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RESEARCH ARTICLE

DRAWING A LINE IN THE SAND: COMMITMENT PROBLEM IN ENDING SOFTWARE SUPPORT[1]

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Appendix

Proofs

Proof of Proposition 1

From (2), we get

$$\frac{d\pi}{dp} = \begin{cases} 1-2p, & \text{for } S \\ 1-\gamma-\frac{2p}{1+\mu}, & \text{for } NS \end{cases}$$

It is immediate that the optimal price and profit are indeed as given by (3). It is also easy to verify that both p_S and p_{NS} in (3) satisfy the required second order conditions. Further, solving $\pi_S = \pi_{NS}$, where π_S and π_{NS} are as in (3), leads to the threshold $c = c_1$; above this threshold, *NS* becomes the dominant strategy as indicated in (4).

Now, after consumers have made their upgrade decisions, the vendor reinstates support if doing so is relatively less costly when compared to financial losses likely to stem from ending support, that is, if

$$c < (\alpha + \beta\mu)\bar{v}$$

When $c > c_1$ and the vendor has chosen *NS*, we ought to have $\bar{v} = \frac{p}{1+\mu}$; see (1). Substituting for $\bar{v}$ and recalling that $\gamma = \frac{\alpha+\beta\mu}{1+\mu}$, we can reduce the above condition to

$$\frac{c}{\gamma} < p$$

Substituting p_{NS} for p, we can further reduce this condition to

$$c < c_2$$

where c_2 is as defined in the statement of the proposition. Thus, when $c_1 < c < c_2$, NS is optimal but not time-consistent.

Finally, as illustrated using examples in the main paper, $c_1 < c_2$ is very much a possibility when γ is relatively low. Therefore, time-consistency of the solution in (4) cannot be guaranteed. ■

Proof of Proposition 2

The prices and profits for S and NS are still as given by (3). For NS-L, we have

$$p^* = p_L = \frac{c}{\gamma} \quad \text{and} \quad \pi^* = \pi_L = c\left(\frac{1}{\gamma}\left(1 - \frac{c}{\gamma(1+\mu)}\right) - 1\right)$$

As explained in the proof of Proposition 1, $c < c_2$ is equivalent to $p_{NS} > \frac{c}{\gamma}$, which violates the condition for NS in (7). So, when $c < c_2$, we only need to compare NS-L with S to see which one is more profitable. Setting $\pi_S = \pi_L$, we get the desired threshold:

$$c_3 = \frac{\gamma(1+\mu)}{2}\left(1 - \sqrt{\frac{\mu}{1+\mu}}\right)$$

with $\pi_L > \pi_S$ for $c > c_3$. On the other hand, if $c \geq c_2$, NS is possible. Since the interior solution must dominate any other solution, $\pi_{NS} \geq \pi_L$ holds for $c \geq c_2$, with the inequality becoming strict for $c > c_2$. Therefore, NS-L is the optimal choice only when $c_3 < c < c_2$.

From Proposition 1, we know that $\pi_{NS} \geq \pi_S$ when $c > c_1$. This, taken together with the discussion in the preceding paragraph, implies that NS is optimal when $c \geq c_2$ as well as $c > c_1$. Now,

$$\frac{c_1 - c_2}{c_3 - c_2} = \frac{1}{2}\left(1 + \frac{1}{\gamma}\sqrt{\frac{\mu}{1+\mu}}\right) > 0$$

implying that $c_1 > c_2$ if and only if $c_3 > c_2$. So, when $c_3 \leq c_2$, $c \geq c_2$ is sufficient since it also implies that c must be at least as large as c_1. Likewise, if $c_3 > c_2$, $c_1 > c_2$ is automatically satisfied, and $c > c_1$ is sufficient to guarantee NS is optimal.

Having delineated the region of optimality for NS-L and subsequently for NS, it is easy to do the same for S, as it has to be the preferred strategy in the portion of the parameter space where the other two are not optimal. Further, since

$$\frac{c_3}{\gamma} = \frac{1+\mu}{2}\left(1 - \sqrt{\frac{\mu}{1+\mu}}\right) < \frac{1}{2}, \quad \forall \mu > 0$$

it is immediate that $p_S > \frac{c}{\gamma}$ holds for $c \leq c_3$. Now, note that $c \leq c_3$ must hold for S to be optimal, since the vendor would choose NS-L otherwise. Therefore, $p_S > \frac{c}{\gamma}$ is satisfied throughout the region of optimality of S, just as required by (7). ■

Proof of Proposition 3

Recall from (4) that $\pi_{NS} > \pi_S$ when $c > c_1$. Further, since the interior solution must dominate any other solution, $\pi_{NS} \geq \pi_L$ holds everywhere, with the inequality becoming strict for $c \neq c_2$. Therefore, when $c_1 < c < c_2$, π_{NS} is higher than both π_S and π_L and, hence, than the no-commitment equilibrium profit in (9).

It is also immediate from (9) that

$$\frac{d\pi_S}{dc} = -1 \qquad \text{and} \qquad \frac{d\pi_{NS}}{dc} = 0$$

implying that the profit is decreasing in c in Region S but independent of c in Region NS. To show that the profit is increasing in c in Region NS-L, note that

$$\frac{d^2\pi_L}{dc^2} = -\frac{2}{\gamma^2(1+\mu)}$$

which is negative. Therefore, $\frac{d\pi_L}{dc}$ attains its minimum at the upper boundary of Region NS-L, which is $c = c_2$. However, the minimum value itself is positive, since

$$\left.\frac{d\pi_L}{dc}\right|_{c=c_2} = \mu > 0$$

So, $\frac{d\pi_L}{dc}$ must be positive everywhere in Region NS-L. ■

Proof of Proposition 4

Let the desired refund be r per consumer. This refund imposes an extra cost burden on the vendor, so it would change the condition in (6) to

$$c + r\left(1 - \frac{p}{1+\mu}\right) \geq \gamma p$$

Setting $p = p_{NS}$, we can reduce the above inequality to

$$r \geq \frac{\gamma(1+\gamma)(1+\mu) - 2c}{1+\gamma}$$

Therefore, the required minimum refund amount is $\rho = \frac{\gamma(1-\gamma)(1+\mu)-2c}{1+\gamma}$. ■

Proof of Proposition 5

From (11), we can see that CS is (weakly) higher for strategy NS when compared to S as long as

$$\phi_{NS} = \frac{(1+\gamma)^2 - 4\mu(1-\gamma)}{8} > \frac{1}{8} = \phi_S$$

Rearranging the terms, we obtain the condition in the theorem, which is

$$\gamma > \sqrt{1+4\mu(2+\mu)} - (1+2\mu) = \Theta(\mu)$$

Clearly, for a given γ and μ, consumers prefer *NS* to *S* under this condition.

The remaining question is whether *NS-L* is also preferable to *S*. To answer this, we note that, according to (11), the consumer surplus for *NS* does not depend on c anywhere, and it also matches that for *NS-L* at the boundary $c = c_2$. However, for *NS-L*, the consumer surplus does depend on c, and its derivative with respect to c is $\frac{c-\gamma(1+\mu)^2}{\gamma^2(1+\mu)^2}$. Now, since $c \le c_2$, we can write

$$\frac{c-\gamma(1+\mu)^2}{\gamma^2(1+\mu)^2} \le \frac{c_2-\gamma(1+\mu)^2}{\gamma^2(1+\mu)^2} = -\frac{1+\gamma+2\mu}{2\gamma(1+\mu)} < 0$$

Because the derivative is always negative within Region *NS-L*, the consumer surplus must be even higher than what *NS* can possibly yield. Thus, if the condition $\gamma > \Theta(\mu)$ holds, consumers would not only prefer *NS* to *S* but would also prefer *NS-L* to *S*.

When $\gamma \le \Theta(\mu)$, consumer clearly prefer *S* to *NS*. However, they would prefer *NS-L* to *S* as long as

$$\phi_L = \frac{c^2-\gamma(1+\mu)^2(2c-\gamma)}{2\gamma^2(1+\mu)^2} > \frac{1}{8} = \phi_S$$

Upon simplification, this leads to

$$\gamma > \frac{2c\left(2(1+\mu)+\sqrt{1+4\mu(2+\mu)}\right)}{3(1+\mu)} = \theta(c,\mu)$$

Note that the expression on the right is increasing in c, and at $c = c_2$, it does coincide with $\Theta(\mu)$. ■