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GOVERNANCE OF CORPORATE TAKEOVERS: TIME FOR SAY-ON-TAKEOVERS?

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Appendix A

Glossary, List of Variables and Parameters, and Proofs

Table A1. Glossary of Governance and Takeover Germs

Term	Description
Hostile takeover	Acquirer makes takeover offer without target firm board consent or approval
Unsolicited takeover	Generally synonymous with hostile takeover. Also means the offer was not requested by the target firm board.
Antitakeover provisions	Target firm provisions in the bylaws or board approved motions to prevent unsolicited takeovers
Poison Pill	Target firm board approved motion to trigger a dilution of shares typically through a dividend issue, if an acquirer secures a threshold of ownership such as a 10% stake
Staggered Board	The separation of directors on the board into separate classes where only one class comes up for reelection each fiscal year. The typical number of classes is three such that it takes at least two years for an acquirer to take control of the board through a proxy vote.
Classified Board	Same as a Staggered Board
Board entrenchment	The characteristic of a board that seeks to maintain their incumbent board position. Entrenchment is enabled through anti-takeover measures such as Poison Pills and Staggered Boards.
Bargaining power hypothesis	The hypothesis that an entrenched board is better able to secure a higher premium from an acquirer than a non-entrenched board.

Table A2. Model Variables and Parameters with Descriptions

	Description
ϕ	Variable denoting informedness.
n_s	Upper support parameter of the range of the shareholder premium preference distribution $U[0,(1-\phi)n_s]$.
$\hat{n}_s$	Random variable associated with the shareholder premium preference distribution.
n_a	Upper support parameter of the range of the acquirer's distribution $U[0,(1-\phi)n_a]$ of the private premium value for the target firm.
$\hat{n}_a$	Random variable associated with the acquirer's private premium value distribution.
c	Variable denoting board entrenchment as a premium demanded by the board under delegated-governance.
k	Bargaining power of the target firm under delegated-governance.
q	Voting rule that determines the minimum proportion of votes required in favor of a successful takeover under owner-governance.
W	Value of benefits to the target firm board as an independent firm.
p	Variable denoting the takeover offer premium.
γ	Parameter that determines reputational value $(\gamma \cdot p)$ to the target firm board from a successful takeover premium.

Proof of Lemma 1

The parties are modeled to maximize the Nash Product of the expected gains from bargaining:

$$\underset{k}{Max}\left\{E\left[\left(\left(k\cdot\hat{n}_a\right)-O_s\right)\right]\cdot E\left[\left(\left(1-k\right)\hat{n}_a-O_a\right)\right]\right\} \tag{1}$$

Which leads to the first order condition:

$$\frac{1}{4}(1-\phi)n_a\left(2\left(O_s-O_a\right)-\left(2k-1\right)(1-\phi)n_a\right)=0 \tag{2}$$

Solving the first order condition generates the solution

$$k=\left(1/2\right)+\left(O_s-O_a\right)/\left(1-\phi\right)n_a \tag{3}$$

This completes the proof.

Proof of Lemma 2

Board's Endogenous Entrenchment

Case 1: When entrenchment is such that $c < k(1-\phi)n_a$, the board maximizes its expected benefits through the objective function, $\underset{c}{Max}\left\{E\left[S_B\right]\right\}$ where

$$E\left[S_B\right]=\int_{c}^{c/k}\frac{c\gamma}{(1-\phi)n_a}d\hat{n}_a+\int_{c/k}^{(1-\phi)n_a}\frac{k\hat{n}_a\gamma}{(1-\phi)n_a}d\hat{n}_a+\int_{0}^{c}\frac{W}{(1-\phi)n_a}d\hat{n}_a \tag{4}$$

Which leads to the first order condition $\dfrac{kW - c(2k-1)\gamma}{k(1-\phi)n_a} = 0$, and solving the first order condition generates the board's optimal entrenchment level and we have $c^* = kW/((2k-1)\gamma)$, and the case 1 condition $c < k(1-\phi)n_a$ is now expressed as $(W/\gamma) < (2k-1)(1-\phi)n_a$. Furthermore, $c^* > 0$ requires that $k > 1/2$.

Case 2: When entrenchment is such that, $k(1-\phi)n_a \le c \le (1-\phi)n_a$, the board maximizes its expected benefits through the objective function, $\underset{c}{Max}\{E[S_B]\}$ where

$$E[S_B] = \int_c^{(1-\phi)n_a} \frac{c\gamma}{(1-\phi)n_a} d\hat{n}_a + \int_0^c \frac{W}{(1-\phi)n_a} d\hat{n}_a \tag{5}$$

Which leads to the first order condition $\gamma + \dfrac{W - 2c\gamma}{(1-\phi)n_a} = 0$, and solving the first order condition generates the board's optimal entrenchment level and we have $c^* = (W/2\gamma) + ((1-\phi)n_a/2)$, and the case 2 condition $c \ge k(1-\phi)n_a$ is now expressed as $(W/\gamma) \ge (2k-1)(1-\phi)n_a$.

Case 3: When entrenchment eliminates all takeovers, we have $c > (1-\phi)n_a$. This condition is outside the interior solution because all takeovers are eliminated. By inspection it is readily seen that the maximum possible entrenchment is at $c^* = (1-\phi)n_a$ at which all takeovers are eliminated and would constitute the boundary condition. The condition $c > (1-\phi)n_a$, outside the boundary, is obtained when we set $(W/2\gamma) + ((1-\phi)n_a/2) > (1-\phi)n_a$, and simplifying we obtain $W/\gamma > (1-\phi)n_a$.
Hence, we generate the three-part solution to the board's optimal entrenchment as

$$c^* = \begin{cases} kW/((2k-1)\gamma), & (W/\gamma) < (2k-1)(1-\phi)n_a \\ (W/2\gamma) + ((1-\phi)n_a/2), & (2k-1)(1-\phi)n_a \le (W/\gamma) \le (1-\phi)n_a \\ (1-\phi)n_a, & (W/\gamma) > (1-\phi)n_a \end{cases} \tag{6}$$

Theoretical Benchmark Entrenchment

Case 1: When entrenchment is such that $c < k(1\text{-}\phi)n_a$, the shareholders expected surplus is maximized, $\underset{c}{Max}\{E[S_S]\}$ where

$$E[S_S] = \int_c^{c/k} \frac{c}{(1-\phi)n_a} d\hat{n}_a + \int_{c/k}^{(1-\phi)n_a} \frac{k\hat{n}_a}{(1-\phi)n_a} d\hat{n}_a = \frac{k(1-\phi)n_a}{2} - \frac{c^2(2k-1)}{2k(1-\phi)n_a} \tag{7}$$

Which leads to the first order condition $-c(2k-1)/(k(1-\phi)n_a) = 0$, and the optimal entrenchment level that maximizes shareholder surplus is $c_s^* = 0$, and the case 1 condition is $0 < k(1-\phi)n_a$.

Case 2: When entrenchment is such that, $c \geq k(1-\phi)n_a$, the shareholders expected surplus is maximized, $\underset{c}{Max}\{E[S_S]\}$ where

$$E[S_S] = \int_{c}^{(1-\phi)n_a} \frac{c}{(1-\phi)n_a} d\hat{n}_a = \frac{c((1-\phi)n_a - c)}{(1-\phi)n_a} \tag{8}$$

Which leads to the first order condition $((1-\phi)n_a - 2c)/((1-\phi)n_a) = 0$, and solving the first order condition generates the optimal entrenchment level that maximizes shareholder surplus, and we have $c_s^* = (1-\phi)n_a / 2$, and the case 2 condition simplifies to $0 < k \leq 1/2$. Therefore, the case 1 condition simplifies to $1/2 < k < 1$.

Hence, we generate the two-part solution to the benchmark entrenchment as $c_S^* = \begin{cases} (1-\phi)n_a / 2, & 0 < k \leq 1/2 \\ 0, & 1/2 < k < 1 \end{cases}$.

This completes the proof.

Proof of Corollary 1

Case 1: When $(W/\gamma) < (2k-1)(1-\phi)n_a$, from Lemma 2, the difference in entrenchment between the board's endogenous level and the benchmark case is

$$kW/(\gamma(2k-1)) - 0 > 0 \tag{9}$$

Case 2: When $(W/\gamma) \geq (2k-1)(1-\phi)n_a$, from Lemma 2, the difference in entrenchment between the board's endogenous entrenchment level and the benchmark case is

$$(W/2\gamma) + (1-\phi)n_a/2 - (1-\phi)n_a/2 = W/2\gamma > 0 \tag{10}$$

This completes the proof.

Proof of Proposition 1

(i) From case 1 in Lemma 2 we have $c^* = kW/((2k-1)\gamma)$ and $(W/\gamma) < (2k-1)(1-\phi)n_a$. Taking the partial derivatives $\frac{\partial c^*}{\partial \phi}$, $\frac{\partial c^*}{\partial k}$ generates the result $\frac{\partial c^*}{\partial \phi} = 0$ and $\frac{\partial c^*}{\partial k} = -\left(\frac{W}{(2k-1)^2 \gamma}\right)$ when $(W/\gamma) < (2k-1)(1-\phi)n_a$.

(ii) From case 2 in Lemma 2 we have $c^* = (W/2\gamma) + ((1-\phi)n_a/2)$ and $(W/\gamma) \geq (2k-1)(1-\phi)n_a$. Taking the partial derivatives $\frac{\partial c^*}{\partial \phi}$, $\frac{\partial c^*}{\partial k}$ generates the result $\frac{\partial c^*}{\partial \phi} = -(n_a/2)$ and $\frac{\partial c^*}{\partial k} = 0$ when $(W/\gamma) \geq (2k-1)(1-\phi)n_a$.

This completes the proof.

Proof of Proposition 2

The proof requires comparison of the shareholder surplus derived using the results from Lemma 2 for the board's endogenous entrenchment level and the theoretical benchmark entrenchment. This comparison generates two cases:

Case 1: When $(W/\gamma)<(2k-1)(1-\phi)n_a$, and bargaining power $k>1/2$: Substitute the values of $c_S^*=0$ and $c^*=kW/((2k-1)\gamma)$ in (7) to generate the difference in the benchmark surplus and the surplus from the board's endogenous entrenchment $\Delta E[S_S]=(E[S_S\mid c=c_s^*])-(E[S_S\mid c=c^*])$. Taking the first derivative of this difference with respect to informedness, we obtain

$$\frac{\partial \Delta E[S_S]}{\partial \phi}=k(W/\gamma)^2/\left(2(2k-1)(1-\phi)^2 n_a\right) \tag{11}$$

It is readily seen that the derivative (11) is always positive. Therefore, the difference is increasing with informedness.

Case 2a: When $(W/\gamma)\geq(2k-1)(1-\phi)n_a$, and bargaining power $k\leq 1/2$: Substitute the values of $c_S^*=(1-\phi)n_a/2$ and $c^*=(W/2\gamma)+((1-\phi)n_a/2)$ in (7) to generate the difference in the benchmark surplus and the surplus from the board's endogenous entrenchment $\Delta E[S_S]$. Taking the first derivative of this difference w.r.t. informedness, we obtain

$$\frac{\partial \Delta E[S_S]}{\partial \phi}=(W/\gamma)^2/\left(4(1-\phi)^2 n_a\right) \tag{12}$$

It is readily seen that the derivative (12) is always positive. Therefore, the difference is increasing with informedness.

Case 2b: When $(W/\gamma)\geq(2k-1)(1-\phi)n_a$, and bargaining power $k>1/2$: Substitute the values of $c_S^*=0$ and $c^*=(W/2\gamma)+((1-\phi)n_a/2)$ in (7) to generate the difference in the benchmark surplus and the surplus from the board's endogenous entrenchment $\Delta E[S_S]$. Taking the first derivative of this difference w.r.t. informedness, we obtain

$$\frac{\partial \Delta E[S_S]}{\partial \phi}=\left(\frac{W}{\gamma}\right)^2\left(\frac{1}{(1-\phi)^2 n_a}\right)-\frac{(2k-1)n_a}{4} \tag{13}$$

It can be seen that the derivative (13) is positive when $k>k_1=\left(1+\left((W/\gamma)/((1-\phi)n_a)\right)^2\right)/2$. It is readily seen that $k_1\geq 1/2$ Therefore, the difference is increasing with informedness when $1/2\leq k<k_1$, and decreasing with informedness when $k\geq k_1$.

This completes the proof.

Proof of Proposition 3

Shareholder surplus is computed from the results of Lemma 2, and there are two cases to be considered.

Case 1: When $(W/\gamma)<(2k-1)(1-\phi)n_a$, and bargaining power $k>1/2$: Substitute the value of $c^*=kW/((2k-1)\gamma)$ in (7) to derive the expected shareholder surplus:

$$E[S_S]=\frac{k}{2}\left((1-\phi)-\left(\frac{W}{\gamma}\right)^2\frac{1}{(2k-1)(1-\phi)n_a}\right) \tag{14}$$

Expected shareholders surplus conditional on a takeover is derived by dividing the expression in (14) by the probability of a takeover:

$$P_{DG} = \int_{c^*}^{(1-\phi)n_a} \left(\frac{1}{(1-\phi)n_a} \right) d\hat{n}_a = 1 - \frac{kW}{(2k-1)\gamma(1-\phi)n_a} \tag{15}$$

Dividing (14) by (15) provides shareholder surplus conditional on a takeover:

$$E[S_S \mid takeover] = \frac{k\left((2k-1)L^2 - W^2\right)}{2\gamma\left(k(2L-W)-L\right)}, \quad L = \gamma(1-\phi)n_a \tag{16}$$

The derivative of (16) is as follows:

$$\frac{\partial}{\partial k} E[S_S \mid takeover] = \frac{L\left((1-2k)^2 L^2 - 2k^2 LW + W^2\right)}{2\left(kW-(2k-1)L\right)^2 \gamma} \tag{17}$$

Examining (17) to derive the conditions on bargaining power when this slope is negative provides two quadratic roots: $k = \frac{L}{(2L-W)} \mp \frac{(L-W)\sqrt{2LW}}{2L(2L-W)}$. Considering the solution $k = \frac{L}{(2L-W)} - \frac{(L-W)\sqrt{2LW}}{2L(2L-W)}$ it is immediately clear that $k < \frac{L}{(2L-W)}$ because the second term is negative. We can also restate the Case 1 condition as $k > \frac{(L+W)}{2L}$, and we must have $\frac{(L+W)}{2L} < k < \frac{L}{(2L-W)}$. Therefore, we must have $\frac{(L+W)}{2L} < \frac{L}{(2L-W)}$, and this requirement reduces to $L < W$. This is a contradiction because we also must have $k > 1/2$ which in turn requires $L > W$. Hence, the solution $k = \frac{L}{(2L-W)} - \frac{(L-W)\sqrt{2LW}}{2L(2L-W)}$ is not feasible and is eliminated leaving the only feasible solution $\tilde{k} = \frac{L}{(2L-W)} + \frac{(L-W)\sqrt{2LW}}{2L(2L-W)}$. We confirm directionality using parameter values $\{n_s = 1, n_a = 1, W = 0.25, \gamma = 2, \phi = 0.5\}$ and we have the feasible solution $\tilde{k} = 0.7230$, and we evaluate (17) above and below $\tilde{k}$ using the values $\tilde{k} = 0.7230 \pm .1$ and we obtain $\left.\frac{\partial}{\partial k} E[S_S \mid takeover]\right|_{\tilde{k}+0.1} = 0.1820 > 0$ and $\left.\frac{\partial}{\partial k} E[S_S \mid takeover]\right|_{\tilde{k}-0.1} = -2.1854 < 0$. Thus, we obtain the result $\frac{\partial}{\partial k} E[S_S \mid takeover] < 0$ when $k < \tilde{k}$. We can restate the results in expanded form as follows:

$$k < \frac{\gamma(1-\phi)n_a}{\left(2\gamma(1-\phi)n_a - W\right)} + \frac{\left(\gamma(1-\phi)n_a - W\right)\sqrt{2W\gamma(1-\phi)n_a}}{2\gamma(1-\phi)n_a\left(2\gamma(1-\phi)n_a - W\right)} = \tilde{k} \tag{18}$$

The next part is to determine the point at which the results switch from Case 1 to Case 2, to determine the threshold of bargaining power $k > 1/2$ above which Case 1 is in force. We obtain this threshold by comparing the case1 and case 2 optimal entrenchment level c^* from the result in Lemma 2 expressed in equation (6) and solving for bargaining power.

$$kW / \left(\left(2k - 1 \right) \gamma \right) < \left(W / 2\gamma \right) + \left(\left(1 - \phi \right) n_a / 2 \right) \tag{19}$$

Solving (19) for bargain power generates the result $k > \left(\frac{1}{2} + \frac{W}{2\gamma \left(1 - \phi \right) n_a} \right) = k_2$

Case 2: When $\left(W / \gamma \right) \geq \left(2k - 1 \right) \left(1 - \phi \right) n_a$ substitute the value of $c^* = \left(W / 2\gamma \right) + \left(\left(1 - \phi \right) n_a / 2 \right)$ in (7) to derive the expected shareholder surplus:

$$E[S_S] = \frac{n_a \left(1 - \phi \right)}{4} - \left(\frac{W}{2\gamma} \right)^2 \frac{1}{\left(1 - \phi \right) n_a} \tag{20}$$

Expected shareholders surplus conditional on a takeover is derived by dividing the expression in (14) by the probability of a takeover:

$$P_{DG} = \int_{c^*}^{(1-\phi)n_a} \left(\frac{1}{\left(1 - \phi \right) n_a} \right) d\hat{n}_a = \frac{1}{2} \left(1 - \frac{W}{\gamma \left(1 - \phi \right) n_a} \right) \tag{21}$$

It can be readily seen that (20) and (21) are independent of bargaining power. Hence, we have

$$\frac{\partial}{\partial k} E[S_S \mid takeover] = 0 \tag{22}$$

Therefore, in this case expected shareholder surplus conditional on a takeover does not change with bargaining power.

This completes the proof.

Proof of Lemma 3

Informed about the random draw of the parameter $\hat{n}_s$, a rational acquirer will always bid $q \times \hat{n}_s$ provided $\hat{n}_a \geq q \times \hat{n}_s$. The expected shareholder surplus is therefore,

$$E[S_s] = \int_0^{(1-\phi)n_s} \left(\int_{q\hat{n}_s}^{(1-\phi)n_a} \left(q\hat{n}_s \right) \frac{1}{\left(1 - \phi \right) n_a} d\hat{n}_a \right) \frac{1}{\left(1 - \phi \right) n_s} d\hat{n}_s = \left(1 - \phi \right) q n_s \left(3n_a - 2qn_s \right) / 6n_a \tag{23}$$

And, solving the shareholders' maximization problem generates the optimal voting rule

$$\underset{q}{Max} \left\{ \left(1 - \phi \right) q n_s \left(3n_a - 2qn_s \right) / 6n_a \right\} \Rightarrow q^* = \begin{cases} 3n_a / 4n_s \quad , & n_a < 4n_s / 3 \\ 1 \quad , & n_a \geq 4n_s / 3 \end{cases} \tag{24}$$

Applying this optimal voting rule to (23) provides the result

$$E[S_s] = \begin{cases} 3\left(1 - \phi \right) n_a / 16 < \left(1 - \phi \right) n_s / 2 \quad , & n_a < 4n_s / 3 \\ \left(1 - \phi \right) n_s \left(1/2 - n_s / 3n_a \right) \geq \left(1 - \phi \right) n_s / 2 \ , & n_a \geq 4n_s / 3 \end{cases} \tag{25}$$

Similarly, the expected acquirer surplus is

$$E[S_a] = \int_0^{(1-\phi)n_s} \left(\int_{q^*\hat{n}_s}^{(1-\phi)n_a} \left(\hat{n}_a - q^*\hat{n}_s\right) \frac{1}{(1-\phi)n_a} d\hat{n}_a \right) \frac{1}{(1-\phi)n_s} d\hat{n}_s$$
$$= \begin{cases} 7(1-\phi)n_a/32\,, & n_a < 4n_s/3 \\ (1-\phi)\left(n_a/2 - n_s\left(1/2 - n_s/6n_a\right)\right) \geq 7(1-\phi)n_a/32, & n_a \geq 4n_s/3 \end{cases} \tag{26}$$

Joint surplus is

$$E[S_t] = E[S_s] + E[S_a] = \begin{cases} 13(1-\phi)n_a/32\,, & n_a < 4n_s/3 \\ (1-\phi)\left(n_a/2 - n_s^2/6n_a\right) \geq 13(1-\phi)n_a/32\,, & n_a \geq 4n_s/3 \end{cases} \tag{27}$$

The total available surplus is $E\left[U\left(0,(1-\phi)n_a\right)\right] = (1-\phi)n_a/2$.

This completes the proof.

Proof of Proposition 4

The proof of Proposition 4 is available in Appendix B.

Proof of Proposition 5

The probability of a takeover under owner-governance is computed as follows:

$$P_{OG} = \int_0^{(1-\phi)n_s} \left(\left(\int_{q^*\hat{n}_s}^{(1-\phi)n_a} \left(\frac{1}{(1-\phi)n_a} \right) d\hat{n}_a \right) \frac{1}{(1-\phi)n_s} \right) d\hat{n}_s \tag{28}$$

The probability of a takeover under delegated-governance is computed as follows:

$$P_{DG} = \int_{c^*}^{(1-\phi)n_a} \left(\frac{1}{(1-\phi)n_a} \right) d\hat{n}_a \tag{29}$$

(i) Substituting the values for c^* and q^* from Lemmas 2 and 3 into (28) and (29) respectively we obtain

$$P_{OG} = \begin{cases} 5/8\,, & n_a < 4n_s/3 \\ 1 - \dfrac{n_s}{2n_a}, & n_a \geq 4n_s/3 \end{cases} \tag{30}$$

$$P_{DG} = \begin{cases} 1 - \dfrac{kW}{(2k-1)\gamma(1-\phi)n_a}, & (W/\gamma) < (2k-1)(1-\phi)n_a \\ \dfrac{1}{2}\left(1 - \dfrac{W}{\gamma(1-\phi)n_a}\right), & (W/\gamma) \geq (2k-1)(1-\phi)n_a \end{cases} \tag{31}$$

It is immediately clear that $\dfrac{\partial P_{OG}}{\partial \phi} = 0$, and we also obtain: $\dfrac{\partial P_{DG}}{\partial \phi} = \begin{cases} -\dfrac{kW}{(2k-1)\gamma(1-\phi)^2 n_a} < 0, & (W/\gamma) < (2k-1)(1-\phi)n_a \\ -\dfrac{W}{2\gamma(1-\phi)^2 n_a} < 0, & (W/\gamma) \geq (2k-1)(1-\phi)n_a \end{cases}$

(ii) Next, compare the probability of a takeover between the governance structures.

Case 1a: $(W/\gamma) < (2k-1)(1-\phi)n_a$, $n_a < 4n_s/3$. The difference in the probability of a takeover between owner-governance and delegated-governance from (30) and (31) is expressed as

$$\frac{kW}{(2k-1)\gamma(1-\phi)n_a} - \frac{3}{8} \tag{32}$$

Solving for **Error! Reference source not found.** to be positive such that $P_{OG} > P_{DG}$ generates the result: $\phi \geq 1 - \dfrac{8kW}{3(2k-1)\gamma n_a}$.

Case 1b: $(W/\gamma) \geq (2k-1)(1-\phi)n_a$, $n_a < 4n_s/3$. The difference in the probability of a takeover between owner-governance and delegated-governance from (30) and (31) is expressed as

$$\frac{1}{8} + \frac{W}{2\gamma(1-\phi)\gamma n_a} \tag{33}$$

This difference is always positive and hence $P_{OG} > P_{DG}$.

Case 2a: $(W/\gamma) < (2k-1)(1-\phi)n_a$, $n_a \geq 4n_s/3$. The difference in the probability of a takeover between owner-governance and delegated-governance from (30) and (31) is expressed as

$$\frac{kW}{(2k-1)\gamma(1-\phi)n_a} - \frac{n_s}{2n_a} \tag{34}$$

Solving for (34) to be positive such that $P_{OG} > P_{DG}$ generates the result: $\phi \geq 1 - \dfrac{2kW}{(2k-1)\gamma n_s}$.

Case 2b: $(W/\gamma) \geq (2k-1)(1-\phi)n_a$, $n_a \geq 4n_s/3$. The difference in the probability of a takeover between owner-governance and delegated-governance from (30) and (31) is expressed as

$$\frac{1}{2} - \frac{n_s}{2n_a} + \frac{W}{2(1-\phi)\gamma n_a} \tag{35}$$

This difference is always positive because $n_a \geq 4n_s / 3$ and hence $P_{OG} > P_{DG}$.

This completes the proof.

Proof of Proposition 6

The proof consists of taking the difference in shareholder surplus between the two governance regimes and solving for informedness. Shareholder surplus under delegated-governance is obtained by substituting the endogenous board entrenchment level from (6) into the expressions for shareholder surplus given by (7) and (8). Shareholder surplus under owner-governance is obtained from Lemma 3 and the expression in (25). This comparison gives rise to four conditions that are analyzed below.

Case 1a: $(W/\gamma) < (2k-1)(1-\phi)n_a$, $n_a < 4n_s / 3$. The difference in shareholder surplus between owner-governance and delegated-governance is as follows:

$$\left(\frac{3(1-\phi)n_a}{16}\right) - \frac{1}{2}\left(k(1-\phi)n_a - \left(\frac{W}{\gamma}\right)^2 \frac{k}{(2k-1)(1-\phi)n_a}\right) \tag{36}$$

Solving for informedness ϕ in (36) generates only one feasible solution because $\phi < 1$:

$$\phi \geq 1 - \frac{2W\sqrt{2k}}{\gamma n_a \sqrt{(2k-1)(8k-3)}}, \quad (W/\gamma) < (2k-1)(1-\phi)n_a \ , \ n_a < 4n_s / 3 \tag{37}$$

Directionality is readily verified using parameter values $\{n_s = 1, n_a = 1, W = 0.25, \gamma = 1, k = 0.8\}$ to show that owner-governance provides greater acquirer surplus than delegated-governance when $\phi \geq 1 - \frac{2W\sqrt{2k}}{\gamma n_a \sqrt{(2k-1)(8k-3)}}$.

Case 1b: $(W/\gamma) \geq (2k-1)(1-\phi)n_a$, $n_a < 4n_s / 3$. The difference in shareholder surplus between owner-governance and delegated-governance is as follows:

$$\left(\frac{3(1-\phi)n_a}{16}\right) - \frac{1}{4}\left((1-\phi)n_a - \left(\frac{W}{\gamma}\right)^2 \frac{1}{(1-\phi)n_a}\right) \tag{38}$$

Solving for informedness ϕ in (38) generates only one feasible solution because $\phi < 1$:

$$\phi \geq 1 - 2W / (\gamma n_a), \qquad \left(\frac{W}{\gamma}\right) \geq (2k-1)(1-\phi)n_a \ , \ n_a < 4n_s / 3 \tag{39}$$

Directionality is readily verified using parameter values $\{n_s = 1, n_a = 1, W = 0.25, \gamma = 1, k = 0.55\}$ to show that owner-governance provides greater acquirer surplus than delegated-governance when $\phi \geq 1 - 2W / (\gamma n_a)$.

Case 2a: $(W/\gamma) < (2k-1)(1-\phi)n_a$, $n_a \geq 4n_s / 3$. The difference in shareholder surplus between owner-governance and delegated-governance is as follows:

$$(1-\phi)n_s\left(\frac{1}{2}-\frac{n_s}{3n_a}\right)-\frac{1}{2}\left(k(1-\phi)n_a-\left(\frac{W}{\gamma}\right)^2\frac{k}{(2k-1)(1-\phi)n_a}\right) \tag{40}$$

Solving for informedness ϕ in (40) generates only one feasible solution because $\phi<1$:

$$\phi\geq 1-\frac{W\sqrt{3k}}{\gamma\sqrt{(2k-1)\left(2n_s^2-3n_sn_a+3kn_a^2\right)}},\quad \left(\frac{W}{\gamma}\right)<(2k-1)(1-\phi)n_a\ ,\ n_a\geq 4n_s/3 \tag{41}$$

We have already seen from Case 1a that owner-governance always provides greater shareholder surplus than delegated-governance when informedness is sufficiently high and $n_a<4n_s/3$. In this case, the only difference with Case 1a is that we have the changed condition $n_a\geq 4n_s/3$. From Lemma 3 expression (25) we know we know that shareholder surplus under owner-governance with condition $n_a\geq 4n_s/3$ is equal to or greater than the shareholder surplus under condition $n_a<4n_s/3$. Hence, based on the result in Case 1a, directionality is such that increasing informedness shifts shareholder preference towards owner-governance.

Case 2b: $(W/\gamma)\geq(2k-1)(1-\phi)n_a\ ,\ n_a\geq 4n_s/3$. The difference in shareholder surplus between owner-governance and delegated-governance is as follows:

$$(1-\phi)n_s\left(\frac{1}{2}-\frac{n_s}{3n_a}\right)-\frac{1}{4}\left((1-\phi)n_a-\left(\frac{W}{\gamma}\right)^2\frac{1}{(1-\phi)n_a}\right) \tag{42}$$

Solving for informedness ϕ in (42) generates only one feasible solution because $\phi<1$:

$$\phi\geq 1-\frac{W\sqrt{3}}{\gamma\sqrt{\left(4n_s^2-6n_sn_a+3n_a^2\right)}},\quad \left(\frac{W}{\gamma}\right)\geq(2k-1)(1-\phi)n_a\ ,\ n_a\geq 4n_s/3 \tag{43}$$

We have already seen from Case 1b that owner-governance always provides greater shareholder surplus than delegated-governance when informedness is sufficiently high and $n_a<4n_s/3$. In this case, the only difference with Case 1b is that we have the changed condition $n_a\geq 4n_s/3$. From Lemma 3 and expression (25) we know that shareholder surplus under owner-governance with condition $n_a\geq 4n_s/3$ is equal to or greater than the shareholder surplus under condition $n_a<4n_s/3$. Hence, based on the result in Case 1b, directionality is such that increasing informedness shifts shareholder preference towards owner-governance.

This completes the proof.

Proof of Proposition 7

The proof consists of taking the difference in acquirer surplus between the two governance structures and solving for informedness. Expected acquirer surplus under owner-governance is given in (26) from Lemma 3. Expected acquirer surplus under delegated-governance is obtained as follows:

$$E[S_A]=\int_c^{c/k}\frac{(\hat{n}_a-c)}{(1-\phi)n_a}d\hat{n}_a+\int_{c/k}^{(1-\phi)n_a}\frac{(1-k)\hat{n}_a}{(1-\phi)n_a}d\hat{n}_a=\frac{(1-k)\left(k\left((1-\phi)n_a\right)^2-c^2\right)}{2k(1-\phi)n_a},\quad c<k(1-\phi)n_a \tag{44}$$

$$E[S_A] = \int_{c}^{(1-\phi)n_a} \frac{(\hat{n}_a - c)}{(1-\phi)n_a} d\hat{n}_a = \frac{((1-\phi)n_a - c)^2}{2(1-\phi)n_a}, \quad c \geq k(1-\phi)n_a \tag{45}$$

Substitute the endogenous board entrenchment level from (6) into the expression for acquirer surplus given by (44) and (45) for delegated-governance in the following comparisons.

Case 1a: $(W/\gamma) < (2k-1)(1-\phi)n_a,\ n_a < 4n_s/3$. The difference in acquirer surplus between owner-governance and delegated-governance is as follows:

$$\frac{7(1-\phi)n_a}{32} - \frac{(1-k)}{2(1-\phi)n_a}\left((1-\phi)^2 n_a^2 - \frac{k}{(2k-1)^2}\left(\frac{W}{\gamma}\right)^2\right) \tag{46}$$

Solving for informedness ϕ in (46) generates only one feasible, real-valued solution when $\phi < 1$.

$$\phi > 1 - \left(\frac{4W}{\gamma(2k-1)n_a}\right)\sqrt{\frac{(1-k)k}{(9-16k)}}, \quad \frac{W}{\gamma} < (2k-1)(1-\phi)n_a,\ n_a < \frac{4n_s}{3},\ k \in \left(0.5,\ \frac{9}{16} = 0.5625\right) \tag{47}$$

Note that the condition $k \in (0.5,\ 0.5625)$ is necessary because a non-negative value is needed under the square root sign in (47) for a real-valued threshold of informedness. Directionality is readily verified using parameter values $\{n_s = 1,\ n_a = 1,\ W = 0.1,\ \gamma = 5,\ k = 0.55\}$ to show that owner-governance provides greater acquirer surplus than delegated-governance when $\phi > 1 - \left(\frac{4W}{\gamma(2k-1)n_a}\right)\sqrt{\frac{(1-k)k}{(9-16k)}}$. In other words, delegated-governance provides greater acquirer surplus when $\phi \leq 1 - \left(\frac{4W}{\gamma(2k-1)n_a}\right)\sqrt{\frac{(1-k)k}{(9-16k)}}$ and $k \in (0.5,\ 0.5625)$.

Case 1b: $(W/\gamma) \geq (2k-1)(1-\phi)n_a$, $n_a < 4n_s/3$. The difference in acquirer surplus between owner-governance and delegated-governance is as follows:

$$\frac{7(1-\phi)n_a}{32} - \frac{(W - \gamma(1-\phi)n_a)^2}{8\gamma^2(1-\phi)n_a} \tag{48}$$

Solving for informedness ϕ in (48) produces two quadratic roots $\phi = 1 + \frac{2\left(2 \pm \sqrt{7}\right)W}{3n_a\gamma} > 1$ for informedness which do not satisfy the feasibility requirement $\phi < 1$. It is readily verified using parameter values $\{n_s = 1, n_a = 1, W = 0.25, \gamma = 1, \phi = 0.5, k = .55\}$ that owner-governance provides greater acquirer surplus than delegated-governance in the feasible region characterized by $\phi < 1$.

Case 2a: $(W/\gamma) < (2k-1)(1-\phi)n_a$, $n_a \geq 4n_s/3$. The difference in acquirer surplus between owner-governance and delegated-governance is as follows:

$$(1-\phi)\left(\frac{n_a}{2} - n_s\left(\frac{1}{2} - \frac{n_s}{6n_a}\right)\right) - \frac{(1-k)}{2(1-\phi)n_a}\left((1-\phi)^2 n_a^2 - \frac{k}{(2k-1)^2}\left(\frac{W}{\gamma}\right)^2\right) \tag{49}$$

Solving for informedness ϕ in (49) generates one feasible, real-valued solution when $\phi < 1$.

$$1-\left(\frac{W}{\gamma(2k-1)}\right)\sqrt{\frac{3(1-k)k}{\left(3n_s n_a - 3kn_a^2 - n_s^2\right)}},\ \ (W/\gamma) < (2k-1)(1-\phi)n_a,\ n_a \geq 4n_s/3 \tag{50}$$

We have already seen from Case 1a that owner-governance always provides greater acquirer surplus than delegated-governance when informedness is sufficiently high and $n_a < 4n_s/3$. In this case, the only difference with Case 1a is that we have the changed condition $n_a \geq 4n_s/3$. From Lemma 3 and expression (26) we know that acquirer surplus under owner-governance with condition $n_a \geq 4n_s/3$ is equal to or greater than the acquirer surplus under condition $n_a < 4n_s/3$. Hence, based on the result in Case 1a, directionality is such that increasing informedness shifts the acquirer's preference toward owner-governance. In other words, delegated-governance provides greater acquirer surplus when $\phi \leq 1-\left(\frac{W}{\gamma(2k-1)}\right)\sqrt{\frac{3(1-k)k}{\left(3n_s n_a - 3kn_a^2 - n_s^2\right)}}$.

Case 2b: $(W/\gamma) \geq (2k-1)(1-\phi)n_a$, $n_a \geq 4n_s/3$. The difference in acquirer surplus between owner-governance and delegated-governance is as follows:

$$(1-\phi)\left(\frac{n_a}{2} - n_s\left(\frac{1}{2} - \frac{n_s}{6n_a}\right)\right) - \frac{\left(W - \gamma(1-\phi)n_a\right)^2}{8\gamma^2(1-\phi)n_a} \tag{51}$$

We have already seen from Case 1b that owner-governance always provides greater acquirer surplus than delegated-governance when $n_a < 4n_s/3$. In this case, the only difference with Case 1b is that we have the changed condition $n_a \geq 4n_s/3$. From Lemma 3 and expression (26) we know that acquirer surplus under owner-governance with condition $n_a \geq 4n_s/3$ is equal to or greater than the acquirer surplus under condition $n_a < 4n_s/3$. Hence, based on the result in Case 1b, owner-governance always provides greater acquirer surplus than delegated-governance in this Case 2b.

This completes the proof.

Proof of Proposition 8

The proof consists of taking the difference in joint surplus between the two governance structures and solving for informedness. Joint surplus is obtained by adding the shareholder surplus and acquirer surplus from the proofs for Proposition 5 and Proposition 6. The entrenchment level applied is the endogenous board entrenchment level from (6). Joint surplus for owner-governance is obtained from Lemma 3. Directionality with respect to increasing informedness for the preference for owner-governance has been established in the proofs for Propositions 6 and 7. Hence joint surplus must have the same directionality because it is the sum of shareholder surplus and acquirer surplus. This comparison gives rise to four conditions that are analyzed below.

Case 1a: $(W/\gamma) < (2k-1)(1-\phi)n_a$, $n_a < 4n_s/3$. The difference in joint surplus between owner-governance and delegated-governance is as follows:

$$\frac{13(1-\phi)n_a}{32} - \frac{1}{2}\left(\left(\frac{W}{\gamma}\right)^2 \frac{k^2}{(2k-1)(1-\phi)n_a}\right) \tag{52}$$

Solving for informedness ϕ in (52) generates only one feasible solution because $\phi < 1$:

$$\phi \geq 1 - \frac{4kW}{\sqrt{3}\left(2k-1\right)\gamma n_a}, \quad \left(W / \gamma\right) < \left(2k-1\right)\left(1-\phi\right) n_a \ , \ n_a < 4n_s / 3 \tag{53}$$

Case 1b: $\left(W / \gamma\right) \geq \left(2k-1\right)\left(1-\phi\right) n_a$, $n_a < 4n_s / 3$. The difference in joint surplus between owner-governance and delegated-governance is as follows:

$$\frac{\left(1-\phi\right) n_a}{32} + \left(\frac{W}{8\gamma}\right)\left(2 + \left(\frac{W}{\gamma}\right)\frac{1}{\left(1-\phi\right) n_a}\right) > 0 \tag{54}$$

It is immediately apparent that (54) is always positive.

Case 2a: $\left(W / \gamma\right) < \left(2k-1\right)\left(1-\phi\right) n_a$, $n_a \geq 4n_s / 3$. The difference in joint surplus between owner-governance and delegated-governance is as follows:

$$\frac{1}{6n_a}\left(\left(\frac{kW}{\left(2k-1\right)\gamma}\right)^2\left(\frac{3}{\left(1-\phi\right)}\right) - n_s^2\left(1-\phi\right)\right) \tag{55}$$

Solving for informedness ϕ in (55) generates only one feasible solution because $\phi < 1$:

$$\phi \geq 1 - \frac{\sqrt{3}kW}{\left(2k-1\right)\gamma n_s}, \quad \left(W / \gamma\right) < \left(2k-1\right)\left(1-\phi\right) n_a \ , \ n_a \geq 4n_s / 3 \tag{56}$$

Case 2b: $\left(W / \gamma\right) \geq \left(2k-1\right)\left(1-\phi\right) n_a$, $n_a \geq 4n_s / 3$. The difference in joint surplus between owner-governance and delegated-governance is as follows:

$$\frac{\left(3n_a^2 - 4n_s^2\right)\left(1-\phi\right)}{24n_a} + \frac{W\left(W + 2\gamma n_a\left(1-\phi\right)\right)}{8\gamma^2 n_a\left(1-\phi\right)} > 0 \tag{57}$$

It is immediately apparent that **Error! Reference source not found.** is always positive.

This completes the proof.

Appendix B

Proof of Proposition 4

Model of Owner-Governance: Shareholder Preference Distribution Parameter $\hat{n}_s$ Is a Random Variable, with an Uninformed Acquirer

In the section analyzing owner-governance within the main body of the paper, the acquirer was informed of the realized shareholder preference distribution $U\left[0,\hat{n}_s\right]$. In this section, all parties are uninformed regarding the realized shareholder preference distribution. The shareholders determine an ex ante voting rule without having the benefit of such information. The sequence of steps is as described in Table 2, in the main body of the article, except for the following change in the information set.

The target firm shareholders and acquirer are uninformed regarding the realized shareholder preference distribution $U\left[0,\hat{n}_s\right]$.

For notational compactness, the distributions $U\left[0,\ (1-\phi)n_s\right]$ and $U\left[0,\ (1-\phi)n_a\right]$ have the upper support denoted as follows: $\theta_s=(1-\phi)n_s$ and $\theta_a=(1-\phi)n_a$. We now have the distributions $U\left[0,\ \theta_s\right]$ and $U\left[0,\ \theta_a\right]$. Lemma B1 reports the owner-governance surplus with an uninformed acquirer.

Lemma B1: Surplus under owner-governance with an *uninformed* acquirer

Under owner-governance, and information uncertainty regarding $\hat{n}_s$, the optimal voting rule is $q^*=\begin{cases}\theta_a/2\theta_s\ , & \theta_a<2\theta_s\\ 1\ , & \theta_a\geq 2\theta_s\end{cases}$, *and optimal expected shareholder surplus is* $E[S_s]=\theta_a/6$, *the acquirer's offer is* $p^*\cdot\hat{n}_a$ *where* $p^*=\begin{cases}1/2\ , & \theta_a<2\theta_s\\ \sqrt{\theta_s/2\theta_a}\ , & \theta_a\geq 2\theta_s\end{cases}$,

the expected acquirer surplus is

$$E[S_a]=\begin{cases}\theta_a/6\ , & \theta_a<2\theta_s\\ \dfrac{(3\theta_a-2\theta_s)\left(2\sqrt{\theta_a}-\sqrt{2\theta_s}\right)}{6\sqrt{\theta_a}}\geq\theta_a/6\ , & \theta_a\geq 2\theta_s\end{cases},$$

and expected joint surplus is

$$E[S_t]=\begin{cases}\theta_a/3\ , & \theta_a<2\theta_s\\ \dfrac{\left(2\theta_a\sqrt{\theta_a}+(3\theta_a-2\theta_s)\left(2\sqrt{\theta_a}-\sqrt{2\theta_s}\right)\right)}{12\sqrt{\theta_a}}\geq\theta_a/3\ , & \theta_a\geq 2\theta_s\end{cases}.$$

Lemma B1 reports the optimal voting rule and the resulting expected surplus for the target shareholders, acquirer surplus, and joint surplus when the acquirer is uninformed about $\hat{n}_s$. Because the acquirer is uninformed, the optimal strategy for the acquirer is to make an offer that is independent of any information regarding the target firm shareholder preferences.

Uncertainty in the shareholder preference distribution hurts target firm shareholders. The intuition behind the decrease in surplus is that there are too many possible low surplus transactions and too few high surplus transactions. High surplus transactions require (1) high valuation for the target firm by the acquirer and (2) a high value of the shareholder preference distribution parameter. Whereas this combination delivers high surplus for the shareholders, such potential transactions are too few in number. In contrast, low surplus transactions occur when there is (1) high valuation for the target firm by the acquirer and (2) a low value of the shareholder preference distribution parameter. This combination delivers low surplus for the shareholders but are more in number compared to high surplus transactions. The net effect is that the low surplus transactions dominate, reducing the expected payoff compared to the previous model with no uncertainty in the distribution parameter.

Counterintuitively, the addition of the stochastic attribute to the shareholder preference distribution parameter increases the acquirer's expected surplus. Uncertainty regarding the shareholder preference distribution benefits the acquirer. The intuition behind the acquirer's surplus increase is from the combination of (1) high valuation for the target firm by the acquirer and (2) a low value of the shareholder preference distribution parameter. This combination delivers high surplus to the acquirer and are large in number relative to low surplus combinations for the acquirer, (1) high valuation for the target firm by the acquirer and (2) a high value of the shareholder preference distribution parameter. The very combination that is surplus-reducing to the target firm shareholders is surplus-enhancing for the acquirer.

Proof of Lemma B1

Because the acquirer has no information regarding $\hat{n}_s$ the solution to the game is not straightforward. The solution requires backward induction in a two-stage game and includes two variables viz. the optimal voting rule and the optimal offer premium. The common information set for both parties are the two distributions $U[0,\ \theta_s]$ and $U[0,\ \theta_a]$. The realization $\hat{n}_s$ is unknown and the realized valuation of the target firm by the acquirer $\hat{n}_a$ is the private information of the acquirer. The sequence of actions is as follows: (1) in the first stage, the shareholders determine an optimal q and (2) in the second stage, the acquirer, given the existence of this q, will determine an optimal premium or price factor p that is applied to the realized valuation $\hat{n}_a$ such that the offer is $(p \cdot \hat{n}_a)$. Unlike the previous cases where q could be assumed to be fixed, here it is varying against p and a one-dimensional approach to the optimization is not applicable. Intuitively, what is taking place is that for any given $q \in [0,1]$, the acquirer has a best response price factor p. Knowing this best response strategy profile of the acquirer, the shareholder will select the optimal q. Both variables p, q are being optimized under random draws from their respective Uniform probability distributions. Therefore, a convolution of the two distributions of the random variables $\hat{n}_s, \hat{n}_a$ is required. In setting up the convolution we take advantage of the fact that a transaction can consummate only if $p \cdot \hat{n}_a \geq q \cdot \hat{n}_s$. Therefore, the new random variable for the convolution operation is defined as $z = (p \cdot \hat{n}_a) - (q \cdot \hat{n}_s)$. The two independent distributions subject to the convolution operation are $U[0, q\theta_s]$ and $U[0, p\theta_a]$. The resulting distribution of the random variable z in the convolution has the support $(-q\theta_s, p\theta_a)$. As noted earlier, the solution is obtained through backward induction.

Backward Induction: Stage 2

Whenever a transaction consummates, the acquirer generates a premium surplus of $(\hat{n}_a - p\hat{n}_a)$. The target firm has a voting rule q in place and the acquirer computes expected surplus using the convolution of two independent Uniform distributions. The general solution is trapezoidal in form (Killmann and von Collani 2001; Olds 1952) and has three terms as follows:

$$E[S_a] = \int_{-q\theta_s}^{0} (\hat{n}_a - p\hat{n}_a)\left(\frac{1}{p\theta_a}\right)dz + \int_{0}^{(p\theta_a - q\theta_s)} (\hat{n}_a - p\hat{n}_a)\left(\frac{1}{p\theta_a}\right)dz + \int_{(p\theta_a - q\theta_s)}^{p\theta_a} (\hat{n}_a - p\hat{n}_a)\left(\frac{p\theta_a - z}{2pq\theta_a\theta_s}\right)dz \qquad (1)$$

The first term disappears as there are no transactions in this region and the second and third terms remain. The surplus from (1) is evaluated over the probability density function of $\hat{n}_a$ and we have the following:

$$E[S_a] = \int_{0}^{\theta_a}\left(\int_{0}^{(p\theta_a - q\theta_s)} (\hat{n}_a - p\hat{n}_a)\left(\frac{1}{p\theta_a}\right)dz + \int_{(pn\theta_a - q\theta_s)}^{p\theta_a} (\hat{n}_a - p\hat{n}_a)\left(\frac{p\theta_a - z}{pq\theta_a\theta_s}\right)dz\right)\frac{1}{\theta_a}d\hat{n}_a \qquad (2)$$

The acquirer's optimization problem is $\underset{p}{Max}\{E[S_a]\}$ where $E[S_a]$ is given by (2) and the solution to this problem is as follows:

$$p^* = \frac{\sqrt{\theta_s}\sqrt{q}}{\sqrt{2\theta_a}} \qquad (3)$$

Depending on the final solution to the price factor *p*, it may appear that there may be the need for setting an upper limit of 1 on *p*, but it is soon evident that this will not be necessary. Thus, we have the optimal response profile for any voting rule that the shareholder may apply.

Backward Induction: Stage 1

The shareholders now know the best response profile of the acquirer from (3) and from this they compute their expected surplus. Note that a consummated transaction delivers a surplus of $p^* \cdot \hat{n}_a$ to the shareholder. The expected shareholder surplus is therefore:

$$E[S_s] = \int_0^{\theta_a} \left(\int_0^{(p^*\theta_a - q\theta_s)} \left(p^*\hat{n}_a\right)\left(\frac{1}{p^*\theta_a}\right) dz + \int_{(p^*\theta_a - q\theta_s)}^{p^*\theta_a} \left(p^*\hat{n}_a\right)\left(\frac{p^*\theta_a - z}{p^* q\theta_a\theta_s}\right) dz \right) \frac{1}{\theta_a} d\hat{n}_a \tag{4}$$

The shareholders optimization problem given the acquirer's best response profile q^* is $\underset{q}{Max}\{E[S_s]\}$ where $E[S_s]$ is given by (4). The solution to this problem results in the following optimal voting rule and price factor:

$$q^* = \begin{cases} \theta_a / 2\theta_s & , \theta_a < 2\theta_s \\ 1 & , \theta_a \geq 2\theta_s \end{cases} \qquad p^* = \begin{cases} 1/2 & , \theta_a < 2\theta_s \\ \sqrt{\theta_s / 2\theta_a} \leq 1/2 & , \theta_a \geq 2\theta_s \end{cases} \tag{5}$$

Clearly the solution $\{q^*,\ p^*\}$ forms a Nash equilibrium as the strategy is the best response to each of the other party's possible strategies. The expected surplus can now be computed from (5). Because the optimal voting rule and price factor are determined, they are not variables henceforward. The appropriate independent distribution may now be used for computing the expected surplus for each party. The convolution variable z is not required any further. The limits of integration for the random variables $\hat{n}_s$ and $\hat{n}_a$ are determined by the following requirement for a successful transaction.

$$p^* \cdot \hat{n}_a \geq q^* \cdot \hat{n}_s \tag{6}$$

The expected surplus is computed as follows:

$$E[S_s] = \int_0^{\theta_a} \left(\int_0^{(p^*/q^*)\hat{n}_a} \left(p^*\hat{n}_a\right)\left(\frac{1}{\theta_s}\right) d\hat{n}_s \right) \frac{1}{\theta_a} d\hat{n}_a = \theta_a / 6 \tag{7}$$

$$E[S_a] = \int_0^{\theta_s} \left(\int_{(q^*/p^*)\hat{n}_s}^{\theta_a} \left(\hat{n}_a - p^*\hat{n}_a\right)\left(\frac{1}{\theta_a}\right) d\hat{n}_a \right) \frac{1}{\theta_s} d\hat{n}_s = \begin{cases} \theta_a / 6 \,, & \theta_a < 2\theta_s \\ \dfrac{(3\theta_a - 2\theta_s)\left(2\sqrt{\theta_a} - \sqrt{2\theta_s}\right)}{12\sqrt{\theta_a}} \geq \dfrac{\theta_a}{6} & ,\theta_a \geq 2\theta_s \end{cases} \tag{8}$$

And joint surplus $E[S_t] = E[S_s] + E[S_a]$.

This completes the proof.

Proof of Proposition 4

The results from Lemma 3 in the article can be restated using the compact notation as follows.

Lemma B2: Surplus under owner-governance

Under owner-governance the optimal voting rule is: $q^* = \begin{cases} 3\theta_a / 4\theta_s & , \theta_a < 4\theta_s / 3 \\ 1 & , \theta_a \geq 4\theta_s / 3 \end{cases}$,

expected shareholder surplus is: $E[S_{s2}] = \begin{cases} 3\theta_a / 16 & , \theta_a < 4\theta_s / 3 \\ \theta_s (1/2 - \theta_s / 3\theta_a) & , \theta_a \geq 4\theta_s / 3 \end{cases}$,

expected acquirer surplus is: $E[S_{a2}] = \begin{cases} 7\theta_a / 32 & , \theta_a < 4\theta_s / 3 \\ \left(\theta_a / 2 - \theta_s (1/2 - \theta_s / 6\theta_a)\right) & , \theta_a \geq 4\theta_s / 3 \end{cases}$,

expected joint surplus is: $E[S_{t2}] = \begin{cases} 13\theta_a / 32 & , \theta_a < 4\theta_s / 3 \\ \left(\theta_a / 2 - \theta_s^2 / 6\theta_a\right) & , \theta_a \geq 4\theta_s / 3 \end{cases}$,

We now compare the results from Lemma B2 with the results from Lemma B1.

Shareholder surplus:

Note that we have $E[S_{s2}] = \begin{cases} 3\theta_a / 16 & , \theta_a < 4\theta_s / 3 \\ \theta_s (1/2 - \theta_s / 3\theta_a) & , \theta_a \geq 4\theta_s / 3 \end{cases}$

Examining the second part, $\theta_s (1/2 - \theta_s / 3\theta_a)$ and applying the constraint $\theta_a \geq 4\theta_s / 3$ it can be readily seen that $\theta_s \left(1/2 - \theta_s / 3\theta_a\right) \geq 3\theta_a / 4\left(1/2 - 3\theta_a / 12\theta_a\right)$ and hence, $\theta_s (1/2 - \theta_s / 3\theta_a) \geq 3\theta_a / 16$.

Taking the difference $E[S_{s2}] - E[S_s]$ we have

$$E[S_{s2}] - E[S_s] = \begin{cases} (3\theta_a / 16) - (\theta_a / 6) & , \theta_a < 4\theta_s / 3 < 2\theta_s \\ \theta_s (1 - 4\theta_s / 3\theta_a) - (\theta_a / 6) & , 4\theta_s / 3 < 2\theta_s \leq \theta_a \end{cases} \quad (9)$$

The first difference $(3\theta_a / 16) - (\theta_a / 6)$ is readily seen to be positive. We have also previously established that $\theta_s (1/2 - \theta_s / 6\theta_a) \geq 3\theta_a / 16$, and therefore the second difference in (9) is positive.

Acquirer surplus:

We have $E[S_{a2}] = \begin{cases} 7\theta_a / 32 & , \theta_a < 4\theta_s / 3 \\ \left(\theta_a / 2 - \theta_s (1/2 - \theta_s / 6\theta_a)\right) & , \theta_a \geq 4\theta_s / 3 \end{cases}$

It is readily seen that when $\theta_a = 4\theta_s / 3$ the two-part solutions equal $7\theta_a / 32$ and acquirer surplus will not exceed $7\theta_s / 24$. Next, it is readily seen that the derivative of the acquirer surplus with respect to θ_a in the second part is $\dfrac{\partial E[S_{a2}]}{\partial \theta_a} = \dfrac{1}{2} - \left(\dfrac{1}{6}\right)\left(\dfrac{\theta_s}{\theta_a}\right)^2$. When the constraint $\theta_a \geq 4\theta_s / 3$ is applied we see that this derivative is positive and we have $\frac{\partial E[S_{a2}]}{\partial \theta_a} > 0$. Therefore, we have $\left(\theta_a / 2 - \theta_s (1 - 2\theta_s / 3\theta_a)\right) \geq 7\theta_a / 32$.

Taking the difference $E[S_{a2}] - E[S_a]$ we have

$$E[S_{a2}] - E[S_a] = \begin{cases} (7\theta_a / 32) - (\theta_a / 6) \ , & \theta_a < 4\theta_s / 3 < 2\theta_s \\ (\theta_a / 2 - \theta_s (1/2 - \theta_s / 6\theta_a)) - (\theta_a / 6) \ , & 4\theta_s / 3 < \theta_a \leq 2\theta_s \\ \frac{\theta_a}{2} - \theta_s \left(\frac{1}{2} - \frac{\theta_s}{6\theta_a} \right) - \frac{(3\theta_a - 2\theta_s)\left(2\sqrt{\theta_a} - \sqrt{2\theta_s}\right)}{6\sqrt{\theta_a}} \ , & 4\theta_s / 3 < 2\theta_s \leq \theta_a \end{cases} \tag{10}$$

The first difference in the first part of (10) expressed as $(7\theta_a / 32) - (\theta_a / 6)$ is readily seen to be positive. We have also established previously that $\left(\theta_a / 2 - \theta_s (1/2 - \theta_s / 6\theta_a)\right) \geq 7\theta_a / 32$. Therefore, the second difference in (10) is positive. To determine the third difference in (10), we take the derivatives of the surplus as follows:

$$\frac{\partial E[S_{a2}]}{\partial \theta_a} = \frac{1}{2} - \frac{A^2}{6}, \qquad A \leq 1/2 \tag{11}$$

$$\frac{\partial E[S_a]}{\partial \theta_a} = \frac{1}{2} - \frac{\sqrt{A}}{4\sqrt{2}} - \frac{A^{3/2}}{6\sqrt{2}}, \quad A \leq 1/2 \tag{12}$$

Where $A = \theta_s / \theta_a$. Taking the difference between (11) and (12) we have $\Delta E\left[\partial S_a\right] = \sqrt{A}\left(3\sqrt{2} + 2\sqrt{2}A - 4A\sqrt{A}\right) / 24$. Applying the constraint $A \leq 1/2$, we see that $\Delta E\left[\partial S_a\right] > 0$. Hence, $E[S_{a2}]$ is increasing faster than $E[S_a]$ with respect to θ_a. We have shown that $E[S_{a2}] > E[S_a]$ when $4\theta_s / 3 < \theta_a \leq 2\theta_s$ and we also shown that $E[S_{a2}]$ is increasing faster than $E[S_a]$ with θ_a. Therefore, the third difference in (10) is also positive and $E[S_{a2}] > E[S_a]$.

Joint surplus:

Joint surplus is the sum of shareholder surplus and acquirer surplus and therefore we have $E[S_{t2}] > E[S_t]$.

References

Killman, F., and von Collani, E. 2001. "A Note on the Convolution of the Uniform and Related Distributions and Their Use in Quality Control," *Economic Quality Control* (16:1), p. 17-41.

Olds, E. G. 1952. "A Note on the Convolution of Uniform Distributions," *The Annals of Mathematical Statistics* (23:2), pp. 282-285.