Single-Sourcing Versus Multisourcing: The Roles of Output Verifiability and Task Modularity

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Appendix

In this appendix we provide mathematical proofs of all lemmas and propositions presented in the paper. It will be useful in what follows to have set, as we do now, $\boldsymbol{\bar{S}} = \boldsymbol{E}[\boldsymbol{S}]$ and $\boldsymbol{\bar{U}} = \boldsymbol{E}[\boldsymbol{U}]$.

Proof of Lemma 1. Let us assume that a contract $f_0(\cdot)$ is optimal (i.e., it maximizes the client's expected profit) and that it induces the optimal efforts $\tilde{e}_1$ and $\tilde{e}_2$ by the vendor. Then the vendor's problem can be represented as

$$\max_{e_1, e_2 \geq 0} E[f_0(S) \mid (e_1, e_2)] - c_1(e_1) - c_2(e_2)$$

The first-order conditions (FOCs) for this problem are

$$c_1'(\tilde{e}_1) = \tilde{e}_1 = \frac{\partial E\left[f_0(S) \mid (e_1,\ e_2)\right]}{\partial \bar{S}} \frac{\partial \bar{S}}{\partial e_1}\bigg|_{\{\tilde{e}_1, \tilde{e}_2\}} \tag{L1.1}$$

$$c_2'(\tilde{e}_2) = \tilde{e}_2 = \frac{\partial E\left[f_0(S) \mid (e_1,\ e_2)\right]}{\partial \bar{S}} \frac{\partial \bar{S}}{\partial e_2}\bigg|_{\{\tilde{e}_1, \tilde{e}_2\}} \tag{L1.2}$$

The FOCs for the vendor's problem under a linear contract $\{\alpha, T\}$ are

$$\alpha \frac{\partial E[S]}{\partial e_1} = e_1$$

$$\alpha \frac{\partial E[S]}{\partial e_2} = e_2$$

Therefore, equations (L1.1) and (L1.2) can be implemented via a linear contract $\{\alpha, T\}$ by setting

$$\alpha = \frac{\partial E[f_0(S) \mid (e_1, e_2)]}{\partial \bar{S}}\Big|_{(\tilde{e}_1, \tilde{e}_2)}$$

Note that the fixed payment T does not affect vendor effort and so can be chosen to make the vendor's participation constraint tight. Checking for second-order conditions (SOCs) under a linear contract yields $\alpha \frac{\partial^2 \bar{S}}{\partial e_i^2} - 1 = -1 < 0$. We have thus established the optimality of the linear contractual form $\{\alpha, T\}$.

Proof of Proposition 1. In the modular tasks case, $S = \phi(e_1 + \gamma e_2) + \varepsilon_2$ because project outcomes do not depend on the client's effort ($\lambda = 0$). Having established in Lemma 1 that the linear contract is optimal, in this proof we need attend only to linear contracts (of the form $T + \alpha S$).

Effort choice. The FOCs for the first-best effort level, as defined in equation (4), are

$$\tilde{e}_1 = \arg\max_{e_1} E[f(S) \mid e_1, e_2] - c_1(e_1) - c_2(e_2)$$
$$= \arg\quad \max_{e_1} T + \alpha\phi[e_1 + \gamma e_2] - \frac{e_1^2}{2} - \frac{e_2^2}{2} = \alpha\phi \quad \text{(P1.1)}$$

and, similarly,

$$\tilde{e}_2 = \alpha\gamma\phi \quad \text{(P1.2)}$$

As shown in the proof of Lemma 1, the SOCs for linear contracts are satisfied. Then, by equation (5), $E[f(S) \mid \tilde{e}_1, \tilde{e}_2] - c_1(\tilde{e}_1) - c_2(\tilde{e}_2) \geq 0$. Hence the client will set T such that $E[f(S) \mid \tilde{e}_1, \tilde{e}_2] - c_1(\tilde{e}_1) - c_2(\tilde{e}_2) = 0$, thereby making the vendor's participation constraint tight and extracting all the surplus. Therefore,

$$T + \alpha E[S \mid \tilde{e}_1, \tilde{e}_2] - c_1(\tilde{e}_1) - c_2(\tilde{e}_2) = 0 \quad \text{(P1.3)}$$

From equation (3) it follows that

$$\tilde{e}_3 = \arg\max_{e_3 \geq 0} E[v(\tilde{e}_1, \tilde{e}_2, e_3)] - c_3(e_3) - E[f(S) \mid \tilde{e}_1, \tilde{e}_2]$$

As a result,

$$\tilde{e}_3 = \arg\max_{e_3 \geq 0} \phi[\tilde{e}_1 + \{\theta + (1-\theta)\gamma\}\tilde{e}_2 + \theta e_3] - \frac{e_3^2}{2} - \frac{\tilde{e}_1^2}{2} - \frac{\tilde{e}_2^2}{2} = \theta\phi \quad \text{(P1.4)}$$

Contract design. According to equation (2), the client's contract design problem can be stated as

$$\max_{f(\cdot)} \Pi_{SS} = E[v(\tilde{e}_1, \tilde{e}_2, \tilde{e}_3)] - c_3(\tilde{e}_3) - E[f(S) \mid (\tilde{e}_1, \tilde{e}_2)]$$

And

$$\max_{\alpha} \Pi_{SS} = \phi[\tilde{e}_1 + \{\theta + (1-\theta)\gamma\}\tilde{e}_2 + \theta\tilde{e}_3] - \frac{\tilde{e}_3^2}{2} - \frac{\tilde{e}_1^2}{2} - \frac{\tilde{e}_2^2}{2}$$

since $E[f(S) \mid (\tilde{e}_1, \tilde{e}_2)] = \frac{\tilde{e}_1^2}{2} + \frac{\tilde{e}_2^2}{2}$. Substituting the values of $\tilde{e}_1$, $\tilde{e}_2$, and $\tilde{e}_3$ from equations (P1.1), (P1.2), and (P1.4) into the contract design problem yields

$$\max_{\alpha} \Pi_{SS} = \phi^2 \left[\alpha + \gamma\alpha\{\theta + (1-\theta)\gamma\} + \theta^2 - \frac{\theta^2}{2} - \frac{\alpha^2}{2} - \frac{\alpha^2\gamma^2}{2}\right]$$

which is a concave function in α. The FOC for the contract design problem gives us $\alpha = \frac{1+\gamma\{\theta+(1-\theta)\gamma\}}{1+\gamma^2}$, and the SOC yields $\frac{\partial^2 \Pi_{SS}}{\partial \alpha^2} = -\phi^2[1 + \gamma^2] < 0$. Substituting into the values of $\tilde{e}_1$, $\tilde{e}_2$, and $\tilde{e}_3$, we obtain $\tilde{e}_1 = \phi[\frac{1+\gamma\{\theta+(1-\theta)\gamma\}}{1+\gamma^2}]$, $\tilde{e}_2 = \phi[\frac{\gamma[1+\gamma\{\theta+(1-\theta)\gamma\}]}{1+\gamma^2}]$, and $\tilde{e}_3 = \phi\theta$. Finally, substituting these efforts into the profit function yields the profits given in Proposition 1.

First-best outcome. From equation (1) in the "Model Description and Assumptions" section, we know that the coordinated solution is $e_1^* = \phi$, $e_2^* = \phi[\theta + (1-\theta)\gamma]$, and $e_3^* = \phi\theta$. We can see that client effort in the SS modular case is $\tilde{e}_3 = \phi\theta$, which is the coordinated solution. For the vendor effort to be first-best, we need $\frac{1+\gamma\{\theta+(1-\theta)\gamma\}}{1+\gamma^2} = 1$ and $\frac{\gamma[1+\gamma\{\theta+(1-\theta)\gamma\}]}{1+\gamma^2} = \theta + (1-\theta)\gamma$. Solving these two equations simultaneously gives us that either $\theta = 0$ or $\gamma = 1$ is both a necessary and sufficient condition for the client to attain the first-best solution in the single-sourcing case.

Proof of Proposition 2. The client offers the contract $\{\alpha_i, T_i\}$ to vendor i, where α_i is the variable term of the linear contract and T_i is fixed. The vendors' optimal efforts are given by

$$\tilde{e}_1 = \arg\max_{e_1 \geq 0} \alpha_1 E[S(e_1, \tilde{e}_2)] - c_1(e_1) + T_1 = \alpha_1 \phi$$

$$\tilde{e}_2 = \arg\max_{e_2 \geq 0} \alpha_2 E[S(\tilde{e}_1, e_2)] - c_2(e_2) + T_2 = \gamma \alpha_2 \phi$$

From Equations (10) and (11), which are the individual rationality constraints, we can see that T_1 and T_2 do not affect vendors' effort decisions. Therefore, we can freely adjust these terms to ensure that the vendor participation constraint is tight. Hence we can write

$$T_1 = -\alpha_1 E[S(\tilde{e}_1, \tilde{e}_2)] + \frac{\tilde{e}_1^2}{2}$$

and

$$T_2 = -\alpha_2 E[S(\tilde{e}_1, \tilde{e}_2)] + \frac{\tilde{e}_2^2}{2}$$

We can now complete the proof by showing that there exist $\{\alpha_1, \alpha_2\}$ such that $\tilde{e}_i = e_i^*$ is the unique Nash equilibrium for the vendor's effort decision. Set $\alpha_1 = 1$ and $\alpha_2 = \frac{\{\theta + \gamma(1-\theta)\}}{\gamma}$ It is easy to check that $\{e_1^*, e_2^*\}$ is a Nash equilibrium outcome and also the first-best solution. The reason is that vendor i's FOC is satisfied at e_i^* when vendor j chooses e_j^*. Since in this case the vendors' effort game is decoupled from client effort, we must show that $\{e_1^*, e_2^*\}$ is a unique Nash equilibrium. For that purpose, the Hessian is computed. We can check that

$$|\mathbf{H}| = \begin{vmatrix} \frac{\alpha_1 \partial^2 ES(e_1,e_2)}{\partial e_1^2} - \frac{\partial^2 c_1(e_1)}{\partial e_1^2} & \frac{\alpha_1 \partial^2 ES(e_1,e_2)}{\partial e_1 \partial e_2} \\ \frac{\alpha_2 \partial^2 ES(e_1,e_2)}{\partial e_1 \partial e_2} & \frac{\alpha_2 \partial^2 ES(e_1,e_2)}{\partial e_2^2} - \frac{\partial^2 c_2(e_2)}{\partial e_2^2} \end{vmatrix}$$
$$= \begin{vmatrix} -1 & 0 \\ 0 & -1 \end{vmatrix} = 1 > 0$$

because $\frac{\partial^2 ES(e_1,e_2)}{\partial e_1^2} = \frac{\alpha_2 \partial^2 ES(e_1,e_2)}{\partial e_2^2} = \frac{\alpha_1 \partial^2 ES(e_1,e_2)}{\partial e_1 \partial e_2} = 0$ and $\frac{\partial^2 c_1(e_1)}{\partial e_1^2} = \frac{\partial^2 c_2(e_2)}{\partial e_2^2} = -1$.Therefore, $\{e_1^*, e_2^*\}$ is a unique Nash equilibrium. Given that T_1 and T_2 are set such that no vendor earns a surplus over its reservation value, we conclude that the client can attain the first-best outcome for itself.

Proof of Lemma 2. Suppose that a contract $f_0(\cdot)$ is optimal and that it induces the optimal efforts $\tilde{e}_1$ and $\tilde{e}_2$ by the vendor and $\tilde{e}_3$ by the client. Then the FOCs for this vendor's problem are

$$c_1'(\tilde{e}_1) = \tilde{e}_1 = \frac{\partial E\left[f_0(S) \mid (e_1, e_2, e_3)\right]}{\partial \bar{S}} \frac{\partial \bar{S}}{\partial e_1}\bigg|_{\{\tilde{e}_1, \tilde{e}_2, \tilde{e}_3\}} \tag{L2.1}$$

$$c_2'(\tilde{e}_2) = \tilde{e}_2 = \frac{\partial E[f_0(S) \mid (e_1, e_2, e_3)]}{\partial \bar{S}} \frac{\partial \bar{S}}{\partial e_2}\bigg|_{\{\tilde{e}_1, \tilde{e}_2, \tilde{e}_3\}} \tag{L2.2}$$

It follows that either $\tilde{e}_1, \tilde{e}_2 \in (0, \infty)$ or $\tilde{e}_1 = \tilde{e}_2 = 0$. The latter case can easily be implemented by setting $\alpha = 0$; we therefore focus on the case $\tilde{e}_1, \tilde{e}_2 \in (0, \infty)$, which renders equations (L2.1) and (L2.2) necessary. The FOCs for the vendor's problem under a linear contract $\{\alpha, T\}$ are

$$\alpha \frac{\partial E[S]}{\partial e_1} = c_1'(e_1)$$

$$\alpha \frac{\partial E[S]}{\partial e_2} = c_2'(e_2)$$

Therefore, equations (L2.1) and (L2.2) can be implemented via a linear contract $\{\alpha, T\}$ by setting

$$\alpha = \frac{\partial E[f_0(S) \mid (e_1,e_2,e_3)]}{\partial \bar{S}}\bigg|_{\{\tilde{e}_1,\tilde{e}_2,\tilde{e}_3\}} \tag{L2.3}$$

We now examine the client's effort decision. If $\tilde{e}_3 > 0$ then, under $f_0(\cdot)$, the FOC for the client's effort choice problem is

$$\frac{\partial v(\tilde{e}_1,\tilde{e}_2,e_3)}{\partial e_3}\bigg|_{e_3=\tilde{e}_3} - \frac{\partial E\left[f_0(S) \mid (e_1,e_2,e_3)\right]}{\partial \bar{S}}\bigg|_{\{\tilde{e}_1,\tilde{e}_2,\tilde{e}_3\}} = c_3'(\tilde{e}_3) = \tilde{e}_3$$

Under the linear contract $\{\alpha, T\}$, the FOC for the client's effort choice problem becomes

$$\frac{\partial v(e_1,e_2,e_3)}{\partial e_3} - \alpha\frac{\partial E[S]}{\partial e_3} = c_3'(e_3)$$

A comparison of the two preceding FOCs shows that the value of α, as given in equation (L2.3), ensures that the client's FOC under linear contracts is satisfied at $\tilde{e}_3$. As in the proof of Lemma 1, $\alpha > 0$; also, $\alpha < 1$ because $\tilde{e}_3 \in (0,\infty)$. All SOCs are (trivially) met. If $\tilde{e}_3 = 0$ then, under $f_0(\cdot)$,

$$\frac{\partial v(\tilde{e}_1,\tilde{e}_2,e_3)}{\partial e_3}\bigg|_{e_3=\tilde{e}_3} - \frac{\partial E\left[f_0(S) \mid (e_1,e_2,e_3)\right]}{\partial \bar{S}}\frac{\partial \bar{S}}{\partial e_3}\bigg|_{\{\tilde{e}_1,\tilde{e}_2,\tilde{e}_3\}} \leq 0$$

Under the linear contract $\{\alpha, T\}$, the derivative of the client's expected profit is

$$\frac{\partial v(e_1,e_2,e_3)}{\partial e_3} - \alpha\frac{\partial E[S]}{\partial e_3} - c_3'(e_3)$$

Substituting the value of α as determined by equation (L2.3) ensures that the client's effort choice is $\tilde{e}_3 = 0$. Furthermore, since $\tilde{e}_3 = 0$ it follows that $\alpha = 1$—thus ensuring satisfaction of the sufficient conditions for the linear contract to implement $\tilde{e}_1$, $\tilde{e}_2$, and $\tilde{e}_3$. Because the fixed payment T does not affect vendor effort, it can (again) be chosen such that the vendor's participation constraint is tight. Hence a linear contract can replicate the performance of any optimal contract and so is itself optimal. We must now establish that the optimal linear contract's performance cannot yield the client's first-best result.

Recall that, when tasks are integrated, the VPM $S = \phi(e_1 + \gamma e_2 + \lambda e_3) + \varepsilon_2$.

Effort choice. The FOCs for the effort devoted to outsourced tasks, as defined in equation (14), are

$$\tilde{e}_1 = \arg\max_{e_1} E[f(S) \mid e_1, e_2, \tilde{e}_3] - c_1(e_1) - c_2(e_2) = \arg\max_{e_1} T + \alpha\phi[e_1 + \gamma e_2 + \lambda e_3] - \frac{e_1^2}{2} - \frac{e_2^2}{2} = \alpha\phi \tag{L2.4}$$

and, similarly,

$$\tilde{e}_2 = \alpha\gamma\phi \tag{L2.5}$$

Equation (5) implies that $E[f(S) \mid \tilde{e}_1, \tilde{e}_2, \tilde{e}_3] - c_1(\tilde{e}_1) - c_2(\tilde{e}_2) \geq 0$. Here the client will set T such that $E[f(S) \mid \tilde{e}_1, \tilde{e}_2, \tilde{e}_3] - c_1(\tilde{e}_1) - c_2(\tilde{e}_2) = 0$, thereby making the vendor's participation constraint tight and extracting all the surplus. Hence

$$T + \alpha E[S] - c_1(\tilde{e}_1) - c_2(\tilde{e}_2) = 0 \tag{L2.6}$$

The equality $\tilde{e}_3 = \arg\max_{e_3 \geq 0} E[v(\tilde{e}_1, \tilde{e}_2, e_3)] - c_3(e_3) - E[f(S) \mid (\tilde{e}_1, \tilde{e}_2, e_3)]$ now follows from equation (13). Therefore,

$$\begin{aligned}\tilde{e}_3 = \arg\max_{e_3 \geq 0}\ & \phi[\tilde{e}_1 + \{\theta + (1-\theta)\gamma\}\tilde{e}_2 + \{\theta + (1-\theta)\lambda - \alpha\lambda\}e_3] \\ & - \frac{e_3^2}{2} - \frac{\tilde{e}_1^2}{2} - \frac{\tilde{e}_2^2}{2} + \alpha\phi\lambda\tilde{e}_3 = \phi[\theta + (1-\theta)\lambda - \lambda\alpha]\end{aligned} \tag{L2.7}$$

<u>Contract design</u>. Equation (12) gives the contract design problem as

$$\max_{f(\cdot)} \Pi_{SS} = E[v(\tilde{e}_1, \tilde{e}_2, \tilde{e}_3)] - c_3(\tilde{e}_3) - E[f(S) \mid (\tilde{e}_1, \tilde{e}_2, \tilde{e}_3)]$$

and we have

$$\max_\alpha \Pi_{SS} = \phi[\tilde{e}_1 + \{\theta + (1-\theta)\gamma\}\tilde{e}_2 + \{\theta + (1-\theta)\lambda\gamma\}\tilde{e}_3] - \frac{\tilde{e}_3^2}{2} - \frac{\tilde{e}_1^2}{2} - \frac{\tilde{e}_2^2}{2}$$

because $E[f(S) \mid (\tilde{e}_1, \tilde{e}_2, \tilde{e}_3)] = \frac{\tilde{e}_1^2}{2} + \frac{\tilde{e}_2^2}{2}$. Substituting the values of $\tilde{e}_1$, $\tilde{e}_2$, and $\tilde{e}_3$ from equations (L2.4), (L2.5), and (L2.7) into the contract design problem yields

$$\max_\alpha \Pi_{SS} = \phi^2[\alpha + \gamma\alpha\{\theta + (1-\theta)\gamma\} + \{\theta + (1-\theta)\lambda\}\{\theta + (1-\theta)\lambda - \lambda\alpha\} - \frac{\{\theta+(1-\theta)\lambda-\lambda\alpha\}^2}{2} - \frac{\alpha^2}{2} - \frac{\alpha^2\gamma^2}{2}]$$

$$= \phi^2\left[\alpha + \gamma\alpha\{\theta + (1-\theta)\gamma\} + \frac{\{\theta + (1-\theta)\lambda\}^2}{2} - \frac{\alpha^2\lambda^2}{2} - \frac{\alpha^2}{2} - \frac{\alpha^2\gamma^2}{2}\right]$$

which is a concave function in α. The FOC for the client's contract design problem now gives us

$$\alpha = \frac{1+\gamma\{\theta+(1-\theta)\gamma\}}{1+\gamma^2+\lambda^2}$$

Substituting into the values of $\tilde{e}_1$, $\tilde{e}_2$, and $\tilde{e}_3$, we obtain

$$\tilde{e}_1 = \phi[\frac{1+\gamma\{\theta+(1-\theta)\gamma\}}{1+\gamma^2+\lambda^2}], \quad \tilde{e}_2 = \phi[\frac{\gamma[1+\gamma\{\theta+(1-\theta)\gamma\}]}{1+\gamma^2+\lambda^2}], \text{ and } \quad \tilde{e}_3 = \phi[\theta + (1-\theta)\lambda - \frac{\lambda[1+\gamma\{\theta+(1-\theta)\gamma\}]}{1+\gamma^2+\lambda^2}]$$

First-best outcome. We know that the coordinated solution is $e_1^* = \phi$, $e_2^* = \phi[\theta + (1-\theta)\gamma]$, and $e_3^* = \phi[\theta + (1-\theta)\lambda]$. It is clear that the first-best efforts can never be achieved, since client effort in the single-sourcing case with integrated tasks is strictly less than the coordinated solution. Substituting the value of α in the SS integrated tasks case gives us

$$\Pi_{SS}^* = \phi^2[\frac{\{\theta + (1-\theta)\lambda\}^2}{2} + \frac{[1 + \gamma\{\theta + (1-\theta)\gamma\}]^2}{2(1 + \gamma^2 + \lambda^2)}]$$

Proof of Lemma 3. We shall start by proving the optimality of linear contracts. Assume that contracts $f_i(\cdot)$ for vendor i are optimal and that they induce optimal efforts $\tilde{e}_1$ and $\tilde{e}_2$ by the vendor and $\tilde{e}_3$ by the client. Note that if $\tilde{e}_i = 0$ for $i \in 1,2$ then $\alpha_i = 0$ trivially implements that effort level; as a consequence, we can restrict our focus to $\tilde{e}_1, \tilde{e}_2 \in (0, \infty)$. The vendors' FOCs are

$$c_1'(\tilde{e}_1) = \tilde{e}_1 = \frac{\partial E[f_1(S) \mid (e_1,e_2,e_3)]}{\partial \bar{S}} \frac{\partial E[S]}{\partial e_1}\Big|_{\{\tilde{e}_1,\tilde{e}_2,\tilde{e}_3\}} \tag{L3.1}$$

$$c_2'(\tilde{e}_2) = \tilde{e}_2 = \frac{\partial E[f_2(S) \mid (e_1,e_2,e_3)]}{\partial \bar{S}} \frac{\partial E[S]}{\partial e_2}\Big|_{\{\tilde{e}_1,\tilde{e}_2,\tilde{e}_3\}} \tag{L3.2}$$

and the FOCs for vendors under linear contracts $\{\alpha_i, T_i\}$ are

$$\alpha_1 \frac{\partial E[S]}{\partial e_1} = c_1'(e_1)$$

$$\alpha_2 \frac{\partial E[S]}{\partial e_2} = c_2'(e_2)$$

Therefore, equations (L3.1) and (L3.2) can be implemented via linear contracts $\{\alpha_i, T_i\}$ by setting

$$\alpha_i = \frac{\partial E[f_i(S) \mid (e_1,e_2,e_3)]}{\partial \bar{S}}\Big|_{\{\tilde{e}_1,\tilde{e}_2,\tilde{e}_3\}} \tag{L3.3}$$

We now check the client's effort decision. If $\tilde{e}_3 > 0$ then, under $f_i(\cdot)$, the FOC for the client's effort choice problem is

$$\frac{\partial E[v(\tilde{e}_1,\tilde{e}_2,e_3)]}{\partial e_3}\Big|_{e_3=\tilde{e}_3} - \sum_{i=1}^2 \frac{\partial E\left[f_i(S) \mid (e_1,e_2,e_3)\right]}{\partial \bar{S}} \frac{\partial \bar{S}}{\partial e_i}\Big|_{\{\tilde{e}_1,\tilde{e}_2,\tilde{e}_3\}} = c_3'(\tilde{e}_3) = \tilde{e}_3$$

Under the linear contracts $\{\alpha_i, T_i\}$, the FOC for the client's effort choice problem becomes

$$\frac{\partial E[v(e_1,e_2,e_3)]}{\partial e_3} - (\alpha_1+\alpha_2)\frac{\partial \bar{S}}{\partial e_3} = c_3'(e_3)$$

Comparing these two FOCs reveals that the value of α_i, as determined in equation (L3.3), ensures that the client's FOC under linear contracts is satisfied at $\tilde{e}_3$. Just as in the proof of Lemma 1, we have $\alpha_i > 0$. Also, since $\tilde{e}_3 \in (0,\infty)$ it follows that $\alpha_1+\alpha_2 < 1$. As before, all SOCs are trivially met. If $\tilde{e}_3 = 0$, then under $f_i(\cdot)$ we have

$$\frac{\partial E[v(\tilde{e}_1,\tilde{e}_2,e_3)]}{\partial e_3}\Big|_{e_3=\tilde{e}_3} - \sum_{i=1}^{2}\frac{\partial E[f_i(S)\mid(e_1,e_2,e_3)]}{\partial \bar{S}}\frac{\partial \bar{S}}{\partial e_i}\Big|_{\{\tilde{e}_1,\tilde{e}_2,\tilde{e}_3\}} \le 0$$

Under the linear contracts $\{\alpha_i, T_i\}$, the derivative of the client's expected profit is

$$\frac{\partial E[v(e_1,e_2,e_3)]}{\partial e_3} - (\alpha_1+\alpha_2)\frac{\partial \bar{S}}{\partial e_3} - c_3'(e_3)$$

Substituting the value of α_i as determined in equation (L3.3) ensures that the client's effort choice is $\tilde{e}_3 = 0$. Similarly to the proof of Lemma 1, we have $\alpha_i > 0$. Also, since $\tilde{e}_3 = 0$ it follows that $\alpha_1+\alpha_2 \ge 1$, thus ensuring that the sufficient conditions for the linear contract to implement $\tilde{e}_3$, $\tilde{e}_3$, and $\tilde{e}_3$ are satisfied. Finally, the fixed payments T_i do not affect vendor effort and can therefore be chosen such that the vendor participation constraints are tight. So again linear contracts can replicate the result of any optimal contract, which means that linear contracts are optimal.

Our next task is to show that the optimal linear contract's performance cannot be the client's first-best result.

Effort choice. The FOCs for effort spent on the outsourced tasks, as defined in equations (18) and (19), are

$$\tilde{e}_1 = \arg\max_{e_1} E[f_1(S)\mid e_1,\tilde{e}_2,\tilde{e}_3] - c_1(e_1)$$

$$\tilde{e}_2 = \arg\max_{e_2} E[f_2(S)\mid \tilde{e}_1,e_2,\tilde{e}_3] - c_2(e_2)$$

$$\tilde{e}_1 = \arg\max_{e_1} T + \alpha_1[\phi(e_1+\gamma e_2+\lambda e_3)] - \frac{e_1^2}{2} = \alpha_1\phi \tag{L3.4}$$

$$\tilde{e}_2 = \arg\max_{e_2} T + \alpha_2[\phi(e_1+\gamma e_2+\lambda e_3)] - \frac{e_2^2}{2} = \gamma\alpha_2\phi \tag{L3.5}$$

From equations (20) and (21) it follows that $E[f_1(S)\mid \tilde{e}_1,\tilde{e}_2,\tilde{e}_3] - c_1(\tilde{e}_1) \ge 0$ and $E[f_2(S)\mid \tilde{e}_1,\tilde{e}_2,\tilde{e}_3] - c_2(\tilde{e}_2) \ge 0$. Hence the client will set T_1 and T_2 such that $E[f_1(S)\mid \tilde{e}_1,\tilde{e}_2,\tilde{e}_3] - c_1(\tilde{e}_1) = 0$ and $E[f_2(S)\mid \tilde{e}_1,\tilde{e}_2,\tilde{e}_3] - c_2(\tilde{e}_2) = 0$, thereby making the vendor's participation constraint tight and extracting all the surplus. Then

$$T_1 + \alpha_1 E[S] - c_1(\tilde{e}_1) = 0 \quad \text{and} \quad T_2 + \alpha_2 E[S] - c_2(\tilde{e}_2) = 0$$

We can now conclude from equation (17) that

$$\tilde{e}_3 = \arg\max_{e_3\ge 0} E[v(\tilde{e}_1,\tilde{e}_2,e_3)] - c_3(e_3) - E[f_1(S)\mid(\tilde{e}_1,\tilde{e}_2,\tilde{e}_3)] - E[f_2(S)\mid(\tilde{e}_1,\tilde{e}_2,\tilde{e}_3)]$$

Therefore,

$$\begin{aligned}\tilde{e}_3 = \arg\max_{e_3\ge 0}\ &\phi[\tilde{e}_1 + \{\theta+(1-\theta)\gamma\}\tilde{e}_2 + \{\theta+(1-\theta)\lambda - \lambda(\alpha_1+\alpha_2)\}e_3] \\ &-\frac{e_3^2}{2} - \frac{\tilde{e}_1^2}{2} - \frac{\tilde{e}_2^2}{2} + \lambda\phi(\alpha_1+\alpha_2)\tilde{e}_3 = \phi[\theta+(1-\theta)\lambda - \lambda(\alpha_1+\alpha_2)]\end{aligned} \tag{L3.6}$$

Contract design. According to equation (16), the client's contract design problem can be stated as

$$\max_{f(\cdot)} \Pi_{\text{MS}} = E[v(\tilde{e}_1,\tilde{e}_2,\tilde{e}_3)] - c_3(\tilde{e}_3) - E[f_1(S)\mid(\tilde{e}_1,\tilde{e}_2,\tilde{e}_3)] - E[f_2(S)\mid(\tilde{e}_1,\tilde{e}_2,\tilde{e}_3)]$$

$$\max_{\alpha_1,\alpha_2} \Pi_{\text{MS}} = \phi[\tilde{e}_1 + \{\theta+(1-\theta)\gamma\}\tilde{e}_2 + \{\theta+(1-\theta)\lambda\}\tilde{e}_3] - \frac{\tilde{e}_3^2}{2} - \frac{\tilde{e}_1^2}{2} - \frac{\tilde{e}_2^2}{2}$$

since $E[f_1(S) \mid (\tilde{e}_1, \tilde{e}_2, \tilde{e}_3)] = \frac{\tilde{e}_1^2}{2}$ and $E[f_2(S) \mid (\tilde{e}_1, \tilde{e}_2, \tilde{e}_3)] = \frac{\tilde{e}_2^2}{2}$. Substituting the values of $\tilde{e}_1$, $\tilde{e}_2$, and $\tilde{e}_3$ from equations (L3.4)–(L3.6) into the contract design problem now gives

$$\max_{\alpha_1,\alpha_2 \geq 0} \Pi_{\mathrm{MS}} = \phi^2[\alpha_1 + \gamma\alpha_2\{\theta + (1-\theta)\gamma\} + \{\theta + (1-\theta)\lambda\}\{\theta + (1-\theta)\lambda - \lambda(\alpha_1 + \alpha_2)\} - \frac{\{\theta + (1-\theta)\lambda - \lambda(\alpha_1 + \alpha_2)\}^2}{2} - \frac{{\alpha_1}^2}{2} - \frac{{\alpha_2}^2\gamma^2}{2}]$$

which is a concave function in α_1 and α_2. The FOC for the client's contract design problem yields the following three cases.

Case (i) If $0 \leq \gamma\{\theta + (1-\theta)\gamma\} < \frac{\lambda^2}{1+\lambda^2}$, then $\alpha_2 = 0$, $\alpha_1 = \frac{1}{1+\lambda^2}$, and $\Pi_{\mathrm{MS}} = \phi^2\left[\frac{\{\theta+(1-\theta)\lambda\}^2}{2} + \frac{1}{2(1+\lambda^2)}\right]$.

In this case it is easy to see that $\tilde{e}_2 = 0$, from which it follows that the client does not attain its first-best outcome.

Case (ii) If $\frac{\lambda^2}{1+\lambda^2} \leq \gamma\{\theta + (1-\theta)\gamma\} < 1 + \frac{\gamma^2}{\lambda^2}$, then $\alpha_1 = \frac{\gamma^2+\lambda^2-\lambda^2\gamma\{\theta+(1-\theta)\gamma\}}{\gamma^2+\lambda^2+\gamma^2\lambda^2}$, $\alpha_2 = \frac{(1+\lambda^2)\gamma\{\theta+(1-\theta)\gamma\}-\lambda^2}{\gamma^2+\lambda^2+\gamma^2\lambda^2}$, and

$$\Pi_{\mathrm{MS}} = \phi^2[\alpha_1 + \gamma\alpha_2\{\theta + (1-\theta)\gamma\} + \{\theta + (1-\theta)\lambda\}\{\theta + (1-\theta)\lambda - \lambda(\alpha_1 + \alpha_2)\} - \frac{\{\theta + (1-\theta)\lambda - \lambda(\alpha_1 + \alpha_2)\}^2}{2} - \frac{\alpha_1^2}{2} - \frac{\alpha_2^2\gamma^2}{2}]$$

$$= \phi^2\left[\frac{\{\theta+(1-\theta)\lambda\}^2}{2} + \frac{\gamma^2+\lambda^2-2\lambda^2\gamma\{\theta+(1-\theta)\gamma\}+(1+\lambda^2)[\gamma\{\theta+(1-\theta)\gamma\}]^2}{2(\gamma^2+\lambda^2+\gamma^2\lambda^2)}\right].$$

Substituting the expressions of α_1 and α_2 into the values of $\tilde{e}_1$, $\tilde{e}_2$, and $\tilde{e}_3$, we obtain

$$\tilde{e}_1 = \phi\left[\frac{\gamma^2+\lambda^2-\lambda^2\gamma\{\theta+(1-\theta)\gamma\}}{\gamma^2+\lambda^2+\gamma^2\lambda^2}\right], \quad \tilde{e}_2 = \phi\left[\frac{\gamma[(1+\lambda^2)\gamma\{\theta+(1-\theta)\gamma\}-\lambda^2]}{\gamma^2+\lambda^2+\gamma^2\lambda^2}\right]$$

$$\text{and} \quad \tilde{e}_3 = \phi\left[\theta + (1-\theta)\lambda - \lambda\left\{\frac{\gamma^2+\lambda^2-\lambda^2\gamma\{\theta+(1-\theta)\gamma\}}{\gamma^2+\lambda^2+\gamma^2\lambda^2} + \frac{(1+\lambda^2)\gamma\{\theta+(1-\theta)\gamma\}-\lambda^2}{\gamma^2+\lambda^2+\gamma^2\lambda^2}\right\}\right]$$

We know that the coordinated solution is $e_1^* = \phi$, $e_2^* = \phi[\theta + (1-\theta)\gamma]$, and $e_3^* = \phi[\theta + (1-\theta)\lambda]$. It is now trivial to deduce that the client's first-best effort in the multisourcing case with integrated tasks is less than in the coordinated solution.

Case (iii) If $\gamma\{\theta + (1-\theta)\gamma\} > 1 + \frac{\gamma^2}{\lambda^2}$, then $\alpha_1 = 0$, $\alpha_2 = \frac{\gamma\{\theta+(1-\theta)\gamma\}}{\gamma^2+\lambda^2}$, and $\Pi_{\mathrm{MS}} = \phi^2\left[\frac{\{\theta+(1-\theta)\lambda\}^2}{2} + \frac{[\gamma\{\theta+(1-\theta)\gamma\}]^2}{2(\gamma^2+\lambda^2)}\right]$.

In this case it trivially follows that $\tilde{e}_1 = 0$, so again the client does not attain its first-best outcome.

Proof of Proposition 3. We shall compare the profits resulting the single-sourcing and multisourcing strategies when tasks are interdependent.

Single-sourcing. From the proof of Lemma 2 we know that the client's profit under the SS strategy is

$$\Pi_{\mathrm{SS}}^* = \phi^2\left[\frac{\{\theta + (1-\theta)\lambda\}^2}{2} + \frac{[1 + \gamma\{\theta + (1-\theta)\gamma\}]^2}{2(1 + \gamma^2 + \lambda^2)}\right]$$

By the proof of Lemma 3, the client's profit under the MS strategy is

$$\Pi_{\mathrm{MS}} = \phi^2[\alpha_1 + \gamma\alpha_2\{\theta + (1-\theta)\gamma\} + \{\theta + (1-\theta)\lambda\}\{\theta + (1-\theta)\lambda - \lambda(\alpha_1 + \alpha_2)\} - \frac{\{\theta + (1-\theta)\lambda - \lambda(\alpha_1 + \alpha_2)\}^2}{2} - \frac{\alpha_1^2}{2} - \frac{\alpha_2^2\gamma^2}{2}]$$

Note that if $\gamma\{\theta + (1-\theta)\gamma\} < \frac{\lambda^2}{1+\lambda^2}$ then $\alpha_2 = 0$ and if $\gamma\{\theta + (1-\theta)\gamma\} > 1 + \frac{\gamma^2}{\lambda^2}$ then $\alpha_1 = 0$. Therefore, under multisourcing we obtain the following results:

Case (i) If $0 \le \gamma\{\theta + (1-\theta)\gamma\} < \frac{\lambda^2}{1+\lambda^2}$, then $\alpha_2 = 0$, $\alpha_1 = \frac{1}{1+\lambda^2}$, and $\Pi_{\text{MS}} = \phi^2\left[\frac{\{\theta+(1-\theta)\lambda\}^2}{2} + \frac{1}{2(1+\lambda^2)}\right]$.

Case (ii) If $\frac{\lambda^2}{1+\lambda^2} \le \gamma\{\theta + (1-\theta)\gamma\} < 1 + \frac{\gamma^2}{\lambda^2}$, then $\alpha_1 = \frac{\gamma^2+\lambda^2-\lambda^2\gamma\{\theta+(1-\theta)\gamma\}}{\gamma^2+\lambda^2+\gamma^2\lambda^2}$ and $\alpha_2 = \frac{(1+\lambda^2)\gamma\{\theta+(1-\theta)\gamma\}-\lambda^2}{\gamma^2+\lambda^2+\gamma^2\lambda^2}$. Hence

$$\Pi_{\text{MS}} = \phi^2[\alpha_1 + \gamma\alpha_2\{\theta + (1-\theta)\gamma\} + \{\theta + (1-\theta)\lambda\}\{\theta + (1-\theta)\lambda - \lambda(\alpha_1 + \alpha_2)\} - \frac{\{\theta+(1-\theta)\lambda-\lambda(\alpha_1+\alpha_2)\}^2}{2} - \frac{\alpha_1^2}{2} - \frac{\alpha_2^2\gamma^2}{2}]$$
$$= \phi^2\left[\frac{\{\theta+(1-\theta)\lambda\}^2}{2} + \frac{\gamma^2+\lambda^2-2\lambda^2\gamma\{\theta+(1-\theta)\gamma\}+(1+\lambda^2)[\gamma\{\theta+(1-\theta)\gamma\}]^2}{2(\gamma^2+\lambda^2+\gamma^2\lambda^2)}\right].$$

Case (iii) If $\gamma\{\theta + (1-\theta)\gamma\} > 1 + \frac{\gamma^2}{\lambda^2}$, then $\alpha_1 = 0$, $\alpha_2 = \frac{\gamma\{\theta+(1-\theta)\gamma\}}{\gamma^2+\lambda^2}$, and $\Pi_{\text{MS}} = \phi^2\left[\frac{\{\theta+(1-\theta)\lambda\}^2}{2} + \frac{[\gamma\{\theta+(1-\theta)\gamma\}]^2}{2(\gamma^2+\lambda^2)}\right]$.

A numerical comparison of the SS- and MS-based profits under different values of θ, γ, and λ now yields the results in the proposition. (These comparisons are plotted in Figure 2 of the main text.)

Proof of Proposition 4. Here we consider only the case when tasks are modular. Also, for this proof we normalize ϕ to 1; doing so does not affect the analysis because it merely acts as a scaling factor in our model.

Single-sourcing. We can state the client's contract problem under SS as follows, where "CE" denotes "certainty equivalent":

$$\max_{f(\cdot)} \Pi_{\text{SS}} = E[v(\tilde{e}_1, \tilde{e}_2, \tilde{e}_3)] - c_3(\tilde{e}_3) - E[f(S) \mid (\tilde{e}_1, \tilde{e}_2)] \tag{P4.1}$$

subject to

$$\tilde{e}_3 = \arg\max_{e_3 \ge 0} E[v(\tilde{e}_1, \tilde{e}_2, e_3)] - c_3(e_3) - E[f(S) \mid (\tilde{e}_1, \tilde{e}_2)], \tag{P4.2}$$

$$\tilde{e}_1, \tilde{e}_2 = \arg\max_{e_1, e_2 \ge 0} \text{CE}[f(S) \mid (e_1, e_2)] - c_1(e_1) - c_2(e_2), \tag{P4.3}$$

$$\text{and} \quad \text{CE}[f(S) \mid (\tilde{e}_1, \tilde{e}_2)] - c_1(\tilde{e}_1) - c_2(\tilde{e}_2) \ge 0. \tag{P4.4}$$

The client is risk neutral and so takes only the *expected value* of the contract into account; in contrast, the vendor is risk averse and therefore, when making its decisions, accounts instead for the *certainty equivalent* of the contract. We first use a CARA model to derive the form of the certainty equivalent for a risk utility function, in which case the uncertainty ε_2 is normally distributed. For the CARA model, $U(x) = 1 - e^{-rx}$, where r is the absolute coefficient of risk aversion. Because the verifiable signal is of the form $S = e_1 + \gamma e_2 + \lambda e_3 + \varepsilon_2$, we seek the certainty equivalent of a general signal of the type $S = A + \varepsilon_2$. Let CE(S) denote the certainty equivalent of signal S. Then

$$\begin{aligned}
1 - e^{-r\,\text{CE}(S)} &= \int_{-\infty}^{\infty}\{1 - e^{-r(A+\varepsilon_2)}\}\frac{1}{\sqrt{2\pi}\sigma}e^{-\varepsilon_2^2/2\sigma^2}\,d\varepsilon_2 \\
&= 1 - e^{-rA}\int_{-\infty}^{\infty}\{e^{-r\varepsilon_2}\}\frac{1}{\sqrt{2\pi}\sigma}e^{-\varepsilon_2^2/2\sigma^2}\,d\varepsilon_2 5 \\
&= 1 - e^{-rA}\int_{-\infty}^{\infty}\frac{1}{\sqrt{2\pi}\sigma}e^{-\varepsilon_2^2/2\sigma^2 - r\varepsilon_2 - \frac{r^2\sigma^4}{2\sigma^2} + \frac{r^2\sigma^4}{2\sigma^2}}\,d\varepsilon_2 \\
&= 1 - e^{-rA + \frac{r^2\sigma^4}{2\sigma^2}}\int_{-\infty}^{\infty}\frac{1}{\sqrt{2\pi}\sigma}e^{-(\varepsilon_2 + r\sigma^2)^2/2\sigma^2}\,d\varepsilon_2
\end{aligned}$$

Yet because $\int_{-\infty}^{\infty}\frac{1}{\sqrt{2\pi}\sigma}e^{-(\varepsilon_2+r\sigma^2)^2/2\sigma^2}\,d\varepsilon_2 = 1$, it follows that

$$r\,\text{CE}(S) = rA - \frac{r^2\sigma^2}{2} \implies \text{CE}(S) = A - \frac{r\sigma^2}{2}$$

.

If tasks are modular, then $S = (e_1 + \gamma e_2) + \varepsilon_2$ because project outcomes do not depend on client effort ($\lambda = 0$). We shall focus on linear contracts of the form $T + \alpha S$.

Effort choice. The FOCs for devoting first-best efforts to the outsourced tasks are

$$\tilde{e}_1 = \arg\max_{e_1} \text{CE}[f(S) \mid e_1, e_2] - c(e_1) - c(e_2)$$
$$= \arg\max_{e_1} T + \alpha[e_1 + \gamma e_2] - \frac{r\sigma^2\alpha^2}{2} - \frac{e_1^2}{2} - \frac{e_2^2}{2} = \alpha$$

and, similarly,

$$\tilde{e}_2 = \alpha\gamma$$

.

Here the participation constraint is expressed as $\text{CE}[f(S) \mid \tilde{e}_1, \tilde{e}_2] - c(\tilde{e}_1) - c(\tilde{e}_2) \geq 0$ and so the client will set T such that $\text{CE}[f(S) \mid \tilde{e}_1, \tilde{e}_2] - c(\tilde{e}_1) - c(\tilde{e}_2) = 0$, which makes the vendor's participation constraint tight and also extracts all the surplus. Therefore,

$$T + \alpha E[S] - \frac{r\sigma^2\alpha^2}{2} - c(\tilde{e}_1) - c(\tilde{e}_2) = 0 \quad \text{and}$$
$$\tilde{e}_3 = \arg\max_{e_3 \geq 0} E[v(\tilde{e}_1, \tilde{e}_2, e_3)] - c_3(e_3) - E[f(S) \mid (\tilde{e}_1, \tilde{e}_2)]$$

It follows that

$$\tilde{e}_3 = \arg\max_{e_3 \geq 0} \tilde{e}_1 + \{\theta + (1-\theta)\gamma\}\tilde{e}_2 + \{\theta + (1-\theta)\lambda\}e_3 - \frac{e_3^2}{2} - \frac{\tilde{e}_1^2}{2} - \frac{\tilde{e}_2^2}{2} - \frac{r\sigma^2\alpha^2}{2}$$
$$= \theta + (1-\theta)\lambda$$

<u>Contract design</u>. According to equation (P4.1), the contract design problem can be stated as

$$\max_{f(\cdot)} \Pi_{\text{SS}} = E[v(\tilde{e}_1, \tilde{e}_2, \tilde{e}_3)] - c_3(\tilde{e}_3) - E[f(S) \mid (\tilde{e}_1, \tilde{e}_2)]$$

$$\text{Max}_\alpha \Pi_{\text{SS}} = \tilde{e}_1 + \{\theta + (1-\theta)\gamma\}\tilde{e}_2 + \{\theta + (1-\theta)\lambda\}\tilde{e}_3 - \frac{\tilde{e}_3^2}{2} - \frac{\tilde{e}_1^2}{2} - \frac{\tilde{e}_2^2}{2} - \frac{r\sigma^2\alpha^2}{2}$$

since $E[f(S) \mid (\tilde{e}_1, \tilde{e}_2, \tilde{e}_3)] = \frac{\tilde{e}_1^2}{2} + \frac{\tilde{e}_2^2}{2}$. Substituting the values of $\tilde{e}_1$, $\tilde{e}_2$, and $\tilde{e}_3$ into the contract design problem then yields

$$\max_\alpha \Pi_{\text{SS}} = \alpha + \gamma\alpha\{\theta + (1-\theta)\gamma\} + \{\theta + (1-\theta)\lambda\}^2$$
$$- \frac{\{\theta + (1-\theta)\lambda\}^2}{2} - \frac{\alpha^2}{2} - \frac{\alpha^2\gamma^2}{2} - \frac{r\sigma^2\alpha^2}{2}$$

which is a concave function in α. The FOC for the contract design problem now gives us

$$\alpha = \frac{1 + \gamma\{\theta + (1-\theta)\gamma\}}{1 + \gamma^2 + r\sigma^2}$$

and the firm's profits under SS are given by

$$\Pi_{\text{SS}} = \frac{[\theta + (1-\theta)\lambda]^2}{2} + \frac{[1 + \gamma\{\theta + (1-\theta)\gamma\}]^2}{2[1 + \gamma^2 + r\sigma^2]}$$

Multisourcing. The client's contract problem in the MS case can be stated as

$$\max_{f_i(\cdot)} \Pi_{\text{MS}} = E[v(\tilde{e}_1, \tilde{e}_2, \tilde{e}_3)] - c_3(\tilde{e}_3) - E[f_1(S) \mid (\tilde{e}_1, \tilde{e}_2)] - E[f_2(S) \mid (\tilde{e}_1, \tilde{e}_2)] \quad \text{(P4.5)}$$

subject to the following conditions:

$$\tilde{e}_3 = \arg\max_{e_3 \geq 0} E[v(\tilde{e}_1, \tilde{e}_2, e_3)] - c_3(e_3) - E[f_1(S) \mid (\tilde{e}_1, \tilde{e}_2)] - E[f_2(S) \mid (\tilde{e}_1, \tilde{e}_2)] \quad \text{(P4.6)}$$

$$\tilde{e}_1 = \arg\max_{e_1 \geq 0} \text{CE}[f_1(S) \mid (e_1, \tilde{e}_2)] - c_1(e_1) \quad \text{(P4.7)}$$

$$\tilde{e}_2 = \arg\max_{e_2 \geq 0} \text{CE}[f_2(S) \mid (\tilde{e}_1, e_2)] - c_2(e_2) \quad \text{(P4.8)}$$

$$\text{CE}[f_1(S) \mid (\tilde{e}_1, \tilde{e}_2)] - c_1(\tilde{e}_1) \geq 0 \quad \text{(P4.9)}$$

$$\text{CE}[f_2(S) \mid (\tilde{e}_1, \tilde{e}_2)] - c_2(\tilde{e}_2) \geq 0 \quad \text{(P4.10)}$$

Effort choice. The FOCs for the first-best efforts on the outsourced tasks are

$$\tilde{e}_1 = \arg\max_{e_1} \text{CE}[f_1(S) \mid e_1, \tilde{e}_2] - c_1(e_1)$$

$$\tilde{e}_2 = \arg\max_{e_2} \text{CE}[f_2(S) \mid \tilde{e}_1, e_2] - c_2(e_2)$$

therefore,

$$\tilde{e}_1 = \arg\max_{e_1} T + \alpha_1(e_1 + \gamma e_2) - \frac{r\sigma^2 {\alpha_1}^2}{2} - \frac{e_1^2}{2} = \alpha_1$$

$$\tilde{e}_2 = \arg\max_{e_2} T + \alpha_2(e_1 + \gamma e_2) - \frac{r\sigma^2 {\alpha_2}^2}{2} - \frac{e_2^2}{2} = \gamma\alpha_2$$

Here the participation constraints are $\text{CE}[f_1(S) \mid \tilde{e}_1, \tilde{e}_2] - c_1(\tilde{e}_1) \geq 0$ and $\text{CE}[f_2(S) \mid \tilde{e}_1, \tilde{e}_2] - c_2(\tilde{e}_2) \geq 0$. The client will set T_1 and T_2 such that $\text{CE}[f_1(S) \mid \tilde{e}_1, \tilde{e}_2] - c_1(\tilde{e}_1) = 0$ and $\text{CE}[f_2(S) \mid \tilde{e}_1, \tilde{e}_2] - c_2(\tilde{e}_2) = 0$, thus making the vendor's participation constraint tight and extracting all the surplus. Hence

$$T_1 + \alpha_1 E[S] - \frac{r\sigma^2 {\alpha_1}^2}{2} - c_1(\tilde{e}_1) = 0 \quad \text{and} \quad T_2 + \alpha_2 E[S] - \frac{r\sigma^2 {\alpha_2}^2}{2} - c_2(\tilde{e}_2) = 0$$

from which we conclude that

$$\begin{aligned}\tilde{e}_3 &= \arg\max_{e_3 \geq 0} E[v(\tilde{e}_1, \tilde{e}_2, e_3)] - c_3(e_3) - E[f_1(S) \mid (\tilde{e}_1, \tilde{e}_2)] - E[f_2(S) \mid (\tilde{e}_1, \tilde{e}_2)] \\ &= \arg\max_{e_3 \geq 0} \tilde{e}_1 + \{\theta + (1-\theta)\gamma\}\tilde{e}_2 + \{\theta + (1-\theta)\lambda\}e_3 \\ &\quad - \frac{e_3^2}{2} - \frac{\tilde{e}_1^2}{2} - \frac{\tilde{e}_2^2}{2} - \frac{r\sigma^2 {\alpha_1}^2}{2} - \frac{r\sigma^2 {\alpha_2}^2}{2} = \theta + (1-\theta)\lambda\end{aligned}$$

Contract design. The client's contract design problem is

$$\max_{f(\cdot)} \Pi_{\text{MS}} = E[v(\tilde{e}_1, \tilde{e}_2, \tilde{e}_3)] - c_3(\tilde{e}_3) - E[f_1(S) \mid (\tilde{e}_1, \tilde{e}_2)] - E[f_2(S) \mid (\tilde{e}_1, \tilde{e}_2)]$$

$$\max_{\alpha_1, \alpha_2} \Pi_{\text{MS}} = \tilde{e}_1 + \{\theta + (1-\theta)\gamma\}\tilde{e}_2 + \{\theta + (1-\theta)\lambda\}\tilde{e}_3 - \frac{\tilde{e}_3^2}{2} - \frac{\tilde{e}_1^2}{2} - \frac{\tilde{e}_2^2}{2} - \frac{r\sigma^2\alpha_1^2}{2} - \frac{r\sigma^2\alpha_2^2}{2}$$

the reason is that $E[f_1(S) \mid (\tilde{e}_1, \tilde{e}_2)] = \frac{\tilde{e}_1^2}{2} + \frac{r\sigma^2\alpha_1^2}{2}$ and $E[f_2(S) \mid (\tilde{e}_1, \tilde{e}_2)] = \frac{\tilde{e}_2^2}{2} + \frac{r\sigma^2\alpha_2^2}{2}$. Substituting the values of $\tilde{e}_1$, $\tilde{e}_2$, and $\tilde{e}_3$ into the contract design problem now yields

$$\begin{aligned}\max_{\alpha_1, \alpha_2 \geq 0} \Pi_{\text{MS}} &= \alpha_1 + \gamma\alpha_2\{\theta + (1-\theta)\gamma\} + \{\theta + (1-\theta)\lambda\}^2 \\ &\quad - \frac{\{\theta + (1-\theta)\lambda\}^2}{2} - \frac{{\alpha_1}^2}{2} - \frac{{\alpha_2}^2\gamma^2}{2} - \frac{r\sigma^2{\alpha_1}^2}{2} - \frac{r\sigma^2{\alpha_2}^2}{2}\end{aligned}$$

which is a concave function in α_1 and α_2. By the FOC for the contract design problem, $\alpha_1 = \frac{1}{1+r\sigma^2}$ and $\alpha_2 = \frac{\{\theta+(1-\theta)\gamma\}\gamma}{\gamma^2+r\sigma^2}$. We can see that the first-best outcomes are not attained under the multisourcing of modular tasks if vendors are risk averse:

$$\Pi_{\text{MS}} = \frac{[\theta + (1-\theta)\lambda]^2}{2} + \frac{1}{2(1+r\sigma^2)} + \frac{\gamma^2\{\theta + (1-\theta)\gamma\}^2}{2(\gamma^2 + r\sigma^2)}$$

Comparing profits from the SS and MS of modular tasks under risk aversion. We are now in a position to compare the profits from single-sourcing and multisourcing. Thus,

$$\Pi_{SS} = \frac{[\theta+(1-\theta)\lambda]^2}{2} + \frac{[1+\gamma\{\theta+(1-\theta)\gamma\}]^2}{2[1+\gamma^2+r\sigma^2]} \quad \text{and} \quad \Pi_{MS} = \frac{[\theta+(1-\theta)\lambda]^2}{2} + \frac{1}{2(1+r\sigma^2)} + \frac{\gamma^2\{\theta+(1-\theta)\gamma\}^2}{2(\gamma^2+r\sigma^2)}$$

These equations confirm our expectations that SS and MS strategies both have lower profits when vendors are risk averse.

We next compare the relative efficacy of these two sourcing strategies as follows:

$$\Pi_{MS} - \Pi_{SS} = \frac{1}{2}\left[\frac{1}{(1+r\sigma^2)} + \frac{\gamma^2\{\theta+(1-\theta)\gamma\}^2}{(\gamma^2+r\sigma^2)} - \frac{[1+\gamma\{\theta+(1-\theta)\gamma\}]^2}{[1+\gamma^2+r\sigma^2]}\right]$$

Put $\zeta = \gamma\{\theta + (1-\theta)\gamma\}$. Then the preceding equation can be rewritten as

$$\Pi_{MS} - \Pi_{SS} = \frac{1}{2}\left[\frac{1}{(1+r\sigma^2)} + \frac{\gamma^2\zeta^2}{(\gamma^2+r\sigma^2)} - \frac{[1+\gamma\zeta]^2}{[1+\gamma^2+r\sigma^2]}\right]$$

and further simplification yields

$$\Pi_{MS} - \Pi_{SS} = \frac{1}{2[1+\gamma^2+r\sigma^2]}\left[\frac{\gamma^2}{(1+r\sigma^2)} + \frac{\gamma^2\zeta^2}{(\gamma^2+r\sigma^2)} - 2\gamma\zeta\right]$$

From the term in brackets, it trivially follows that there exist $\overline{r\sigma^2} > 0$ such that $\Pi_{MS} - \Pi_{SS} \geq 0$ for all $r\sigma^2 \leq \overline{r\sigma^2}$ and $\Pi_{MS} - \Pi_{SS} < 0$ for all $r\sigma^2 > \overline{r\sigma^2}$. Here $\overline{r\sigma^2}$ is the positive root of the quadratic equation (with $r\sigma^2$ as the variable) $\left[\frac{\gamma^2}{1+r\sigma^2} + \frac{\gamma^2\zeta^2}{\gamma^2+r\sigma^2} - 2\gamma\zeta\right] = 0$.

Proof of Proposition 5. Proving this proposition will require that we compute the optimal efforts of the vendor(s) as a simultaneous effort decision, since their costs of coordination depend on the efforts exerted on both tasks. We first compute the first-best efforts with inter-dependent costs and modular tasks. We normalize $\phi = 1$ to simplify the calculations; this has no effect on the insights that we derive.

Modular tasks. The coordinated firm solves the problem

$$\max_{e_1,e_2,e_3} e_1 + \{\theta + (1-\theta)\gamma\}e_2 + \{\theta + (1-\theta)\lambda\}e_3 - \frac{e_1^2}{2} - \frac{e_2^2}{2} - \frac{e_3^2}{2} - ae_1e_2$$

This function is concave with respect to effort, as can be verified from the Hessian. The FOCs for the first-best efforts are

$$1 = e_1^* + ae_2^*, \qquad \{\theta + (1-\theta)\gamma\} = e_2^* + ae_1^*, \qquad e_3^* = \{\theta + (1-\theta)\lambda\}$$

Solving the first two equations simultaneously gives the following coordinated solution:

$$e_1^* = \frac{1-a\{\theta+(1-\theta)\gamma\}}{1-a^2}, \qquad e_2^* = \frac{\{\theta+(1-\theta)\gamma\}-a}{1-a^2}, \qquad e_3^* = \{\theta + (1-\theta)\lambda\}$$

Depending on the relative values of γ and a, the coordinated firm may decide to invest in only one of tasks 1 and 2.

Case (i) If $0 < \{\theta + (1-\theta)\gamma\} < a$, then $e_1^* = 1$, $e_2^* = 0$, and $e_3^* = \{\theta + (1-\theta)\lambda\}$.

Case (ii) If $a < \{\theta + (1-\theta)\gamma\} < \frac{1}{a}$, then $e_1^* = \frac{1-a\{\theta+(1-\theta)\gamma\}}{1-a^2}$, $e_2^* = \frac{\{\theta+(1-\theta)\gamma\}-a}{1-a^2}$, and $e_3^* = \{\theta + (1-\theta)\lambda\}$. In this case, the firm invests effort on all three tasks.

Case (iii) If $\{\theta + (1-\theta)\gamma\} > \frac{1}{a}$, then $e_1^* = 0$, $e_2^* = \{\theta + (1-\theta)\gamma\}$, and $e_3^* = \{\theta + (1-\theta)\lambda\}$.

We now compare the efficacy of single- and multisourcing strategies when interdependent tasks are costly. We first compute vendor effort in the SS case.

Single-sourcing. The client's contract design problem can be stated as

$$\max_{f(\cdot)} \Pi_{SS} = E[v(\tilde{e}_1, \tilde{e}_2, \tilde{e}_3)] - c_3(\tilde{e}_3) - E[f(S) \mid (\tilde{e}_1, \tilde{e}_2)]$$

subject to

$$\tilde{e}_3 = \arg\max_{e_3 \geq 0} E[v(\tilde{e}_1, \tilde{e}_2, e_3)] - c_3(e_3) - E[f(S) \mid (\tilde{e}_1, \tilde{e}_2)]$$

$$\tilde{e}_1, \tilde{e}_2 = \arg\max_{e_1, e_2 \geq 0} E[f(S) \mid (e_1, e_2)] - c_1(e_1) - c_2(e_2) - ae_1e_2,$$
$$\text{and} \quad E[f(S) \mid (\tilde{e}_1, \tilde{e}_2)] - c_1(\tilde{e}_1) - c_2(\tilde{e}_2) - ae_1e_2 \geq 0$$

For the modular tasks case, we have $S = (e_1 + \gamma e_2) + \varepsilon_2$ and focus on linear contracts.

Effort choice. The FOCs for the first-best efforts on the outsourced tasks are

$$\tilde{e}_1 = \arg\max_{e_1} E[f(S) \mid e_1, e_2] - c(e_1) - c(e_2) - ae_1e_2$$
$$= \arg\max_{e_1} T + \alpha[(e_1 + \gamma e_2)] - \frac{e_1^2}{2} - \frac{e_2^2}{2} - ae_1e_2$$

and, similarly,

$$\tilde{e}_2 = \arg\max_{e_2} T + \alpha[(e_1 + \gamma e_2)] - \frac{e_1^2}{2} - \frac{e_2^2}{2} - ae_1e_2$$

and $\tilde{e}_3 = \theta$. The participation constraint is written as $E[f(S) \mid \tilde{e}_1, \tilde{e}_2] - c(\tilde{e}_1) - c(\tilde{e}_2) - a\tilde{e}_1\tilde{e}_2 \geq 0$. The client will set T such that $E[f(S) \mid \tilde{e}_1, \tilde{e}_2] - c(\tilde{e}_1) - c(\tilde{e}_2) - a\tilde{e}_1\tilde{e}_2 = 0$, making the vendor's participation constraint tight and extracting all the surplus. Therefore, $T + \alpha E[S] - a\tilde{e}_1\tilde{e}_2 - c(\tilde{e}_1) - c(\tilde{e}_2) = 0$.

So in order to see whether single-sourcing will attain the *client's* first-best *outcome*, we need only check for the existence of an α that can yield the *vendor's* first-best *efforts*. Given that the first-best efforts maximize the function $e_1 + \{\theta + (1-\theta)\gamma\}e_2 + \theta e_3 - \frac{e_1^2}{2} - \frac{e_2^2}{2} - \frac{e_3^2}{2} - ae_1e_2$, it is easy to see that SS will yield the first-best outcome for the client if and only if $\theta = 0$ or $\gamma = 1$.

Multisourcing. Because we assume that the cost of task interdependence is borne by the primary vendor (and thus we assume, without loss of generality, that the primary vendor performs the second task), the client's contract design problem can be stated as

$$\text{Max}_{f_i(\cdot)} \Pi_{MS} = E[v(\tilde{e}_1, \tilde{e}_2, \tilde{e}_3)] - c_3(\tilde{e}_3) - E[f_1(S) \mid (\tilde{e}_1, \tilde{e}_2)] - E[f_2(S) \mid (\tilde{e}_1, \tilde{e}_2)]$$

subject to the following conditions:

$$\tilde{e}_3 = \arg\max_{e_3 \geq 0} E[v(\tilde{e}_1, \tilde{e}_2, e_3)] - c_3(e_3) - E[f_1(S) \mid (\tilde{e}_1, \tilde{e}_2)] - E[f_2(S) \mid (\tilde{e}_1, \tilde{e}_2)]$$

$$\tilde{e}_1 = \arg\max_{e_1 \geq 0} E[f_1(S) \mid (e_1, \tilde{e}_2)] - c_1(e_1)$$

$$\tilde{e}_2 = \arg\max_{e_2 \geq 0} E[f_2(S) \mid (\tilde{e}_1, e_2)] - c_2(e_2) - ae_1e_2$$

$$E[f_1(S) \mid (\tilde{e}_1, \tilde{e}_2)] - c_1(\tilde{e}_1) \geq 0$$

$$E[f_2(S) \mid (\tilde{e}_1, \tilde{e}_2)] - c_2(\tilde{e}_2) - a\tilde{e}_1\tilde{e}_2 \geq 0$$

Effort choice. The FOCs for the first-best efforts on the outsourced tasks are

$$\tilde{e}_1 = \arg\max_{e_1} E[f_1(S) \mid e_1, e_2] - c(e_1) = \alpha_1, \text{ and } \tilde{e}_3 = \theta,$$

and, similarly, $\tilde{e}_2 = \max\{\alpha_2\gamma - a\tilde{e}_1, 0\} = (\alpha_2\gamma - a\alpha_1)^+$. Here the participation constraints are $E[f_1(S) \mid \tilde{e}_1, \tilde{e}_2] - c(\tilde{e}_1) \geq 0$ and $E[f_2(S) \mid \tilde{e}_1, \tilde{e}_2] - c(\tilde{e}_2) - a\tilde{e}_1\tilde{e}_2 \geq 0$. The client will set T_i such that the vendor's participation constraint is tight, thereby extracting all the surplus; hence $T_1 + \alpha_1 E[S] - c_1(\tilde{e}_1) = 0$ and $T_2 + \alpha_2\{S\} - a\tilde{e}_1\tilde{e}_2 - c_2(\tilde{e}_2) = 0$. Therefore, the client can attain its first-best outcome if it can set feasible values for α_1 and α_2 that also yield first-best efforts by vendors. We now demonstrate that the client can indeed set such values.

Case (i) If $0 < \{\theta + (1-\theta)\gamma\} < a$, then $e_1^* = 1$, $e_2^* = 0$, and $e_3^* = \theta$. In this case, the client can set $\alpha_2 = 0$ and $\alpha_1 = 1$ to attain the first-best outcome.

Case (ii) If $a < \{\theta + (1-\theta)\gamma\} < \frac{1}{a}$, then $e_1^* = \frac{1-a\{\theta+(1-\theta)\gamma\}}{1-a^2}$, $e_2^* = \frac{\{\theta+(1-\theta)\gamma\}-a}{1-a^2}$, and $e_3^* = \theta$. Now, setting the contract parameter values such that $\alpha_1 = \frac{1-a\{\theta+(1-\theta)\gamma\}}{1-a^2}$ and $\alpha_2 = \frac{\{\theta+(1-\theta)\gamma\}}{\gamma}$ results in the client attaining its first-best outcome.

Case (iii) If $\{\theta + (1-\theta)\gamma\} > \frac{1}{a}$, then $e_1^* = 0$, $e_2^* = \{\theta + (1-\theta)\gamma\}$, and $e_3^* = \theta$. Here the client can set $\alpha_2 = \frac{\{\theta+(1-\theta)\gamma\}}{\gamma}$ and $\alpha_1 = 0$ to attain the first-best outcome.

We therefore conclude that the multisourcing strategy attains the first-best outcome for the client.