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PRICE EFFECTS IN ONLINE PRODUCT REVIEWS: AN ANALYTICAL MODEL AND EMPIRICAL ANALYSIS

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Appendix A

Derivation of Optimal Price Functions for the Monopoly Setting

We apply backward induction to derive optimal price functions. In the second period, given first period price p_1, the firm selects second period price p_2 ($p_2 < Max\{q, R\}$) to maximize its second period profit:

$$\pi_2 = n\left((1-a)p_2\text{Max}\left\{0,\frac{q-p_2}{t}\right\} + ap_2\text{Max}\left\{0,\frac{\text{Max}\left\{0,\text{Min}\left\{1,q-b\left(p_1-\frac{q}{2}\right)\right\}\right\}-p_2}{t}\right\}\right)$$

The profit function can be reduced to four possibilities based on the value of p_1:

$$\pi_2 = \begin{cases} n\left((1-a)p_2\text{Max}\left\{0,\frac{q-p_2}{t}\right\} + ap_2\frac{1-p_2}{t}\right) & if\ 0 < p_1 < \frac{(2+b)q-2}{2b} \\ n\left((1-a)p_2\text{Max}\left\{0,\frac{q-p_2}{t}\right\} + ap_2\frac{q-b\left(p_1-\frac{q}{2}\right)-p_2}{t}\right) & if\ \frac{(2+b)q-2}{2b} \le p_1 < \frac{q}{2} \\ n\left((1-a)p_2\frac{q-p_2}{t} + ap_2\text{Max}\left\{0,\frac{q-b\left(p_1-\frac{q}{2}\right)-p_2}{t}\right\}\right) & if\ \frac{q}{2} \le p_1 \le \frac{(2+b)q}{2b} \\ n\left((1-a)p_2\frac{q-p_2}{t}\right) & if\ \frac{(2+b)q}{2b} < p_1 < q^e \end{cases}$$

By maximizing profit in each of the four cases, we can derive the optimal second period price p_2 as a function of first period price p_1:

$$p_2^*(p_1) = \begin{cases} \frac{a+(1-a)q}{2} & if\ 0 < p_1 < \frac{(2+b)q-2}{2b} \\ \frac{q}{2} + \frac{ab(q-2p_1^*)}{4} & if\ \frac{(2+b)q-2}{2b} < p_1 < \frac{(2-2\sqrt{1-a}+ab)q}{2ab} \\ \frac{q}{2} & if\ \frac{(2-2\sqrt{1-a}+ab)q}{2ab} < p_1 < q^e \end{cases}$$

The corresponding second period profit as a function of p_1 is

$$\pi_2^*(p_1) = \begin{cases} \frac{n(a+(1-a)q)^2}{4t} & if\ 0 < p_1 < \frac{(2+b)q-2}{2b} \\ \frac{n\left(2q+ab(q-2p_1)\right)^2}{16t} & if\ \frac{(2+b)q-2}{2b} < p_1 < \frac{(2-2\sqrt{1-a}+ab)q}{2ab} \\ \frac{n(1-a)q^2}{4t} & if\ \frac{(2-2\sqrt{1-a}+ab)q}{2ab} < p_1 < q^e \end{cases}$$

Back in the first period, given $\pi_2^*(p_1)$, the firm selects a first period price p_1 ($p_1 < q^e$) to maximize its total profit in both periods: $\frac{p_1(q^e - p_1)}{t} + \pi_2^*(p_1)$. By comparing optimal profit in different ranges of p_1, we can derive the optimal first period price for different values of q:

$$p_1^* = \begin{cases} \frac{q^e}{2} & \text{if } 0 < q \le \bar{Q}_1\ or\ \frac{2+bq^e}{2+b} \le q < 1 \\ \text{Max}\left\{0, \frac{(2+b)q-2}{2b}\right\} & \text{if Max}\left\{\bar{Q}_1, \text{Min}\left\{\bar{Q}_2, \frac{4\,q^e}{2\,a\,b\,n+a^2b^2\,n}\right\}\right\} \le q < \frac{2+bq^e}{2+b} \\ \frac{q^e}{2} - \frac{ab\left((2+ab)q-abq^e\right)n}{2(4-a^2b^2n)} & \text{if } \bar{Q}_1 < q < \text{Max}\left\{\bar{Q}_1, \text{Min}\left\{\bar{Q}_2, \frac{4\,q^e}{2\,a\,b\,n+a^2b^2\,n}\right\}\right\} \end{cases}$$

Combining p_1^* and $p_2^*(p_1)$, we can derive that $p_2^* = \begin{cases} \frac{a+(1-a)q}{2} & \text{if Max}\left\{\bar{Q}_2, \frac{2}{2+b}\right\} < q < 1 \\ \frac{q}{2} + \frac{ab(q-2p_1^*)}{4} & \text{if } \bar{Q}_1 < q < \text{Max}\left\{\bar{Q}_2, \frac{2}{2+b}\right\} \\ \frac{q}{2} & \text{if } 0 < q < \bar{Q}_1 \end{cases}$.

Appendix B

Derivation of Optimal Price Functions for the Duopoly Setting

We first utilize the case of $q_1 = \frac{1}{2}$ to explain in detail how to derive equilibrium prices and then follow the same procedure to solve equilibria for the other two cases ($q_1 = 1$ and $q_1 = 0$).

We apply backward induction to derive optimal price functions. In the second period, given the first period prices, p_{11} and p_{21} ($p_{j1} < q_j^e = \frac{1}{2}$), the ratings of the two products are $R_1 = Min\left\{1, Max\left\{0, \frac{3-4p_{11}}{4}\right\}\right\}$ and $R_2 = Min\left\{1, Max\left\{0, \frac{3q_2-2p_{21}}{2}\right\}\right\}$. Given $a = 1$, $b = 1$, $n = 3$, $t = \frac{1}{6}$, and $q_1^e = q_2^e = \frac{1}{2}$, if all second-period consumers purchase from one of the two firms, the second period profits are $\pi_{12} = 3\left(p_{12}\,Min\left\{1, Max\left\{0, 3\left(R_1 - R_2 - p_{12} + p_{22} + \frac{1}{6}\right)\right\}\right\}\right)$ and $\pi_{22} = 3\left(p_{22}\,Min\left\{1, Max\left\{0, 3\left(R_2 - R_1 - p_{22} + p_{12} + \frac{1}{6}\right)\right\}\right\}\right)$. If some second-period consumers expect negative utility from both firms and do not buy from either firm, the profit functions are $\pi_{12} = 3\left(p_{12}\,Min\left\{1, Max\left\{0, 6\left(R_1 - p_{12}\right)\right\}\right\}\right)$ and $\pi_{22} = 3\left(p_{22}\,Min\left\{1, Max\left\{0, 6\left(R_2 - p_{22}\right)\right\}\right\}\right)$. Then back in the first period, firms select p_{11} and p_{21} to maximize their total profits in both periods: $\pi_1 = 3p_{11}\,Min\left\{1, Max\left\{0, -p_{11} + p_{21} + \frac{1}{6}\right\}\right\} + \pi_{12}^*$ and

$\pi_2 = 3p_{21}Min\left\{1, Max\left\{0, -p_{21} + p_{11} + \frac{1}{6}\right\}\right\} + \pi_{22}^*$. It can be proved that in this scenario all of the second-period consumers will purchase one of the two products in equilibrium. Thus, firms' second period profit functions are:

$$\pi_{12} = 3\left(p_{12}Min\left\{1, 3\left(\tfrac{3-4p_{11}}{4} - Min\left\{1, Max\left\{0, \tfrac{3q_2-2p_{21}}{2}\right\}\right\} - p_{12} + p_{22} + \tfrac{1}{6}\right)\right\}\right),$$
$$\pi_{22} = 3\left(p_{22}Max\left\{0, 3\left(Min\left\{1, Max\left\{0, \tfrac{3q_2-2p_{21}}{2}\right\}\right\} - \tfrac{3-4p_{11}}{2} - p_{22} + p_{12} + \tfrac{1}{6}\right)\right\}\right).$$

We can then derive the optimal second period prices as functions of the first period prices:

$$p_{12}^*(p_{11}, p_{21}) = \begin{cases} 0 & if\ p_{11} + Min\left\{1, Max\left\{0, \frac{3q_2}{2} - p_{21}\right\}\right\} > \frac{5}{4} \\ \frac{5-4\left(p_{11}+Min\left\{1, Max\left\{0, \frac{3q_2-2p_{21}}{2}\right\}\right\}\right)}{3} & if \frac{1}{4} < p_{11} + Min\left\{1, Max\left\{0, \frac{3q_2}{2} - p_{21}\right\}\right\} < \frac{5}{4}, \\ \frac{7-12\left(p_{11}+Min\left\{1, Max\left\{0, \frac{3q_2-2p_{21}}{2}\right\}\right\}\right)}{12} & if\ p_{11} + Min\left\{1, Max\left\{0, \frac{3q_2}{2} - p_{21}\right\}\right\} < \frac{1}{4} \end{cases}$$

$$p_{22}^*(p_{11}, p_{21}) = \begin{cases} \frac{-11-12\left(p_{11}+Min\left\{1, Max\left\{0, \frac{3q_2-2p_{21}}{2}\right\}\right\}\right)}{12} & if\ p_{11} + Min\left\{1, Max\left\{0, \frac{3q_2}{2} - p_{21}\right\}\right\} > \frac{5}{4} \\ \frac{-1+4\left(p_{11}+Min\left\{1, Max\left\{0, \frac{3q_2-2p_{21}}{2}\right\}\right\}\right)}{12} & if \frac{1}{4} < p_{11} + Min\left\{1, Max\left\{0, \frac{3q_2}{2} - p_{21}\right\}\right\} < \frac{5}{4}. \\ 0 & if\ p_{11} + Min\left\{1, Max\left\{0, \frac{3q_2}{2} - p_{21}\right\}\right\} < \frac{1}{4} \end{cases}$$

The corresponding second period profits as functions of the first period prices thus are:

$$\pi_{12}^*(p_{11}, p_{21}) = \begin{cases} 0 & if\ p_{11} + Min\left\{1, Max\left\{0, \frac{3q_2}{2} - p_{21}\right\}\right\} > \frac{5}{4} \\ \frac{\left(\frac{5-4p_{11}}{4} - Min\left\{1, Max\left\{0, \frac{3q_2-2p_{21}}{2}\right\}\right\}\right)^2}{3} & if \frac{1}{4} < p_{11} + Min\left\{1, Max\left\{0, \frac{3q_2}{2} - p_{21}\right\}\right\} < \frac{5}{4}, \\ \frac{7-12\left(p_{11}+Min\left\{1, Max\left\{0, \frac{3q_2-2p_{21}}{2}\right\}\right\}\right)}{4} & if\ p_{11} + Min\left\{1, Max\left\{0, \frac{3q_2}{2} - p_{21}\right\}\right\} < \frac{1}{4} \end{cases}$$

$$\pi_{22}^*(p_{11}, p_{21}) = \begin{cases} \frac{-11-12\left(p_{11}+Min\left\{1, Max\left\{0, \frac{3q_2-2p_{21}}{2}\right\}\right\}\right)}{4} & if\ p_{11} + Min\left\{1, Max\left\{0, \frac{3q_2}{2} - p_{21}\right\}\right\} > \frac{5}{4} \\ \frac{\left(\frac{1-4p_{11}}{4} - Min\left\{1, Max\left\{0, \frac{3q_2-2p_{21}}{2}\right\}\right\}\right)^2}{3} & if \frac{1}{4} < p_{11} + Min\left\{1, Max\left\{0, \frac{3q_2}{2} - p_{21}\right\}\right\} < \frac{5}{4}. \\ 0 & if\ p_{11} + Min\left\{1, Max\left\{0, \frac{3q_2}{2} - p_{21}\right\}\right\} < \frac{1}{4} \end{cases}$$

Then back in the first period, firms select the first period prices to maximize their total profits in both periods:

$$\pi_1 = 3p_{11}Min\left\{1, Max\left\{0, -p_{11} + p_{21} + \tfrac{1}{6}\right\}\right\} + \pi_{12}^*(p_{11}, p_{21}),$$
$$\pi_2 = 3p_{21}Min\left\{1, Max\left\{0, -p_{21} + p_{11} + \tfrac{1}{6}\right\}\right\} + \pi_{22}^*(p_{11}, p_{21}).$$

By comparing profits in different ranges of p_{11} and p_{21}, we can derive the optimal first period prices for different values of q_2:

$$p_{11}^* = \begin{cases} \frac{1}{10} & if\ \frac{34}{45} < q_2 < 1 \\ \frac{3(3q_2-2)}{8} & if \frac{2}{3} < q_2 < \frac{34}{45} \\ 0 & if\ 0 < q_2 < \frac{2}{3} \end{cases}, \quad p_{21}^* = \begin{cases} \frac{2}{15} & if\ \frac{34}{45} < q_2 < 1 \\ \frac{3q_2-2}{2} & if \frac{2}{3} < q_2 < \frac{34}{45} \\ 0 & if \frac{1}{3} < q_2 < \frac{2}{3} \\ \frac{-3q_2+1}{4} & if \frac{2}{9} < q_2 < \frac{1}{3} \\ \frac{1}{12} & if\ 0 < q_2 < \frac{2}{9} \end{cases}.$$

Combining p_{11}^*, p_{21}^*, $p_{12}^*(p_{11}, p_{21})$, and $p_{22}^*(p_{11}, p_{21})$, we can derive that:

$$p_{12}^* = \begin{cases} \frac{1}{20} & if \frac{34}{45} < q_2 < 1 \\ \frac{8-9q_2}{24} & if \frac{2}{3} < q_2 < \frac{34}{45} \\ \frac{5-6q_2}{12} & if \frac{1}{3} < q_2 < \frac{2}{3} \\ \frac{1}{2} - \frac{3q_2}{4} & if \frac{2}{9} < q_2 < \frac{1}{3} \\ \frac{2}{3} - \frac{3q_2}{2} & if \frac{1}{18} < q_2 < \frac{2}{9} \\ \frac{7}{12} & if\ 0 < q_2 < \frac{1}{18} \end{cases}, \quad p_{22}^* = \begin{cases} \frac{17}{60} & if \frac{34}{45} < q_2 < 1 \\ \frac{3q_2}{8} & if \frac{2}{3} < q_2 < \frac{34}{45} \\ \frac{6q_2-1}{12} & if \frac{1}{3} < q_2 < \frac{2}{3} \\ -\frac{1}{6} + \frac{3q_2}{4} & if \frac{2}{9} < q_2 < \frac{1}{3} \\ 0 & if\ 0 < q_2 < \frac{2}{9} \end{cases}.$$

Following similar procedure, we can derive the optimal price functions for $q_1 = 1$:

$$p_{11}^* = \begin{cases} \frac{1}{6} & if \frac{7}{9} < q_2 < 1 \\ \frac{9q_2-5}{12} & if \frac{2}{3} < q_2 < \frac{7}{9} \\ \frac{1}{12} & if \frac{7}{12} < q_2 < \frac{2}{3} \\ \frac{13-18q_2}{30} & if \frac{4}{9} < q_2 < \frac{7}{12} \\ \frac{1}{6} & if\ 0 < q_2 < \frac{4}{9} \end{cases}, \quad p_{21}^* = \begin{cases} \frac{1}{6} & if \frac{7}{9} < q_2 < 1 \\ \frac{3q_2-2}{2} & if \frac{2}{3} < q_2 < \frac{7}{9} \\ 0 & if \frac{7}{12} < q_2 < \frac{2}{3} \\ \frac{7-12q_2}{10} & if \frac{4}{9} < q_2 < \frac{7}{12} \\ \frac{1}{6} & if\ 0 < q_2 < \frac{4}{9} \end{cases}.$$

$$p_{12}^* = \begin{cases} \frac{1}{6} & if \frac{2}{3} < q_2 < 1 \\ \frac{1-q_2}{2} & if \frac{7}{12} < q_2 < \frac{2}{3} \\ \frac{22-27q_2}{30} & if \frac{4}{9} < q_2 < \frac{7}{12} \\ 1 - \frac{3q_2}{2} & if\ 0 < q_2 < \frac{4}{9} \end{cases}, \quad p_{22}^* = \begin{cases} \frac{1}{6} & if \frac{2}{3} < q_2 < 1 \\ \frac{3q_2-1}{6} & if \frac{7}{12} < q_2 < \frac{2}{3} \\ \frac{9q_2-4}{10} & if \frac{4}{9} < q_2 < \frac{7}{12} \\ 0 & if\ 0 < q_2 < \frac{4}{9} \end{cases}.$$

Similarly, the optimal price functions for $q_1 = 0$ are

$$p_{11}^* = \begin{cases} \frac{1}{6} & if \frac{7}{9} < q_2 < 1 \\ \frac{9q_2-5}{12} & if \frac{2}{3} < q_2 < \frac{7}{9} \\ \frac{1}{12} & if \frac{2}{9\sqrt{6}} < q_2 < \frac{2}{3} \\ \frac{1}{6} & if\ 0 < q_2 < \frac{2}{9\sqrt{6}} \end{cases}, \quad p_{21}^* = \begin{cases} \frac{1}{6} & if \frac{7}{9} < q_2 < 1 \\ \frac{3q_2-2}{2} & if \frac{2}{3} < q_2 < \frac{7}{9} \\ 0 & if \frac{2}{9\sqrt{6}} < q_2 < \frac{2}{3} \\ \frac{1}{6} & if\ 0 < q_2 < \frac{2}{9\sqrt{6}} \end{cases}.$$

$$p_{12}^* = 0, \quad p_{22}^* = \begin{cases} \frac{5}{6} & if \frac{2}{3} < q_2 < 1 \\ \frac{3q_2}{2} - \frac{1}{6} & if \frac{2}{9} < q_2 < \frac{2}{3} \\ \frac{3q_2}{4} & if \frac{2}{9\sqrt{6}} < q_2 < \frac{2}{9} \\ 0 & if\ 0 < q_2 < \frac{2}{9\sqrt{6}} \end{cases}.$$

In the benchmark scenario, the firms select p_{11}, p_{21}, p_{12}, and p_{22} to maximize their total profits:

$$\pi_1 = 3p_{11} Min\left\{1, Max\left\{0, -p_{11} + p_{21} + \tfrac{1}{6}\right\}\right\} + 3\left(p_{12} Min\left\{1, Max\left\{0, 3\left(q_1 - q_2 - p_{12} + p_{22} + \tfrac{1}{6}\right)\right\}\right\}\right),$$

$$\pi_2 = 3p_{21} Min\left\{1, Max\left\{0, -p_{21} + p_{11} + \tfrac{1}{6}\right\}\right\} + 3\left(p_{22} Min\left\{1, Max\left\{0, 3\left(q_2 - q_1 = p_{22} + p_{12} + \tfrac{1}{6}\right)\right\}\right\}\right).$$

It can be shown that the optimal first period prices are both $\frac{1}{6}$, the second period prices are:

(1) If $q_1 = 1, p_{21}^* = \begin{cases} \frac{1}{6} + \frac{1-q_2}{3} & if\ \frac{1}{2} < q_2 < 1 \\ \frac{5}{6} - q_2 & if\ 0 < q_2 < \frac{1}{2} \end{cases}, p_{22}^* = \begin{cases} \frac{1}{6} - \frac{1-q_2}{3} & if\ \frac{1}{2} < q_2 < 1 \\ 0 & if\ 0 < q_2 < \frac{1}{2} \end{cases}$.

(2) If $q_1 = \frac{1}{2}, p_{21}^* = \frac{1}{6} + \frac{\frac{1}{2} - q_2}{3}, p_{22}^* = \frac{1}{6} - \frac{\frac{1}{2} - q_2}{3}$.

(3) If $q_1 = 0, p_{21}^* = 0, p_{22}^* = \begin{cases} \frac{q_2}{2} & if\ 0 < q_2 < \frac{1}{3} \\ q_2 - \frac{1}{6} & if\ \frac{1}{3} < q_2 < 1 \end{cases}$.