NS}+\frac{va\overline{w}_{NS}}{1-a\overline{w}_{NS}}\right]>0$ and (ii) $\overline{w}_{NS}-2w_L+\frac{va\overline{w}_{NS}}{1-a\overline{w}_{NS}}<0$. Let $\alpha=a\overline{w}_{NS}$.

The seller's profit from participating consumer is

$$\pi_1^v(p_1^v)=\begin{cases}[1-\lambda]p_1^v\left[\beta+[1-\beta]g(w)\left[\overline{w}_{NS}-\dfrac{p_1^v-va\overline{w}_{NS}}{1-a\overline{w}_{NS}}\right]\right] \ \text{ if } p_1^v\leq[1-a\overline{w}_{NS}]\widehat{w}+va\overline{w}_{NS} \\ [1-\lambda]p_1^v[1-\beta]g(w)\left[\overline{w}_{NS}-\dfrac{p_1^v-va\overline{w}_{NS}}{1-a\overline{w}_{NS}}\right] \ \text{ otherwise}\end{cases}$$

Given the assumption we have

$$\frac{\partial\pi_1^v(p_1^v)}{\partial p_1^v}=\begin{cases}[1-\lambda]\left[\beta+[1-\beta]g(w)\left[\overline{w}_{NS}-\dfrac{2p_1^v-va\overline{w}_{NS}}{1-a\overline{w}_{NS}}\right]\right]>0 \\ [1-\lambda][1-\beta]g(w)\left[\overline{w}_{NS}-\dfrac{2p_1^v-va\overline{w}_{NS}}{1-a\overline{w}_{NS}}\right]<0\end{cases}$$

Therefore,

$$p_1^v = [1 - a\bar{w}_{NS}]\hat{w} + va\bar{w}_{NS}$$

Solving $\frac{\partial \pi_0^v(p_0^v)}{\partial p_0^v} = \frac{w_H - [1-\lambda]\bar{w}_{NS} - 2\lambda p_0^v}{w_H - w_L} = 0$ and $U_0^v(w = \bar{w}_{NS}, NS) = U_1^v(w = \bar{w}_{NS}, NS)$ together, we have

$$p_0^{v^*} = \frac{w_H - [1-\lambda]\bar{w}_{NS}}{2\lambda}$$

and

$$\bar{w}_{NS} = \frac{a[1-\beta]\lambda w_L - [1+\beta\lambda] + \sqrt{4a[1-\beta]\lambda[w_H - [1-\beta]\lambda w_L] + [1+\beta\lambda - a[1-\beta]\lambda w_L]^2}}{2a[1-\beta]\lambda}$$

We show $p_0^{v^*} - p^{b^*} = \frac{[1-\lambda][w_H - \bar{w}_{NS}]}{2\lambda} \geq 0$; hence, compared to no profiling, under voluntary profiling, all nonparticipating consumers are not better off. Also, given that there is a $\bar{w}_{NS} \leq w_H$ such that $U_1^v(w = \bar{w}_{NS}, NS) = U_0^v(w = \bar{w}_{NS}, NS)$, we have: some participating consumers are not better off; therefore, aggregate consumer surplus and social welfare under voluntary profiling can be higher or lower.

Generic Search Support for Nonparticipating Consumer

We analyze the scenario when a participating consumer's search cost is $[1-\alpha]c$ and a nonparticipating consumer's search cost (and search cost under no profiling) is $[1-\alpha_0]c$, where $\alpha_0 = \alpha - \varepsilon$. We restrict our analysis to the high-cost equilibrium.

Under no profiling, the seller maximizes the expected profit given by

$$\max_{p^b} \pi^b(p^b) = p^b \int_{\frac{p^b - \alpha_0 v}{1-\alpha_0}}^{w_H} f(w)\, dw$$

and it yields

$$p^{b^*} = \frac{[1-\alpha_0]w_H + \alpha_0 v}{2}$$

Under voluntary profiling, we have $p_1^{v^*} = [1-\alpha]\hat{w} + \alpha v$. Further, solving $\frac{\partial \pi_0^v(p_0^v)}{\partial p_0^v} = \frac{[1-\alpha_0]w_H - [1-\lambda][1-\alpha_0]\bar{w}_{NS} - 2\lambda p_0^v + \lambda\alpha_0 v}{[1-\alpha_0][w_H - w_L]} = 0$ and $U_0^v(w = \bar{w}_{NS}, NS) = U_1^v(w = \bar{w}_{NS}, NS)$ together, we have

$$p_0^{v^*} = \frac{[1-\alpha_0][w_H - [1-\lambda]\bar{w}_{NS}] + \lambda\alpha_0 v}{2\lambda}$$

and

$$\bar{w}_{NS} = \frac{[1-\alpha_0]w_H - \lambda[1-\beta][1-\alpha]w_L - \lambda\alpha_0 v}{1 - \alpha_0 + \lambda[\alpha - \alpha_0 - \alpha\beta + \beta]}$$

We show $p_0^{v^*} \geq p^{b^*}$; hence, compared to no profiling, under voluntary profiling, all nonparticipating consumers are not better off. Also, given that there is a $\bar{w}_{NS} \leq w_H$ such that $U_1^v(w = \bar{w}_{NS}, NS) = U_0^v(w = \bar{w}_{NS}, NS)$, we have some participating consumers are not better off, and therefore, aggregate consumer surplus and social welfare under voluntary profiling can be higher or lower.