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RESEARCH ARTICLE

PLATFORM OR WHOLESALE? A STRATEGIC TOOL FOR ONLINE RETAILERS TO BENEFIT FROM THIRD-PARTY INFORMATION

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Appendix

Derivation of Conditional Expectation of Misfit

Proof. Cumulative density function of s, conditional on the consumer's true degree of misfit λ being z, can be formulated as $P(s \le y \mid \lambda = z) = (1-\beta)y + \beta H(y-z)$, where $H(\cdot)$ is the Heaviside step function that evaluates to zero if the argument is negative, and to one otherwise. The corresponding probability density function is $P(s = y \mid \lambda = z) = (1-\beta) + \beta\delta(y-z)$, where $\delta(x)$ is the Dirac delta distribution that satisfies

$$\int_{-\infty}^{\infty} \delta(x)dx = 1 \text{ and } \delta(x) = \begin{cases} 0 & \text{for } x \neq 0 \\ \infty & \text{for } x = 0 \end{cases}$$

Using the Bayes' Law,

$$P(\lambda = z|s = y) = \frac{P(s = y|\lambda = z)P(\lambda = z)}{P(s = y)} = (1-\beta) + \beta\delta(y-z) \tag{44}$$

and the conditional expectation is

$$\mathrm{E}(\lambda|s = y) = \int_0^1 \lambda\left[(1-\beta) + \beta\delta(y-\lambda)\right]d\lambda = \int_0^1 (1-\beta)\lambda d\lambda + \beta y = \frac{1-\beta}{2} + \beta y$$

Proof of Lemma 1

Proof. We denote $a_A \equiv 1/2 + (1-\gamma)x_R/(2\tau)$, $a_B \equiv 1/2 - (1-\gamma)x_R/(2\tau)$, $b \equiv 1/(2\tau) + \alpha$, and $c \equiv 1/(2\tau)$. The demand functions in Equation (8) then can be rewritten as:

$$\begin{aligned} D_A &= a_A - bp_A + cp_B \\ D_B &= a_B - bp_B + cp_A \end{aligned} \tag{45}$$

The retailer's optimization problem in stage 1 is characterized by the first-order conditions as follows:

$$\frac{\partial \pi_R}{\partial k} = p_A\left(a_A - bp_A + cp_B\right) + p_B\left(a_B + cp_A - bp_B\right) \text{ and } \frac{\partial \pi_R}{\partial m} = 2 \tag{46}$$

The manufacturers' optimization problems in stage 2, given k and m are characterized by the first-order conditions of Equation (9):

$$\begin{aligned} \frac{\partial \pi_A}{\partial p_A} &= (1-k)\left(a_A - 2bp_A + cp_B\right) = 0 \\ \frac{\partial \pi_B}{\partial p_B} &= (1-k)\left(a_B - 2bp_B + cp_A\right) = 0 \end{aligned}$$

from which we can derive the manufacturers' optimal retail prices:

$$\begin{aligned} p_A &= \frac{2a_A b + a_B c}{4b^2 - c^2} \\ p_B &= \frac{2a_B b + a_A c}{4b^2 - c^2} \end{aligned} \tag{47}$$

Substituting the retail prices, we can characterize the retailer's equilibrium profit and the manufacturers' equilibrium profits as:

$$\pi_R = k\left(p_A D_A + p_B D_B\right) + 2m = \frac{k\left[b(a_A^2 + a_B^2)(4b^2 + c^2) + 8a_A a_B b^2 c\right]}{(4b^2 - c^2)^2} + 2m \tag{48}$$

$$\begin{aligned} \pi_A &= (1-k)p_A D_A - m = \frac{b(1-k)\left(2ba_A + ca_B\right)^2}{(4b^2 - c^2)^2} - m \\ \pi_B &= (1-k)p_B D_B - m = \frac{b(1-k)\left(ca_A + 2ba_B\right)^2}{(4b^2 - c^2)^2} - m \end{aligned} \tag{49}$$

Anticipating the manufacturers' participation incentives of selling on the retailer's platform ($\pi_i \geq \mu_i$), the retailer sets the optimal k and m by solving the binding constraints, $\pi_i = \mu_i$, simultaneously:

$$\begin{aligned} k &= 1 - \frac{(4b^2 - c^2)(\mu_A - \mu_B)}{b(a_A^2 - a_B^2)} \\ m &= \frac{\mu_A(ca_A + 2ba_B)^2}{(4b^2 - c^2)(a_A^2 - a_B^2)} - \frac{\mu_B(2ba_A + ca_B)^2}{(4b^2 - c^2)(a_A^2 - a_B^2)} \end{aligned} \tag{50}$$

Substituting the above optimal retail prices, the commission rate, and fixed fee into the retailer's profit:

$$\pi_R = \frac{b(4b^2 + c^2)(a_A^2 + a_B^2) + 8b^2 c a_A a_B}{(4b^2 - c^2)^2} - \mu_A - \mu_B \tag{51}$$

Lemma 1 follows by substituting a_A, a_B, b, and c into the above optimal retail prices, the commission rate, and fixed fee. Similarly, by substituting a_A, a_B, b, and c into Equation (51), the retailer's profit can be derived as

$$\pi_R = \frac{(1+2\alpha\tau)}{\tau}\left[\frac{\tau^2}{(1+4\alpha\tau)^2} + \frac{(1-\gamma)^2 x_R^2}{(3+4\alpha\tau)^2}\right] - \mu_A - \mu_B \tag{52}$$

Proof of Proposition 1

Proof. We notice

$$\frac{\partial p_B}{\partial(1-\gamma)} = -\frac{\varepsilon}{(1+4\alpha\gamma\varepsilon)^2} - \frac{(3+4\alpha\varepsilon)x_R}{(3+4\alpha\gamma\varepsilon)^2} < 0$$

$\partial p_A / \partial(1-\gamma) < 0$ if and only if

$$\frac{\partial p_A}{\partial(1-\gamma)} = -\frac{\varepsilon}{(1+4\alpha\gamma\varepsilon)^2} + \frac{(3+4\alpha\varepsilon)x_R}{(3+4\alpha\gamma\varepsilon)^2} < 0$$

which leads to the condition in Inequality (15).

Substituting $\tau = \gamma\varepsilon$ into Equation (52), we have the retailer's profit. $\partial \pi_R / \partial(1-\gamma) < 0$ if and only if

$$\frac{\partial \pi_R}{\partial(1-\gamma)} = -\frac{\varepsilon}{(1+4\alpha\gamma\varepsilon)^3} + \frac{(1-\gamma)x_R^2\left[3+\gamma(3+4\alpha\varepsilon(3+\gamma(2+4\alpha\varepsilon)))\right]}{\gamma^2\varepsilon(3+4\alpha\gamma\varepsilon)^3} < 0$$

which leads to the condition in Inequality (16).

Proof of Corollary 2

Proof. Throughout the appendix, we denote $x_A = x_B \equiv x$. Consumer surplus (CS_i) derived from product i can be formulated as:

$$\begin{aligned} CS_A &= \frac{\overline{D}_A - \alpha p_A}{\overline{D}_A}\int_0^1\int_{\frac{p_A - p_B - (1-\gamma)x_R - (1-2y)\beta t}{\gamma}}^{\varepsilon} \frac{1}{2\varepsilon}\left[x - \left(\beta y + \frac{1-\beta}{2}\right)t - p_A\right]dxdy \\ CS_B &= \frac{\overline{D}_B - \alpha p_B}{\overline{D}_B}\int_0^1\int_{-\varepsilon}^{\frac{p_A - p_B - (1-\gamma)x_R - (1-2y)\beta t}{\gamma}} \frac{1}{2\varepsilon}\left[x - \left(\beta(1-y) + \frac{1-\beta}{2}\right)t - p_B\right]dxdy \end{aligned} \tag{53}$$

where $\overline{D}_A$ and $\overline{D}_B$ are defined as in Equation (5). The total consumer surplus $CS = CS_A + CS_B$. By substituting the optimal retail prices from Lemma 1, we have

$$CS = \frac{(1+2\alpha\gamma\varepsilon)\left[(1+4\alpha\gamma\varepsilon)(3+4\alpha\gamma\varepsilon)^2(6\gamma\varepsilon x-3\gamma\varepsilon t+\beta^2t^2)-6\gamma^2\varepsilon^2(3+4\alpha\gamma\varepsilon)^2-6(1+4\alpha\gamma\varepsilon)^2(1-\gamma)^2x_R^2\right]}{6\gamma\varepsilon(1+4\alpha\gamma\varepsilon)^2(3+4\alpha\gamma\varepsilon)^2}$$

Similarly, we can derive the social welfare from product i as:

$$\begin{aligned} SW_A &= \frac{\overline{D}_A-\alpha p_A}{\overline{D}_A}\int_0^1\int_{\frac{p_A-p_B-(1-\gamma)x_R-(1-2y)\beta t}{\gamma}}^{\varepsilon}\frac{1}{2\varepsilon}\left[x-\left(\beta y+\frac{1-\beta}{2}\right)t\right]dxdy \\ SW_B &= \frac{\overline{D}_B-\alpha p_B}{\overline{D}_B}\int_0^1\int_{-\varepsilon}^{\frac{p_A-p_B-(1-\gamma)x_R-(1-2y)\beta t}{\gamma}}\frac{1}{2\varepsilon}\left[x-\left(\beta(1-y)+\frac{1-\beta}{2}\right)t\right]dxdy \end{aligned} \tag{54}$$

The total social welfare $SW = SW_A + SW_B$. By substituting the optimal retail prices from Lemma 1, we have

$$SW = \frac{\left[3\gamma\varepsilon(2x-t)+t^2\beta^2\right](1+2\alpha\gamma\varepsilon)}{6\gamma\varepsilon(1+4\alpha\gamma\varepsilon)}$$

When $x_R = 0$, we notice:

$$\frac{\partial CS}{\partial(1-\gamma)} = \frac{6\gamma^2\varepsilon^2\alpha(2x-t)(1+4\alpha\gamma\varepsilon)+6\gamma^2\varepsilon^2+t^2\beta^2(1+4\alpha\gamma\varepsilon)[1+8\alpha\gamma\varepsilon(1+\alpha\gamma\varepsilon)]}{6\gamma^2\varepsilon(1+4\alpha\gamma\varepsilon)^3}$$

$$\frac{\partial SW}{\partial(1-\gamma)} = \frac{6(2x-t)\alpha\gamma^2\varepsilon^2+t^2\beta^2\left[1+8\alpha\gamma\varepsilon(1+\alpha\gamma\varepsilon)\right]}{6\gamma^2\varepsilon(1+4\alpha\gamma\varepsilon)^2}$$

We can verify that both are positive using a condition that even the consumer with a signal indicating the largest degree of misfit has incentive to purchase.

Proof of Proposition 2

Proof. We notice

$$\frac{\partial p_B}{\partial\beta} = \frac{t}{(1+4\alpha\beta t)^2}+\frac{4\alpha t(1-\gamma)x_R}{(3+4\alpha\beta t)^2} > 0$$

$\partial p_A/\partial\beta > 0$ if and only if

$$\frac{\partial p_A}{\partial\beta} = \frac{t}{(1+4\alpha\beta t)^2}-\frac{4\alpha t(1-\gamma)x_R}{(3+4\alpha\beta t)^2} > 0$$

which leads to the condition in Inequality (18).

Substituting $\tau = \beta t$ into Equation (52), we have the retailer's profit. $\partial\pi_R/\partial\beta > 0$ if and only if

$$\frac{\partial\pi_R}{\partial\beta} = \frac{t}{(1+4\alpha\beta t)^3}-\frac{(1-\gamma)^2x_R^2\left[3+4\alpha\beta t(3+4\alpha\beta t)\right]}{\beta^2t(3+4\alpha\beta t)^3} > 0$$

which leads to the condition in Inequality (19).

Proof of Corollary 4

Proof. Consumer surplus (CS_i) derived from product i can be formulated as:

$$
\begin{aligned}
CS_A &= \frac{\overline{D}_A - \alpha p_A}{\overline{D}_A}\left(\int_0^{y_A}\left[x-\left(\beta y+\frac{1-\beta}{2}\right)t-p_A\right]dy+\int_{y_A}^{y_B}\int_{\frac{p_A-p_B-(1-\gamma)x_R-(1-2y)\beta t}{\gamma}}^{\varepsilon}\frac{1}{2\varepsilon}\left[x-\left(\beta y+\frac{1-\beta}{2}\right)t-p_A\right]dxdy\right)\\
CS_B &= \frac{\overline{D}_B - \alpha p_B}{\overline{D}_B}\left(\int_{y_A}^{y_B}\int_{-\varepsilon}^{\frac{p_A-p_B-(1-\gamma)x_R-(1-2y)\beta t}{\gamma}}\frac{1}{2\varepsilon}\left[x-\left(\beta(1-y)+\frac{1-\beta}{2}\right)t-p_B\right]dxdy+\int_{y_B}^{1}\left[x-\left(\beta(1-y)+\frac{1-\beta}{2}\right)t-p_B\right]dy\right)
\end{aligned}
\tag{55}
$$

where y_A and y_B are defined as in Equation (6) and $\overline{D}_A$ and $\overline{D}_B$ are defined as in Equation (7). The total consumer surplus $CS = CS_A + CS_B$. By substituting the optimal retail prices from Lemma 1, we have

$$
CS = \frac{(1+2\alpha\beta t)\left[(3+4\alpha\beta t)^2\left[(1+4\alpha\beta t)(12\beta tx-\gamma^2\varepsilon^2)-12\alpha(2-\beta)\beta^2t^3-3(3\beta+2)\beta t^2\right]-3(1+4\alpha\beta t)^2(5+4\alpha\beta t)(1-\gamma)^2x_R^2\right]}{12\beta t(1+4\alpha\beta t)^2(3+4\alpha\beta t)^2}
$$

Similarly, we can derive the social welfare (SW_i) derived from product i as:

$$
\begin{aligned}
SW_A &= \frac{\overline{D}_A - \alpha p_A}{\overline{D}_A}\left(\int_0^{y_A}\left[x-\left(\beta y+\frac{1-\beta}{2}\right)t\right]dy+\int_{y_A}^{y_B}\int_{\frac{p_A-p_B-(1-\gamma)x_R-(1-2y)\beta t}{\gamma}}^{\varepsilon}\frac{1}{2\varepsilon}\left[x-\left(\beta y+\frac{1-\beta}{2}\right)t\right]dxdy\right)\\
SW_B &= \frac{\overline{D}_B - \alpha p_B}{\overline{D}_B}\left(\int_{y_A}^{y_B}\int_{-\varepsilon}^{\frac{p_A-p_B-(1-\gamma)x_R-(1-2y)\beta t}{\gamma}}\frac{1}{2\varepsilon}\left[x-\left(\beta(1-y)+\frac{1-\beta}{2}\right)t\right]dxdy+\int_{y_B}^{1}\left[x-\left(\beta(1-y)+\frac{1-\beta}{2}\right)t\right]dy\right)
\end{aligned}
\tag{56}
$$

The total social welfare $SW = SW_A + SW_B$. By substituting the optimal retail prices from Lemma 1, we have

$$
SW = \frac{(1+2\alpha\beta t)\left[(3+4\alpha\beta t)^2\left(3\beta t(4x-t(2-\beta))-\gamma^2\varepsilon^2\right)-3(1+4\alpha\beta t)^2(1-\gamma)^2x_R^2\right]}{12\beta t(1+4\alpha\beta t)(3+4\alpha\beta t)^2}
$$

When $x_R = 0$, $\partial CS / \partial\beta < 0$ if and only if

$$
\frac{\partial CS}{\partial\beta} = -\frac{2\alpha tx}{(1+4\alpha\beta t)^2} - \frac{t(9+4\alpha\beta t)}{8(1+4\alpha\beta t)^3} + \frac{t[3+8\alpha t(1+\beta+2\alpha\beta^2 t)]}{8(1+4\alpha\beta t)^2} + \frac{[1+8\alpha\beta t(1+\alpha\beta t)]\gamma^2\varepsilon^2}{12\beta^2 t(1+4\alpha\beta t)^2} < 0
$$

which leads to the condition in Inequality (21). $\partial SW / \partial\beta < 0$ if and only if

$$
\frac{\partial SW}{\partial\beta} = -\frac{2\alpha tx}{(1+4\alpha\beta t)^2} + \frac{t[1+4\alpha t+4\alpha\beta t(1+2\alpha\beta t)]}{4(1+4\alpha\beta t)^2} + \frac{[1+8\alpha\beta t(1+\alpha\beta t)]\gamma^2\varepsilon^2}{12\beta^2 t(1+4\alpha\beta t)^2} < 0
$$

which leads to the condition in Inequality (22).

Proof of Lemma 2

Proof. We denote $a_A \equiv 1/2 + (1-\gamma)x_R/(2\tau)$, $a_B \equiv 1/2 - (1-\gamma)x_R/(2\tau)$, $b \equiv 1/(2\tau) + \alpha$, and $c \equiv 1/(2\tau)$. The demand functions in Equation (8) then can be rewritten as

$$D_A = a_A - bp_A + cp_B$$
$$D_B = a_B - bp_B + cp_A \tag{57}$$

The retailer's optimization problem in stage 2 is characterized by the first-order conditions of Equation (23):

$$\frac{\partial \pi_R}{\partial p_A} = a_A - bp_A + cp_B + c(p_B - w_B) - b(p_A - w_A) = 0$$
$$\frac{\partial \pi_R}{\partial p_B} = a_B - bp_B + cp_A + c(p_A - w_A) - b(p_B - w_B) = 0$$

from which we can derive the retailer's optimal retail prices as functions of the wholesale prices:

$$p_A = \frac{w_A}{2} + \frac{a_A b + a_B c}{2(b^2 - c^2)}$$
$$p_B = \frac{w_B}{2} + \frac{a_B b + a_A c}{2(b^2 - c^2)} \tag{58}$$

The manufacturers' optimization problems in stage 1 are characterized by the first-order conditions of Equation (24):

$$\frac{\partial \pi_A}{\partial w_A} = \frac{1}{2}\left(a_A - 2bw_A + cw_B\right) = 0$$
$$\frac{\partial \pi_B}{\partial w_B} = \frac{1}{2}\left(a_B - 2bw_B + cw_A\right) = 0$$

from which we can derive the optimal wholesale prices:

$$w_A = \frac{2a_A b + a_B c}{4b^2 - c^2}$$
$$w_B = \frac{2a_B b + a_A c}{4b^2 - c^2} \tag{59}$$

Substituting the above optimal wholesale prices into Equation (58), we derive the optimal retail prices:

$$p_A = \frac{2a_A b + a_B c}{2(4b^2 - c^2)} + \frac{a_A b + a_B c}{2(b^2 - c^2)}$$
$$p_B = \frac{2a_B b + a_A c}{2(4b^2 - c^2)} + \frac{a_B b + a_A c}{2(b^2 - c^2)} \tag{60}$$

With the above equilibrium demands, the optimal wholesale prices in Equation (59), and the optimal retail prices in Equation (60), we have the retailer's equilibrium profit:

$$\pi_R = (p_A - w_A)D_A + (p_B - w_B)D_B = \frac{(a_A - a_B)^2 b^3}{4(b^2 - c^2)(2b + c)^2} + \frac{(2a_A b + a_B c)(2a_B b + a_A c)b^2}{2(b - c)(4b^2 - c^2)^2} \tag{61}$$

Lemma 2 follows by substituting a_A, a_B, b, and c into the above optimal wholesale prices and retail prices. Similarly, by substituting a_A, a_B, b, and c into Equation (61), the retailer's profit can be derived as

$$\pi_R = \frac{(1+2\alpha\tau)^2}{8\alpha(1+4\alpha\tau)^2} + \frac{(1+2\alpha\tau)^2(1-\gamma)^2 x_R^2}{8\tau(1+\alpha\tau)(3+4\alpha\tau)^2} \tag{62}$$

Proof of Proposition 3

Proof. (a) We notice

$$\frac{\partial w_B}{\partial(1-\gamma)} = -\frac{\varepsilon}{(1+4\alpha\gamma\varepsilon)^2} - \frac{(3+4\alpha\varepsilon)x_R}{(3+4\alpha\gamma\varepsilon)^2} < 0$$

$\partial w_A / \partial(1-\gamma) < 0$ if and only if

$$\frac{\partial w_A}{\partial(1-\gamma)} = -\frac{\varepsilon}{(1+4\alpha\gamma\varepsilon)^2} + \frac{(3+4\alpha\varepsilon)x_R}{(3+4\alpha\gamma\varepsilon)^2} < 0$$

which leads to the condition in Inequality (29).

(b) Substituting $\tau = \gamma\varepsilon$ into Equation (62), we have the retailer's profit. $\partial\pi_R / \partial(1-\gamma) > 0$ because

$$\frac{\partial\pi_R}{\partial(1-\gamma)} = \frac{(1+2\alpha\gamma\varepsilon)}{8\varepsilon}\left(\frac{4\varepsilon^2}{(1+4\alpha\gamma\varepsilon)^3} + \frac{(1-\gamma)x_R^2\left[3+\gamma(3+2\alpha\varepsilon(6+\gamma(7+2\alpha\varepsilon(6+\gamma(3+4\alpha\varepsilon)))))\right]}{\gamma^2(1+\alpha\gamma\varepsilon)^2(3+4\alpha\gamma\varepsilon)^3}\right) > 0$$

We notice

$$\frac{\partial p_B}{\partial(1-\gamma)} = -\frac{\varepsilon}{2(1+4\alpha\gamma\varepsilon)^2} - \frac{[15+\alpha\varepsilon(17+2\gamma(18+\alpha\varepsilon(20+\gamma(11+12\alpha\varepsilon))))]x_R}{4(1+\alpha\gamma\varepsilon)^2(3+4\alpha\gamma\varepsilon)^2} < 0$$

$\partial p_A / \partial(1-\gamma) < 0$ if and only if

$$\frac{\partial p_A}{\partial(1-\gamma)} = -\frac{\varepsilon}{2(1+4\alpha\gamma\varepsilon)^2} + \frac{[15+\alpha\varepsilon(17+2\gamma(18+\alpha\varepsilon(20+\gamma(11+12\alpha\varepsilon))))]x_R}{4(1+\alpha\gamma\varepsilon)^2(3+4\alpha\gamma\varepsilon)^2} < 0$$

which leads to the condition in Inequality (30).

Proof of Corollary 6

Proof. As under the platform scheme, we can similarly derive the consumer surplus (CS) and social welfare (SW) by substituting the optimal retail prices in Lemma 2 into Equations (53) and (54), respectively:

$$\begin{aligned} CS = {} & \frac{(1+2\alpha\gamma\varepsilon)x}{2(1+4\alpha\gamma\varepsilon)} - \frac{t}{2} - \frac{(1+2\alpha\gamma\varepsilon)(1+6\alpha\gamma\varepsilon)}{8\alpha(1+4\alpha\gamma\varepsilon)^2} + \frac{t\alpha(1+6\alpha\gamma\varepsilon)}{4\alpha\left(1+4\alpha\gamma\varepsilon\right)} - \frac{(1-\gamma)^2(1+2\alpha\gamma\varepsilon)(5+6\alpha\gamma\varepsilon)x_R^2}{8\gamma\varepsilon(1+\alpha\gamma\varepsilon)(3+4\alpha\gamma\varepsilon)^2} \\ & + \frac{t^2\beta^2}{6\gamma\varepsilon} - \frac{t^2\beta^2\left[2\gamma\varepsilon(1+\alpha\gamma\varepsilon)^2(3+4\alpha\gamma\varepsilon)^2(1+6\alpha\gamma\varepsilon) - \alpha(1-\gamma)^2(1+4\alpha\gamma\varepsilon)(5+6\alpha\gamma\varepsilon)(1+8\alpha\gamma\varepsilon(1+\alpha\gamma\varepsilon))x_R^2\right]}{6(1+4\alpha\gamma\varepsilon)\left[4\gamma^2\varepsilon^2(1+\alpha\gamma\varepsilon)^2(3+4\alpha\gamma\varepsilon)^2 - (1-\gamma)^2(1+8\alpha\gamma\varepsilon(1+\alpha\gamma\varepsilon))^2x_R^2\right]} \end{aligned}$$

$$SW = \frac{(1+2\alpha\gamma\varepsilon)}{12\gamma\varepsilon}\left[\frac{6\gamma\varepsilon x}{1+4\alpha\gamma\varepsilon} - \frac{3\gamma\varepsilon t}{1+4\alpha\gamma\varepsilon} + \frac{2\beta^2 t^2(1+\alpha\gamma\varepsilon)\left(2\gamma^2\varepsilon^2(1+\alpha\gamma\varepsilon)(3+4\alpha\gamma\varepsilon)^2 - (1-\gamma)^2(1+4\alpha\gamma\varepsilon)(1+8\alpha\gamma\varepsilon(1+\alpha\gamma\varepsilon))x_R^2\right)}{(1+4\alpha\gamma\varepsilon)\left(4\gamma^2\varepsilon^2(1+\alpha\gamma\varepsilon)^2(3+4\alpha\gamma\varepsilon)^2 - (1-\gamma)^2(1+8\alpha\gamma\varepsilon(1+\alpha\gamma\varepsilon))^2 x_R^2\right)}\right]$$

When $x_R = 0$, $\partial CS / \partial(1-\gamma) > 0$ because

$$\frac{\partial CS}{\partial(1-\gamma)} = \frac{\alpha\varepsilon x}{(1+4\alpha\gamma\varepsilon)^2} + \frac{\beta^2 t^2\left[1+8\alpha\gamma\varepsilon(1+\alpha\gamma\varepsilon)\right]}{12\gamma^2\varepsilon(1+4\alpha\gamma\varepsilon)^2} - \frac{12\alpha\gamma^3\varepsilon^3 + t(1+4\alpha\gamma\varepsilon)(6\alpha\gamma^2\varepsilon^2)}{12\gamma^2\varepsilon(1+4\alpha\gamma\varepsilon)^3}$$

and we can verify that the above is positive using a condition that even the consumer with a signal indicating the largest degree of misfit has incentive to purchase.

$\partial SW / \partial(1-\gamma) > 0$ because

$$\frac{\partial SW}{\partial(1-\gamma)} = \frac{(2x-t)\alpha\varepsilon}{2(1+4\alpha\gamma\varepsilon)^2} + \frac{t^2\beta^2\left[1+8\alpha\gamma\varepsilon(1+\alpha\gamma\varepsilon)\right]}{12\gamma^2\varepsilon(1+4\alpha\gamma\varepsilon)^2} > 0$$

Proof of Proposition 4

Proof. (a) We notice

$$\frac{\partial w_B}{\partial\beta} = \frac{t}{(1+4t\alpha\beta)^2} + \frac{4\alpha t(1-\gamma)x_R}{(3+4t\alpha\beta)^2} > 0$$

$\partial w_A / \partial\beta > 0$ if and only if

$$\frac{\partial w_A}{\partial\beta} = \frac{t}{(1+4t\alpha\beta)^2} - \frac{4\alpha t(1-\gamma)x_R}{(3+4t\alpha\beta)^2} > 0$$

which leads to the condition in Inequality (31).

(b) Substituting $\tau = \beta t$ into Equation (62), we have the retailer's profit. $\partial\pi_R / \partial\beta < 0$ because

$$\frac{\partial\pi_R}{\partial\beta} = -\frac{(1+2\alpha\beta t)}{8}\left(\frac{(1-\gamma)^2 x_R^2\left[3+4\alpha\beta t(3+2\alpha\beta t(3+2\alpha\beta t))\right]}{\beta^2 t(1+\alpha\beta t)^2(3+4\alpha\beta t)^3} + \frac{4t}{(1+4\alpha\beta t)^3}\right) < 0$$

We notice

$$\frac{\partial p_B}{\partial\beta} = \frac{t}{2(1+4\alpha\beta t)^2} + \frac{\alpha t[17+8\alpha\beta t(5+3\alpha\beta t)](1-\gamma)x_R}{4(1+\alpha\beta t)^2(3+4\alpha\beta t)^2} > 0$$

$\partial p_A / \partial\beta > 0$ if and only if

$$\frac{\partial p_A}{\partial\beta} = \frac{t}{2(1+4\alpha\beta t)^2} - \frac{\alpha t[17+8\alpha\beta t(5+3\alpha\beta t)](1-\gamma)x_R}{4(1+\alpha\beta t)^2(3+4\alpha\beta t)^2} > 0$$

which leads to the condition in Inequality (32).

Proof of Corollary 8

Proof. As under the platform scheme, we can similarly derive the consumer surplus (CS) and social welfare (SW) by substituting the optimal retail prices in Lemma 2 into Equations (55) and (56), respectively:

$$CS = \frac{(1+2\alpha\beta t)x}{2(1+4\alpha\beta t)} - \frac{t}{2} + \frac{\beta t}{4} - \frac{(1+2\alpha\beta t)(1+6\alpha\beta t)}{8\alpha(1+4\alpha\beta t)^2} + \frac{t(2-\beta)(1+6\alpha\beta t)}{8(1+4\alpha\beta t)} - \frac{(1+2\alpha\beta t)[11+4\alpha\beta t(5+2\alpha\beta t)](1-\gamma)^2 x_R^2}{16\beta t(1+\alpha\beta t)(3+4\alpha\beta t)^2}$$
$$- \frac{\gamma^2\varepsilon^2}{12\beta t} + \frac{\gamma^2\varepsilon^2\left[2\beta t(1+\alpha\beta t)^2(3+4\alpha\beta t)^2(1+6\alpha\beta t) - \alpha(1+4\alpha\beta t)(5+6\alpha\beta t)(1+8\alpha\beta t(1+\alpha\beta t))(1-\gamma)^2 x_R^2\right]}{12(1+4\alpha\beta t)\left[4\beta^2 t^2(1+\alpha\beta t)^2(3+4\alpha\beta t)^2 - (1+8\alpha\beta t(1+\alpha\beta t))^2(1-\gamma)^2 x_R^2\right]}$$

$$SW = \frac{(1+2\alpha\beta t)}{2(1+4\alpha\beta t)}\Big[x - \frac{(1+\alpha\beta t)\left(3(2-\beta)\beta t^2+\gamma^2\varepsilon^2\right)\left(2\beta^2 t^2(1+\alpha\beta t)(3+4\alpha\beta t)^2 - (1+4\alpha\beta t)(1+8\alpha\beta t(1+\alpha\beta t))(1-\gamma)^2 x_R^2\right)}{6\beta t\left(4\beta^2 t^2(1+\alpha\beta t)^2(3+4\alpha\beta t)^2 - (1+8\alpha\beta t(1+\alpha\beta t))^2(1-\gamma)^2 x_R^2\right)}$$
$$- \frac{(1+8\alpha\beta t(1+\alpha\beta t))\left(\beta t(3+4\alpha\beta t)+(1+4\alpha\beta t)(1-\gamma)x_R\right)\left(4t(1+\alpha\beta t)(3+4\alpha\beta t)+(1+8\alpha\beta t(1+\alpha\beta t))(1-\gamma)x_R\right)(1-\gamma)x_R}{16\beta t(1+\alpha\beta t)(3+4\alpha\beta t)^2\left(2\beta t(1+\alpha\beta t)(3+4\alpha\beta t)+(1+8\alpha\beta t(1+\alpha\beta t))(1-\gamma)x_R\right)}$$
$$+ \frac{(1+8\alpha\beta t(1+\alpha\beta t))\left(\beta t(3+4\alpha\beta t)-(1+4\alpha\beta t)(1-\gamma)x_R\right)\left(4t(1+\alpha\beta t)(3+4\alpha\beta t)-(1+8\alpha\beta t(1+\alpha\beta t))(1-\gamma)x_R\right)(1-\gamma)x_R}{16\beta t(1+\alpha\beta t)(3+4\alpha\beta t)^2\left(2\beta t(1+\alpha\beta t)(3+4\alpha\beta t)-(1+8\alpha\beta t(1+\alpha\beta t))(1-\gamma)x_R\right)}\Big]$$

When $x_R = 0$, $\partial CS / \partial\beta < 0$ if and only if

$$\frac{\partial CS}{\partial\beta} = -\frac{\alpha t x}{(1+4\alpha\beta t)^2} + \frac{[1+8\alpha\beta t(1+\alpha\beta t)]\gamma^2\varepsilon^2}{24\beta^2 t(1+4\alpha\beta t)^2} + \frac{t[1+4\alpha t(1+2\beta(2+\alpha t(2+\beta(3+4\alpha\beta t))))]}{8(1+4\alpha\beta t)^3} < 0$$

which leads to the condition in Inequality (33).

$\partial SW / \partial\beta < 0$ if and only if

$$\frac{\partial SW}{\partial\beta} = -\frac{\alpha t x}{(1+4\alpha\beta t)^2} + \frac{3\beta^2 t^2\left[1+4\alpha t(1+\beta+2\alpha\beta^2 t)\right] + \left[1+8\alpha\beta t(1+\alpha\beta t)\right]\gamma^2\varepsilon^2}{24\beta^2 t(1+4\alpha\beta t)^2} < 0$$

which leads to the condition in Inequality (34).

Proof of Proposition 5

Proof. When $x_R = 0$, the platform scheme generates more profit for the retailer than wholesale scheme if and only if

$$\pi_R^w - \pi_R^p = \frac{(1+2\alpha\tau)(1-6\alpha\tau) + 8\alpha(1+4\alpha\tau)^2(\mu_A+\mu_B)}{8\alpha(1+4\alpha\tau)^2} < 0 \tag{63}$$

which leads to the condition in Inequality (35).

When $x_R > 0$, we can rewrite Inequality (63) as

$$\pi_R^w - \pi_R^p = \frac{(1-6\alpha\tau)(1+2\alpha\tau)}{8\alpha(1+4\alpha\tau)^2} - \frac{(1-\gamma)^2(7+6\alpha\tau)(1+2\alpha\tau)x_R^2}{8\tau(1+\alpha\tau)(3+4\alpha\tau)^2} + (\mu_A+\mu_B) < 0$$

Therefore, $\pi_R^w < \pi_R^p$ if and only if

$$x_R^2 > \frac{\tau(1+\alpha\tau)(3+4\alpha\tau)^2\left[(1+2\alpha\tau)(1-6\alpha\tau)+8\alpha(1+4\alpha\tau)^2(\mu_A+\mu_B)\right]}{\alpha(1+4\alpha\tau)^2(1-\gamma)^2(1+2\alpha\tau)(7+6\alpha\tau)}$$

Proof of Proposition 6

Proof. (a) Under platform scheme, based on Equation (52), we have

$$\frac{\partial\pi_R^P}{\partial x_R} = \frac{2(1-\gamma)^2(1+2\alpha\tau)x_R}{\tau(3+4\alpha\tau)^2} > 0$$

Under wholesale scheme, based on Equation (62), we have

$$\frac{\partial\pi_R^W}{\partial x_R} = \frac{(1-\gamma)^2(1+2\alpha\tau)^2 x_R}{4\tau(1+\alpha\tau)(3+4\alpha\tau)^2} > 0$$

(b) From the above derivatives, we have

$$\frac{\partial(\pi_R^P-\pi_R^W)}{\partial x_R} = \frac{(1-\gamma)^2(1+2\alpha\tau)x_R}{\tau(3+4\alpha\tau)^2}\left[2-\frac{(1+2\alpha\tau)}{4(1+\alpha\tau)}\right] > 0$$

Proof of Proposition 7

Proof. In the quality-dominates-fit case with $x_R = 0$, $CS^W < CS^P$ because

$$CS^W - CS^P = \frac{(1+2\alpha\gamma\varepsilon)\left[-2\alpha(1+4\alpha\gamma\varepsilon)(6\gamma\varepsilon x+\beta^2t^2)-3\gamma\varepsilon+6\alpha\gamma^2\varepsilon^2+6\alpha\gamma\varepsilon t(1+4\alpha\gamma\varepsilon)\right]}{24\alpha\gamma\varepsilon(1+4\alpha\gamma\varepsilon)^2}$$

and we can verify that the above is negative using a condition that even the consumer with a signal indicating the largest degree of misfit has incentive to purchase. $SW^W < SW^P$ because

$$SW^W - SW^P = \frac{\left[-3(2x-t)\gamma\varepsilon-\beta^2t^2\right](1+2\alpha\gamma\varepsilon)}{12\gamma\varepsilon(1+4\alpha\gamma\varepsilon)} < 0$$

In the fit-dominates-quality case with $x_R = 0$, $CS^W < CS^P$ because

$$CS^W - CS^P = \frac{(1+2\alpha\beta t)\left[-12\alpha\beta t(1+4\alpha\beta t)x-3\beta t+3\beta t(\alpha t(2+(1+4\alpha t(2-\beta))\beta))+\alpha(1+4\alpha\beta t)\gamma^2\varepsilon^2\right]}{24\alpha\beta t(1+4\alpha\beta t)^2}$$

and we can verify that the above is negative using a condition that even the consumer with a signal indicating the largest degree of misfit has incentive to purchase. $SW^W < SW^P$ because

$$SW^W - SW^P = \frac{(1+2\alpha\beta t)\left[-12\beta tx+3\beta t^2(2-\beta)+\gamma^2\varepsilon^2\right]}{24\beta t(1+4\alpha\beta t)} \le -\frac{(1+2\alpha\beta t)\left[3\beta t+9\alpha(3+4\alpha\beta t)\beta^2t^2-\alpha(1+4\alpha\beta t)\gamma^2\varepsilon^2\right]}{24\alpha\beta t(1+4\alpha\beta t)^2} < 0$$

where the second inequality is by applying a condition that even the consumer with a signal indicating the largest degree of misfit has incentive to purchase.

Analysis of Model Extensions

Proof of Lemma 3 and Results on the Effects of Precision Improvement under Non-Zero Quality Difference

Proof. The proof is the same as that of Lemma 1, except that we now have denote $a_A \equiv 1/2 + [\gamma\delta + (1-\gamma)x_R]/(2\tau)$ and $a_B \equiv 1/2 - [\gamma\delta + (1-\gamma)x_R]/(2\tau)$. By substituting a_A, a_B, b, and c into the optimal retail prices, commission rate, and fixed fee derived in the proof of Lemma 1, we obtain the results in Lemma 3. Similarly, we can derive the retailer's profit as

$$\pi_R = \frac{(1+2\alpha\tau)}{\tau}\left[\frac{\tau^2}{(1+4\alpha\tau)^2} + \frac{\gamma^2\delta^2}{(3+4\alpha\tau)^2} + \frac{(1-\gamma)x_R\left[2\gamma\delta + (1-\gamma)x_R\right]}{(3+4\alpha\tau)^2}\right] - \mu_A - \mu_B$$

(a) In quality-dominates-fit case with $x_R = \delta$:

$\partial p_B / \partial(1-\gamma) < 0$ because

$$\frac{\partial p_B}{\partial(1-\gamma)} = -\frac{\varepsilon}{(1+4\alpha\gamma\varepsilon)^2} - \frac{4\alpha\varepsilon\delta}{(3+4\alpha\gamma\varepsilon)^2} < 0$$

$\partial p_A / \partial(1-\gamma) < 0$ if and only if

$$\frac{\partial p_A}{\partial(1-\gamma)} = -\frac{\varepsilon}{(1+4\alpha\gamma\varepsilon)^2} + \frac{4\alpha\varepsilon\delta}{(3+4\alpha\gamma\varepsilon)^2} < 0$$

That is, $\partial p_A / \partial(1-\gamma) < 0$ if and only if $\delta < (3+4\alpha\gamma\varepsilon)^2 / [4\alpha(1+4\alpha\gamma\varepsilon)^2]$.

$\partial \pi_R / \partial(1-\gamma) < 0$ if and only if

$$\frac{\partial \pi_R}{\partial(1-\gamma)} = -\frac{\varepsilon}{(1+4\alpha\gamma\varepsilon)^3} + \frac{\delta^2\left[3+4\alpha\gamma\varepsilon(3+4\alpha\gamma\varepsilon)\right]}{\gamma^2\varepsilon(3+4\alpha\gamma\varepsilon)^3} < 0$$

That is, $\partial \pi_R / \partial(1-\gamma) < 0$ if and only if $\delta^2 < \gamma^2\varepsilon^2(3+4\alpha\gamma\varepsilon)^3 / [(1+4\alpha\gamma\varepsilon)^3(3+12\alpha\gamma\varepsilon+16\alpha^2\gamma^2\varepsilon^2)]$.

All together, we can conclude that the results on the effects of precision improvement remain the same as in Proposition 1 when δ is small.

(b) In fit-dominates-quality case with $x_R = \delta$:

$\partial p_B / \partial\beta > 0$ because

$$\frac{\partial p_B}{\partial\beta} = \frac{t}{(1+4\alpha\beta t)^2} + \frac{4\alpha t\delta}{(3+4\alpha\beta t)^2} > 0$$

$\partial p_A / \partial\beta > 0$ if and only if

$$\frac{\partial p_A}{\partial\beta} = \frac{t}{(1+4\alpha\beta t)^2} - \frac{4\alpha t\delta}{(3+4\alpha\beta t)^2} > 0$$

That is, $\partial p_A / \partial \beta > 0$ if and only if $\delta < (3+4\alpha\beta t)^2 / [4\alpha(1+4\alpha\beta t)^2]$.

$\partial \pi_R / \partial \beta > 0$ if and only if

$$\frac{\partial \pi_R}{\partial \beta} = \frac{t}{(1+4\alpha\beta t)^3} - \frac{\delta^2\left[3+4\alpha\beta t(3+4\alpha\beta t)\right]}{\beta^2 t(3+4\alpha\beta t)^3} > 0 .$$

That is, $\partial \pi_R / \partial \beta > 0$ if and only if $\delta^2 < \beta^2 t^2 (3+4\alpha\beta t)^3 / [(1+4\alpha\beta t)^3 (3+12\alpha\beta t+16\alpha^2\beta^2 t^2)]$.

All together, we can conclude that the results on the effects of precision improvement remain the same as in Proposition 2 when δ is small.

Proof of Lemma 4 and Results on the Effects of Precision Improvement under Non-Zero Quality Difference

Proof. The proof is the same as that of Lemma 2, except that we now have $a_A \equiv 1/2 + [\gamma\delta + (1-\gamma)x_R]/(2\tau)$ and $a_B \equiv 1/2 - [\gamma\delta + (1-\gamma)x_R]/(2\tau)$. By substituting a_A, a_B, b, and c into the optimal wholesale prices, retail prices, and retailer profit derived in the proof of Lemma 2, we obtain the results as in Lemma 4. Similarly, we can derive the retailer's profit as

$$\pi_R = \frac{(1+2\alpha\tau)^2}{8\alpha(1+4\alpha\tau)^2} + \frac{(1+2\alpha\tau)^2\left[\gamma\delta + (1-\gamma)x_R\right]^2}{8\tau(1+\alpha\tau)(3+4\alpha\tau)^2}$$

(a) In quality-dominates-fit case with $x_R = \delta$:

$\partial w_B / \partial(1-\gamma) < 0$ because

$$\frac{\partial w_B}{\partial(1-\gamma)} = -\frac{\varepsilon}{(1+4\alpha\gamma\varepsilon)^2} - \frac{4\alpha\varepsilon\delta}{(3+4\alpha\gamma\varepsilon)^2} < 0$$

$\partial w_A / \partial(1-\gamma) < 0$ if and only if

$$\frac{\partial w_A}{\partial(1-\gamma)} = -\frac{\varepsilon}{(1+4\alpha\gamma\varepsilon)^2} + \frac{4\alpha\varepsilon\delta}{(3+4\alpha\gamma\varepsilon)^2} < 0$$

That is, $\partial w_A / \partial(1-\gamma) < 0$ if and only if $\delta < (3+4\alpha\gamma\varepsilon)^2 / [4\alpha(1+4\alpha\gamma\varepsilon)^2]$.

$\partial p_B / \partial(1-\gamma) < 0$ because

$$\frac{\partial p_B}{\partial(1-\gamma)} = -\frac{\varepsilon}{2(1+4\alpha\gamma\varepsilon)^2} - \frac{\alpha\varepsilon\delta[17+8\alpha\gamma\varepsilon(5+3\alpha\gamma\varepsilon)]}{4(1+\alpha\gamma\varepsilon)^2(3+4\alpha\gamma\varepsilon)^2} < 0$$

$\partial p_A / \partial(1-\gamma) < 0$ if and only if

$$\frac{\partial p_A}{\partial(1-\gamma)} = -\frac{\varepsilon}{2(1+4\alpha\gamma\varepsilon)^2} + \frac{\alpha\varepsilon\delta[17+8\alpha\gamma\varepsilon(5+3\alpha\gamma\varepsilon)]}{4(1+\alpha\gamma\varepsilon)^2(3+4\alpha\gamma\varepsilon)^2} < 0 .$$

That is, $\partial p_A / \partial(1-\gamma) < 0$ if and only if $\delta < 2(1+\alpha\gamma\varepsilon)^2(3+4\alpha\gamma\varepsilon)^2 / [\alpha(1+4\alpha\gamma\varepsilon)^2(17+40\alpha\gamma\varepsilon+24\alpha^2\gamma^2\varepsilon^2)]$.

$\partial\pi_R / \partial(1-\gamma) > 0$ because

$$\frac{\partial \pi_R}{\partial(1-\gamma)} = \frac{\varepsilon(1+2\alpha\gamma\varepsilon)}{2(1+4\alpha\gamma\varepsilon)^3} + \frac{(1+2\alpha\gamma\varepsilon)[3+4\alpha\gamma\varepsilon(3+2\alpha\gamma\varepsilon(3+2\alpha\gamma\varepsilon))]\delta^2}{8\gamma^2\varepsilon(1+\alpha\gamma\varepsilon)^2(3+4\alpha\gamma\varepsilon)^3} > 0$$

All together, we can conclude that the results on the effects of precision improvement remain the same as in Proposition 3 when δ is small.

(b) In fit-dominates-quality case with $x_R = \delta$:

$\partial w_B / \partial\beta > 0$ because

$$\frac{\partial w_B}{\partial\beta} = \frac{t}{(1+4\alpha\beta t)^2} + \frac{4\alpha t\delta}{(3+4\alpha\beta t)^2} > 0$$

$\partial w_A / \partial\beta > 0$ if and only if

$$\frac{\partial w_A}{\partial\beta} = \frac{t}{(1+4\alpha\beta t)^2} - \frac{4\alpha t\delta}{(3+4\alpha\beta t)^2} > 0$$

That is, $\partial w_A / \partial\beta > 0$ if and only if $\delta < (3+4\alpha\beta t)^2 / [4\alpha(1+4\alpha\beta t)^2]$.

$\partial p_B / \partial\beta > 0$ because

$$\frac{\partial p_B}{\partial\beta} = \frac{t}{2(1+4\alpha\beta t)^2} + \frac{\alpha t\delta[17+8\alpha\beta t(5+3\alpha\beta t)]}{4(1+\alpha\beta t)^2(3+4\alpha\beta t)^2} > 0$$

$\partial p_A / \partial\beta > 0$ if and only if

$$\frac{\partial p_A}{\partial\beta} = \frac{t}{2(1+4\alpha\beta t)^2} - \frac{\alpha t\delta[17+8\alpha\beta t(5+3\alpha\beta t)]}{4(1+\alpha\beta t)^2(3+4\alpha\beta t)^2} > 0 .$$

That is, $\partial p_A / \partial\beta > 0$ if and only if $\delta < 2(1+\alpha\beta t)^2(3+4\alpha\beta t)^2 / [\alpha(1+4\alpha\beta t)^2(17+40\alpha\beta t+24\alpha^2\beta^2 t^2)]$.

$\partial\pi_R / \partial\beta < 0$ because

$$\frac{\partial \pi_R}{\partial\beta} = -\frac{t(1+2\alpha\beta t)}{2(1+4\alpha\beta t)^3} - \frac{(1+2\alpha\beta t)[3+4\alpha\beta t(3+2\alpha\beta t(3+2\alpha\beta t))]\delta^2}{8\beta^2 t(1+\alpha\beta t)^2(3+4\alpha\beta t)^3} < 0$$

All together, we can conclude that the results on the effects of precision improvement remain the same as in Proposition 4 when δ is small.

Proof and Results on the Effects of Precision Improvement under Different Precisions Among Consumers

Proof. (a) Effect of Third-Party Information under Platform Scheme

Denoting the notations as $a_A = a_B \equiv (a_H + a_L)/2$, $b \equiv (a_H + a_L)/(2\overline{\tau}) + (a_H + a_L)\alpha$, and $c \equiv (a_H + a_L)/(2\overline{\tau})$ when $x_R = 0$, we can replicate the same process as in the proof of Lemma 1. Because $a_H + a_L = 1$, it is easy to see that all terms are reduced to those in our baseline model by replacing τ with a new term $\overline{\tau}$, $\overline{\tau} = (a_H + a_L)\tau_H \tau_L / [a_L \tau_H + a_H \tau_L]$, where $\tau_H \in \{\gamma_H \varepsilon, \beta_H t\}$ and $\tau_L \in \{\gamma_L \varepsilon, \beta_L t\}$ with $\tau_i = \gamma_i \varepsilon$ for the quality-dominates-fit case and $\tau_i = \beta_i t$ for the fit-dominates-quality case. Therefore, we can obtain the optimal retail prices and the retailer's profit:

$$p_A = p_B = \frac{\overline{\tau}}{1 + 4\alpha\overline{\tau}}$$

$$\pi_R = \frac{(1 + 2\alpha\overline{\tau})\overline{\tau}}{(1 + 4\alpha\overline{\tau})^2} - \mu_A - \mu_B$$

Notice that in Lemma 1 in our base line model, $\tau = \gamma\varepsilon$ in the quality-dominates-fit case and $\tau = \beta t$ in the fit-dominates-quality case, where $\partial\tau / \partial\gamma = \varepsilon > 0$ in the quality-dominates-fit case and $\partial\tau / \partial\beta = t > 0$ in the fit-dominates-quality case. In the case with different precisions across consumers, it is also found that $\partial\overline{\tau} / \partial\gamma_H = a_H (a_H + a_L)\gamma_L^2 \varepsilon / (a_L \gamma_H + a_H \gamma_L)^2 > 0$ and $\partial\overline{\tau} / \partial\gamma_L = a_L (a_H + a_L)\gamma_H^2 \varepsilon / (a_L \gamma_H + a_H \gamma_L)^2 > 0$ in the quality-dominates-fit case and $\partial\overline{\tau} / \partial\beta_H = a_H (a_H + a_L)\beta_L^2 t / (a_L \beta_H + a_H \beta_L)^2 > 0$ and $\partial\overline{\tau} / \partial\beta_L = a_L (a_H + a_L)\beta_H^2 t / (a_L \beta_H + a_H \beta_L)^2 > 0$ in the fit-dominates-quality case. Because the signs of the impact of precision parameters on τ in the baseline model and $\overline{\tau}$ in the the model with heterogeneous precisions are the same, and the equilibrium quantities in the two models differ only on this variable, it is easy to see the effect of the improved precision from the third-party information is qualitatively same in both these models. We next show the detailed analysis under the platform scheme.

(a.1) In the quality-dominates-fit case:

$\partial p_i / \partial(1 - \gamma_H) < 0$ because

$$\frac{\partial p_i}{\partial(1 - \gamma_H)} = -\frac{a_H \varepsilon \gamma_L^2}{\left(a_H \gamma_L + a_L \gamma_H + 4\alpha\varepsilon\gamma_H \gamma_L\right)^2} < 0$$

$\partial p_i / \partial(1 - \gamma_L) < 0$ because

$$\frac{\partial p_i}{\partial(1 - \gamma_L)} = -\frac{a_L \varepsilon \gamma_H^2}{\left(a_L \gamma_H + a_H \gamma_L + 4\alpha\varepsilon\gamma_H \gamma_L\right)^2} < 0$$

$\partial \pi_R / \partial(1 - \gamma_H) < 0$ because

$$\frac{\partial \pi_R}{\partial(1 - \gamma_H)} = -\frac{a_H \varepsilon \gamma_L^2 \left(a_L \gamma_H + a_H \gamma_L\right)}{\left(a_L \gamma_H + a_H \gamma_L + 4\alpha\varepsilon\gamma_H \gamma_L\right)^3} < 0$$

$\partial \pi_R / \partial(1 - \gamma_L) < 0$ because

$$\frac{\partial \pi_R}{\partial(1-\gamma_L)} = -\frac{a_L \varepsilon \gamma_H^2 \left(a_L \gamma_H + a_H \gamma_L\right)}{\left(a_L \gamma_H + a_H \gamma_L + 4\alpha\varepsilon\gamma_H \gamma_L\right)^3} < 0$$

(a.2) In the fit-dominates-quality case:

$\partial p_i / \partial \beta_H > 0$ because

$$\frac{\partial p_i}{\partial \beta_H} = \frac{a_H \beta_L^2 t}{\left(a_L \beta_H + a_H \beta_L + 4\alpha\beta_H \beta_L t\right)^2} > 0$$

$\partial p_i / \partial \beta_L > 0$ because

$$\frac{\partial p_i}{\partial \beta_L} = \frac{a_L \beta_H^2 t}{\left(a_L \beta_H + a_H \beta_L + 4\alpha\beta_H \beta_L t\right)^2} > 0$$

$\partial \pi_R / \partial \beta_H > 0$ because

$$\frac{\partial \pi_R}{\partial \beta_H} = \frac{a_H \beta_L^2 t \left(a_L \beta_H + a_H \beta_L\right)}{\left(a_L \beta_H + a\beta_L + 4\alpha\beta_H \beta_L t\right)^3} > 0$$

$\partial \pi_R / \partial \beta_L > 0$ because

$$\frac{\partial \pi_R}{\partial \beta_L} = \frac{a_L \beta_H^2 t \left(a_L \beta_H + a_H \beta_L\right)}{\left(a_L \beta_H + a_H \beta_L + 4\alpha\beta_H \beta_L t\right)^3} > 0$$

(b) Effect of Third-Party Information under Wholesale Scheme

Similarly, using the notations defined $a_A = a_B \equiv (a_H + a_L)/2$, $b \equiv (a_H + a_L)/(2\overline{\tau}) + (a_H + a_L)\alpha$, and $c \equiv (a_H + a_L)/(2\overline{\tau})$ when $x_R = 0$, we can replicate the same process as in the proof of Lemma 2. Replacing τ with $\overline{\tau} = (a_H + a_L)\tau_H \tau_L / [a_L \tau_H + a_H \tau_L]$, where $\tau_H \in \{\gamma_H \varepsilon, \beta_H t\}$ and $\tau_L \in \{\gamma_L \varepsilon, \beta_L t\}$ with $\tau_i = \gamma_i \varepsilon$ for the quality-dominates-fit case and $\tau_i = \beta_i t$ for the fit-dominates-quality case, we can similarly derive the equilibrium outcomes. The optimal wholesale prices, retail prices, and the retailer's profit are obtained:

$$w_A = w_B = \frac{\overline{\tau}}{1 + 4\alpha\overline{\tau}}$$

$$p_A = p_B = \frac{1 + 6\alpha\overline{\tau}}{4\alpha(1 + 4\alpha\overline{\tau})}$$

$$\pi_R = \frac{(1 + 2\alpha\overline{\tau})^2}{8\alpha(1 + 4\alpha\overline{\tau})^2}$$

As shown under the platform scheme, we show the analysis under the wholesale scheme.

(b.1) In the quality-dominates-fit case:

$\partial w_i / \partial(1-\gamma_H) < 0$ because

$$\frac{\partial w_i}{\partial(1-\gamma_H)} = -\frac{a_H \varepsilon \gamma_L^2}{\left(a_H\gamma_L + a_L\gamma_H + 4\alpha\varepsilon\gamma_H\gamma_L\right)^2} < 0$$

$\partial w_i / \partial(1-\gamma_L) < 0$ because

$$\frac{\partial w_i}{\partial(1-\gamma_L)} = -\frac{a_L \varepsilon \gamma_H^2}{\left(a_L\gamma_H + a_H\gamma_L + 4\alpha\varepsilon\gamma_H\gamma_L\right)^2} < 0$$

$\partial p_i / \partial(1-\gamma_H) < 0$ because

$$\frac{\partial p_i}{\partial(1-\gamma_H)} = -\frac{a_H \varepsilon \gamma_L^2}{2\left(a_L\gamma_H + a_H\gamma_L + 4\alpha\varepsilon\gamma_H\gamma_L\right)^2} < 0$$

$\partial p_i / \partial(1-\gamma_L) < 0$ because

$$\frac{\partial p_i}{\partial(1-\gamma_L)} = -\frac{a_L \varepsilon \gamma_H^2}{2\left(a_L\gamma_H + a_H\gamma_L + 4\alpha\varepsilon\gamma_H\gamma_L\right)^2} < 0$$

$\partial \pi_R / \partial(1-\gamma_H) > 0$ because

$$\frac{\partial \pi_R}{\partial(1-\gamma_H)} = \frac{a_H \varepsilon \gamma_L^2\left(a_L\gamma_H + a_H\gamma_L + 2\alpha\varepsilon\gamma_H\gamma_L\right)}{2\left(a_L\gamma_H + a_H\gamma_L + 4\alpha\varepsilon\gamma_H\gamma_L\right)^3} > 0$$

$\partial \pi_R / \partial(1-\gamma_L) > 0$ because

$$\frac{\partial \pi_R}{\partial(1-\gamma_L)} = \frac{a_L \varepsilon \gamma_H^2\left(a_L\gamma_H + a_H\gamma_L + 2\alpha\varepsilon\gamma_H\gamma_L\right)}{2\left(a_L\gamma_H + a_H\gamma_L + 4\alpha\varepsilon\gamma_H\gamma_L\right)^3} > 0$$

(b.2) In the fit-dominates-quality case:

$\partial w_i / \partial\beta_H > 0$ because

$$\frac{\partial w_i}{\partial\beta_H} = \frac{a_H \beta_L^2 t}{\left(a_L\beta_H + a_H\beta_L + 4\alpha\beta_H\beta_L t\right)^2} > 0$$

$\partial w_i / \partial\beta_L > 0$ because

$$\frac{\partial w_i}{\partial\beta_L} = \frac{a_L \beta_H^2 t}{\left(a_L\beta_H + a_H\beta_L + 4\alpha\beta_H\beta_L t\right)^2} > 0$$

$\partial p_i / \partial\beta_H > 0$ because

$$\frac{\partial p_i}{\partial\beta_H} = \frac{a_H \beta_L^2 t}{2\left(a_L\beta_H + a_H\beta_L + 4\alpha\beta_H\beta_L t\right)^2} > 0$$

$\partial p_i / \partial \beta_L > 0$ because

$$\frac{\partial p_i}{\partial \beta_L} = \frac{a_L \beta_H^2 t}{2\left(a_L \beta_H + a_H \beta_L + 4\alpha \beta_H \beta_L t\right)^2} > 0$$

$\partial \pi_R / \partial \beta_H < 0$ because

$$\frac{\partial \pi_R}{\partial \beta_H} = -\frac{a_H \beta_L^2 t\left(a_H \beta_L + a_L \beta_H + 2\alpha \beta_H \beta_L t\right)}{2\left(a_L \beta_H + a_H \beta_L + 4\alpha \beta_H \beta_L t\right)^3} < 0$$

$\partial \pi_R / \partial \beta_L < 0$ because

$$\frac{\partial \pi_R}{\partial \beta_L} = -\frac{a_L \beta_H^2 t\left(a_H \beta_L + a_L \beta_H + 2\alpha \beta_H \beta_L t\right)}{2\left(a_L \beta_H + a_H \beta_L + 4\alpha \beta_H \beta_L t\right)^3} < 0$$

All together, we can conclude that the main results from the baseline model stay the same quantitatively and the insights carry over to the case with different consumer precisions.