# Price Discrimination in E-Commerce? An Examination of Dynamic Pricing in Name-Your-Own Price Markets 

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## Appendix A

## Optimal Offers for Two-Offer Case for Market with Fixed Threshold Price

A buyer who can make a maximum of two offers maximizes the expected consumer surplus:

$$
\begin{equation*}
\max _{p_{0}, p_{1}} E C S_{0}=\frac{1}{U B-L B}\left[\left(W T P-p_{0}\right) \cdot\left(p_{0}-L B\right)+\delta_{B}^{1}\left(W T P-p_{1}\right) \cdot\left(p_{1}-p_{0}\right)\right] \tag{A1}
\end{equation*}
$$

The (unrestricted) optimization of equation (A1) for the two-offer model results in the following equations for the optimal first and second offers:

$$
\begin{aligned}
& \frac{d E C S_{0}}{d p_{0}}=W T P-2 p_{0}+L B-\delta_{B}^{1}\left(W T P-p_{1}\right) \stackrel{!}{=} 0 \Rightarrow 2 p_{0}=W T P+L B-\delta_{B}^{1}\left(W T P-p_{1}\right) \\
& \frac{d E C S_{0}}{d p_{1}}=\delta_{B}^{1}\left(W T P-2 p_{1}+p_{0}\right) \stackrel{!}{=} 0 \Rightarrow p_{1}=\frac{W T P+p_{0}}{2}
\end{aligned}
$$

Mutual insertion yields for the optimal first and second offer:

$$
p_{0}=\frac{W T P\left(1-\frac{1}{2} \delta_{B}\right)+L B}{\left(2-\frac{1}{2} \delta_{B}\right)}, p_{1}=\frac{W T P\left(3-\delta_{B}\right)+L B}{\left(4-\delta_{B}\right)}
$$

The model can be generalized to the $n$-offer case where the offers $p_{0}$ to $p_{n}$ are given by the following equations:

$$
\begin{aligned}
& p_{0}=\frac{L B+W T P-\delta_{B}\left(W T P-p_{1}\right)}{2} \\
& p_{j}=\frac{1}{2} p_{j-1}+\frac{1}{2} \delta_{B} p_{j+1}+\frac{W T P \cdot\left(1-\delta_{B}\right)}{2} \quad \text { for } 0<\mathrm{j}<\mathrm{n} \\
& p_{n}=\frac{p_{n-1}+W T P}{2} .
\end{aligned}
$$

It can also be shown that the model converges since $p_{0}$ monotonically decreases and $p_{n}$ monotonically increases with increasing $n$ and $p_{0}$ and $p_{n}$ are bounded.

## Appendix B

## Seller's Decision in Terms of Offer Acceptance

We want to show that any seller with valuation lower than $s$ strictly prefers to accept the offer $p_{j}$ now instead of waiting for better offers in the future if a seller with valuation $s$ is willing to accept the offer.

Define $V(s)$ to be the equilibrium payoff at time $j+1$ of a seller with valuation s and let $Q(s)$ be the discounted probability of a trade for the seller $s$. Suppose a seller with valuation $s$ chooses to accept a offer $p_{j}$, then $p_{j}-s \geq \delta_{S} V(s)$ must hold.

Now consider a seller with a valuation $s^{\prime}<s$. We want to show that a seller with valuation $s^{\prime}$ is also better off to accept the offer $p_{j}$ now and thus $p_{j}-s^{\prime} \geq \delta_{S} V\left(s^{\prime}\right)$. The seller with valuation $s$ imitates $s^{\prime}$ only if the expected payoff is at least as high as the payoff he expects when he is following his own equilibrium behavior. Thus

$$
V(s) \geq V\left(s^{\prime}\right)-Q\left(s^{\prime}\right) \cdot\left(s-s^{\prime}\right)
$$

and since $Q\left(s^{\prime}\right) \leq 1$

$$
\begin{aligned}
& V(s) \geq V\left(s^{\prime}\right)-Q\left(s^{\prime}\right) \cdot\left(s-s^{\prime}\right) \geq V\left(s^{\prime}\right)-s+s^{\prime} \\
& \Leftrightarrow p_{j}-s \geq \delta_{S}\left(V\left(s^{\prime}\right)-s+s^{\prime}\right) \\
& p_{j} \geq \delta_{S} V\left(s^{\prime}\right)+s\left(1-\delta_{S}\right)+\delta_{S^{\prime}} s^{\prime}
\end{aligned}
$$

since $s^{\prime} \leq s\left(1-\delta_{S}\right)+\delta_{S} s^{\prime}$

$$
\begin{aligned}
& p_{j} \geq \delta_{S} V\left(s^{\prime}\right)+s^{\prime} \\
& p_{j}-s^{\prime} \geq \delta_{S} V\left(s^{\prime}\right) \quad \text { q.e.d. }
\end{aligned}
$$

Intuitively, it is clear that a seller with valuation $s^{\prime}\left(\right.$ remember $\left.s^{\prime}<s\right)$ loses more money due to the discount factor than a seller with valuation $s$ and thus the seller with valuation $s^{\prime}$ will always accept an offer if a seller with valuation $s$ is willing to accept the offer.

## Appendix C

## Behavior in Markets with N and Infinite Offers and One-Sided Uncertainty

The equilibrium is described as a collection of functions $\left\{p_{j}(\cdot), \beta_{j}(\cdot), \sigma_{j}(\cdot), \mu_{j}(\cdot)\right\}_{j=0}^{\infty}$, where $\mu_{j}(\cdot)$ is a probability distribution representing the seller's conjectures about the buyer's valuation for the events when the buyer's offer is not in the range of the equilibrium offer schedule $p_{j} .{ }^{1}$ The solution strategy for this bargaining model is to reduce the two-sided uncertainty problem (seller's valuation, buyer's $W T P$ ) to a case with one-sided uncertainty where the buyer's $W T P$ is known. Subsequently, the two-sided uncertainty case is solved.

For the one-sided case the seller knows the buyer's valuation $W T P$, but the buyer only knows that the seller's valuation is uniformly distributed on $[L B, U B] .^{2}$ The buyer chooses an offer that maximizes the present value of current and future surplus, given her knowledge of the seller's valuation and subject to the constraint that the seller will accept the offer only if his valuation is sufficiently low so that the seller is better off accepting now than waiting for higher offers in the future. An offer that is rejected provides learning; $\sigma_{j-1}$ is then used as a new lower bound of the distribution assumption applying Bayes' rule. With $i$ periods remaining in the $n$-stage bargaining game, define $j$ to be $n+1-i$, so the buyers chooses to maximize her expected gain $\mathbf{u}\left(W T P, \sigma_{j-1}\right)$ given the seller's valuation is uniformly distributed on $\left[\sigma_{j-1}, U B\right]$ :

$$
u_{j}\left(W T P, \sigma_{j-1}\right)=\max _{p} \frac{1}{U B-b_{j-1}}\left[(W T P-p) \cdot\left(\sigma_{j}-\sigma_{j-1}\right)+\delta_{B}\left(U B-\sigma_{j}\right) u_{j+1}\left(W T P, \sigma_{j}\right)\right]
$$

such that

$$
p_{j}-\sigma_{j}=\delta_{s}\left(p_{j+1}-\sigma_{j}\right)
$$

where $\sigma_{j}$ is the indifference valuation in $j$.

This yields the following function for the buyer's optimal offering behavior and the seller's cutoff valuations:

$$
\begin{gathered}
p_{j}\left(W T P, \sigma_{j-1}\right)=\frac{\left(1-\delta_{S}+\delta_{S} c_{j+1}\right)^{2}}{2 \cdot\left(1-\delta_{S}+\delta_{S} c_{j+1}-\frac{1}{2} \delta_{B} c_{j+1}\right)} \cdot\left(\sigma_{j-1}-W T P\right)+W T P \\
\sigma_{j}=\frac{\left(1-\delta_{S}+\delta_{S} c_{j+1}\right)}{2 \cdot\left(1-\delta_{S}+\delta_{S} c_{j+1}-\frac{1}{2} \delta_{B} c_{j+1}\right)} \cdot\left(\sigma_{j-1}-W T P\right)+W T P \\
\text { with } c_{n}=\frac{1}{2} ; \text { for } i>1: c_{j}=\frac{\left(1-\delta_{S}+\delta_{S} c_{j+1}\right)^{2}}{2\left(1-\delta_{S}+\delta_{S} c_{j+1}\right)-\delta_{B} c_{j+1}} .
\end{gathered}
$$

The proof is by induction on $n$. With one period remaining, the buyer wishes to choose $p$ according to the following program:

[^0]\[

$$
\begin{aligned}
& u_{n}\left(W T P, \sigma_{n-1}\right)=\max _{p} \frac{1}{U B-\sigma_{n-1}}\left[(W T P-p) \cdot\left(p-\sigma_{n-1}\right)\right]=\max _{p} \frac{p W T P-p^{2}-\sigma_{n-1} W T P+p \sigma_{n-1}}{U B-\sigma_{n-1}} \\
& \frac{d u_{n}\left(W T P, \sigma_{n-1}\right)}{d p} \\
& 0 \vdots=W T P-2 p+\sigma_{n-1} \\
& p=\frac{W T P+\sigma_{n-1}}{2}
\end{aligned}
$$
\]

so

$$
\begin{aligned}
& u_{n}\left(W T P, \sigma_{n-1}\right)=\frac{1}{U B-\sigma_{n-1}}\left[\left(W T P-\frac{W T P+\sigma_{n-1}}{2}\right) \cdot\left(\frac{W T P+\sigma_{n-1}}{2}-\sigma_{n-1}\right)\right] \\
& =\frac{\left(W T P-\sigma_{n-1}\right)^{2}}{4\left(U B-\sigma_{n-1}\right)}
\end{aligned}
$$

With $i$ periods remaining, the buyer's expected consumer surplus is given by

$$
u_{j}\left(W T P, \sigma_{j-1}\right)=\max _{p} \frac{1}{U B-\sigma_{j-1}}\left[(W T P-p) \cdot\left(\sigma_{j}-\sigma_{j-1}\right)+\delta_{B}\left(U B-\sigma_{j}\right) u_{j+1}\left(W T P, \sigma_{j}\right)\right]
$$

Assume by the induction hypothesis that

$$
\begin{aligned}
& u_{j+1}\left(W T P, \sigma_{j}\right)=\frac{1}{2} c_{j+1} \frac{\left(W T P-\sigma_{j}\right)^{2}}{U B-\sigma_{j}} \\
& p_{j+1}\left(W T P, \sigma_{j}\right)=c_{j+1}\left(\sigma_{j}-W T P\right)+W T P
\end{aligned}
$$

with $c_{n}=\frac{1}{2}$; for $i>1: c_{j}=\frac{\left(1-\delta_{S}+\delta_{S} c_{j+1}\right)^{2}}{2\left(1-\delta_{S}+\delta_{S} c_{j+1}\right)-\delta_{B} c_{j+1}}$
then

$$
\begin{align*}
& p=\sigma_{j}\left(1-\delta_{S}\right)+\delta_{S}\left(c_{j+1}\left(\sigma_{j}-W T P\right)+W T P\right) \\
& \Leftrightarrow p=\left(1-\delta_{S}+\delta_{S} c_{j+1}\right)\left(\sigma_{j}-W T P\right)+W T P \tag{C1}
\end{align*}
$$

Substituting yields

$$
\begin{align*}
& u_{j}\left(W T P, \sigma_{j-1}\right)= \\
& \max _{\sigma_{j}} \frac{1}{U B-\sigma_{j-1}}\left[\left(W T P-\left(1-\delta_{S}\right) \sigma_{j}-\delta_{S}\left[c_{j+1}\left(\sigma_{j}-W T P\right)+W T P\right]\right) \cdot\left(\sigma_{j}-\sigma_{j-1}\right)+\delta_{B} \frac{1}{2} c_{j+1}\left(W T P-\sigma_{j}\right)^{2}\right] \\
& =\max _{\sigma_{j}} \frac{1}{U B-\sigma_{j-1}}\left[\left(W T P\left(1+\delta_{S} c_{j+1}-\delta_{S}\right)-\left(1-\delta_{S}+\delta_{S} c_{j+1}\right) \sigma_{j}\right) \cdot\left(\sigma_{j}-\sigma_{j-1}\right)+\delta_{B} \frac{1}{2} c_{j+1}\left(W T P-\sigma_{j}\right)^{2}\right]  \tag{C2}\\
& \frac{d u_{j}\left(W T P, \sigma_{j-1}\right)}{d \sigma_{j}}=W T P\left(1+\delta_{S} c_{j+1}-\delta_{S}\right)-2 \sigma_{j}\left(1-\delta_{S}+\delta_{S} c_{j+1}\right)+\left(1-\delta_{S}+\delta_{S} c_{j+1}\right) \sigma_{j-1}-2 \delta_{B} \frac{1}{2} c_{j+1}\left(W T P-\sigma_{j}\right) \stackrel{!}{=} 0
\end{align*}
$$

which has an unique maximum when

$$
2 \sigma_{j}\left(1-\delta_{S}+\delta_{S} c_{j+1}-\delta_{B} \frac{1}{2} c_{j+1}\right)=W T P\left(1+\delta_{S} c_{j+1}-\delta_{S}-2 \delta_{B} \frac{1}{2} c_{j+1}\right)+\left(1-\delta_{S}+\delta_{S} c_{j+1}\right) \sigma_{j-1}
$$

and for strict concavity of u when $c_{j+1}\left(\delta_{B}-\delta_{S}\right)+\delta_{S} \leq 1$
which is clearly satisfied since $0<c_{j+1}, \delta_{B}, \delta_{S}<1$

$$
\begin{align*}
& \sigma_{j}=\frac{W T P\left(1+\delta_{S} c_{j+1}-\delta_{S}-\delta_{B} c_{j+1}\right)+\sigma_{j-1}\left(1-\delta_{S}+\delta_{S} c_{j+1}\right)}{2 \cdot\left(1-\delta_{S}+\delta_{S} c_{j+1}-\frac{1}{2} \delta_{B} c_{j+1}\right)}  \tag{C3}\\
& \sigma_{j}=\frac{\left(1-\delta_{S}+\delta_{S} c_{j+1}\right)}{2 \cdot\left(1-\delta_{S}+\delta_{S} c_{j+1}-\frac{1}{2} \delta_{B} c_{j+1}\right)} \cdot\left(\sigma_{j-1}-W T P\right)+W T P
\end{align*}
$$

Then by substituting (C3) into (C1) and (C2) we get

$$
p_{j}\left(W T P, \sigma_{j-1}\right)=\frac{\left(1-\delta_{S}+\delta_{S} c_{j+1}\right)^{2}}{2 \cdot\left(1-\delta_{S}+\delta_{S} c_{j+1}-\frac{1}{2} \delta_{B} c_{j+1}\right)} \cdot\left(\sigma_{j-1}-W T P\right)+W T P
$$

q.e.d.
and

$$
u_{j}\left(W T P, \sigma_{j-1}\right)=\frac{1}{2} \frac{\left(1-\delta_{S}+\delta_{S} c_{j+1}\right)^{2}}{2 \cdot\left(1-\delta_{S}+\delta_{S} c_{j+1}-\frac{1}{2} \delta_{B} c_{j+1}\right)} \frac{\left(W T P-\sigma_{j}\right)^{2}}{U B-\sigma_{j}}=\frac{1}{2} c_{j} \frac{\left(W T P-\sigma_{j}\right)^{2}}{U B-\sigma_{j}} \quad \text { q.e.d. }
$$

as required by the induction hypothesis.
Fudenberg et al. (1985) show that the latter-described equilibrium is a unique equilibrium in the infinite-horizon game. The buyer's equilibrium offer $p_{j}$, the seller's indifference valuation $\sigma_{j}$, and the expected consumer surplus $u_{j}$ for period j in the infinite horizon game is given by

$$
\begin{aligned}
& p_{j}=c \cdot(L B-W T P) \cdot d^{j-1}+W T P \\
& \sigma_{j}=(L B-W T P) \cdot d^{j-1}+W T P \\
& u=\frac{1}{2} c \cdot \frac{(L B-W T P)^{2}}{U B-L B}
\end{aligned}
$$

where

$$
\begin{aligned}
& c=\frac{\left(1-\delta_{S}+\delta_{S} c\right)^{2}}{2\left(1-\delta_{S}+\delta_{S} c\right)-\delta_{B} c} \\
& d=\frac{1}{\delta_{B}}\left(1-\sqrt{1-\delta_{B}}\right)=\frac{c}{1-\delta_{S}+\delta_{S} c}
\end{aligned}
$$

## Appendix D

## Behavior in Markets with Infinite Offers and Two-Sided Uncertainty

In the case of two-sided uncertainty, the assumption of a known buyer's valuation WTP is relaxed. The seller only assesses the buyer's valuation to be given by the distribution $\mathrm{F}(W T P)$ with a positive density $\mathrm{f}(W T P)$ on $\left[W T P_{\text {low }}, W T P_{\text {high }}\right]$. In this case, the buyer must be concerned about the information that her offer reveals to the seller, and the seller must interpret this offer as an indication of the buyer's true willingness-to-pay carefully.

The separating equilibrium that distinguishes high valuation buyers from low valuation buyers is achieved through discounting over time. The class of high valuation buyers is described by a valuation that is higher than a certain cutoff value $\beta_{j}$ in $j$ : $W T P>\beta_{j}$. To determine the equilibrium, we must follow an iterative procedure: First, compute the offer sequence and indifference values that result after the buyer's willingness-to-pay has been revealed according to the case with one-sided uncertainty. Determine the offer sequence for the buyer with the highest willingness-to-pay $W T P=W T P_{\text {high }}$ and her optimal number of offers.

Second, stepwise decrease $W T P$ from $W T P_{\text {high }}$ by some small amount $\triangle W T P>0$ so that the buyer $W T P$ is indifferent between offering $\mathrm{p}(W T P)$ and $\mathrm{p}(W T P-\triangle W T P)$. With decreasing $W T P$ there will come a point $\beta_{1}$ at which no seller will accept the offer $\mathrm{p}\left(\beta_{1}\right)$. All buyers with $W T P<$ $\beta_{1}$ will thereby offer too low and in this way signal their low willingness-to-pay. For a buyer with willingness-to-pay $W T P>\beta_{1}$ the value of the subsequent offers can easily be calculated since she already revealed her private information. All buyers with $W T P<\beta_{1}$ will wait for subsequent rounds to reveal their true willingness-to-pay. For these buyers, we go back to the first step and determine the offer sequence and the optimal number of offers for a buyer with willingness-to-pay $W T P=\beta_{1}$. This process is repeated until the offers of all buyers $W T P \in$ $\left[W T P_{\text {low }}, W T P_{\text {high }}\right]$ are determined. By this procedure the NYOP seller has disjunctive classes of buyers that reveal their true willingness-to-pay in the first round (e.g., buyers with $\left.W T P \in\left(\beta_{1}, W T P_{\text {high }}\right]\right)$, in the second round (e.g. ,buyers with $\left.W T P \in\left(\beta_{2}, \beta_{1}\right]\right)$, and so on and so forth.

In equilibrium, high valuation buyers reveal their private information by submitting an offer that is strictly increasing in $W T P$. The seller can then infer the buyer's $W T P$ by inverting the offer schedule $\mathrm{p}(W T P)$ by calculating $W T P=p^{-1}(p)$. Proof: Suppose buyer $W T P$ chooses pretending to be buyer $W T P_{\text {shade }}$ by offering the offer $p$. This means she is trying to imitate the behavior of a low valuation buyer although she actually has a higher valuation of $W T P$ for the product offered. Then her expected consumer surplus is determined by the first offer utilizing $L B$ as the starting point plus all other discounted surpluses that make use of the belief being updated by a rejection in the preceding offering round:

$$
\begin{equation*}
u_{B}\left(W T P_{\text {shade }}, p\right)=\frac{1}{U B-L B} \cdot\left[(\sigma-L B)(W T P-p)+\sum_{j=1}^{\infty} \delta_{B}^{n}\left(\sigma_{j}-\sigma_{j-1}\right)\left(W T P-p_{j}\right)\right] \tag{D1}
\end{equation*}
$$

where the future offers and indifference valuations are given by

$$
\begin{aligned}
& p_{j}=c \cdot\left(\sigma-W T P_{\text {shade }}\right) \cdot d^{j-1}+W T P_{\text {shade }} \\
& \sigma_{j}=\left(\sigma-W T P_{\text {shade }}\right) \cdot d^{j}+W T P_{\text {shade }}
\end{aligned}
$$

with

$$
\sigma=\frac{\left(p-W T P_{\text {shade }}\right)}{1-\delta_{S}+\delta_{S} c}+W T P_{\text {shade }}
$$

Thus,

$$
\begin{aligned}
& \sigma_{j}-\sigma_{j-1}=\left(\sigma-W T P_{\text {shade }}\right) \cdot d^{j}+W T P_{\text {shade }}-\left[\left(\sigma-W T P_{\text {shade }}\right) \cdot d^{j-1}+W T P_{\text {shade }}\right] \\
& =\left(\sigma-W T P_{\text {shade }}\right) \cdot d^{j-1} \cdot(d-1)
\end{aligned}
$$

$$
\begin{aligned}
& W T P-p_{j}=W T P-\left(c \cdot\left(\sigma-W T P_{\text {shade }}\right) \cdot d^{j-1}+W T P_{\text {shade }}\right) \\
& =W T P-W T P_{\text {shade }}-c \cdot\left(\sigma-W T P_{\text {shade }}\right) \cdot d^{j-1}
\end{aligned}
$$

so that

$$
\begin{align*}
& \left(\sigma_{j}-\sigma_{j-1}\right)\left(W T P-p_{j}\right) \\
& =\left(\sigma-W T P_{\text {shade }}\right) \cdot d^{j-1} \cdot(d-1)\left(W T P-W T P_{\text {shade }}-c \cdot\left(\sigma-W T P_{\text {shade }}\right) \cdot d^{j-1}\right)  \tag{D2}\\
& =\left(\sigma-W T P_{\text {shade }}\right) \cdot(d-1) \cdot\left[\left(W T P-W T P_{\text {shade }}\right) \cdot d^{j-1}-c \cdot\left(\sigma-W T P_{\text {shade }}\right) \cdot\left(d^{2}\right)^{j-1}\right]
\end{align*}
$$

Substituting into (D1) yields

$$
\begin{aligned}
& u_{B}\left(W T P_{\text {shade }}, p\right)=\frac{1}{U B-L B} \cdot\left[(\sigma-L B)(W T P-p)+\sum_{j=1}^{\infty} \delta_{B}^{j}\left(\sigma_{j}-\sigma_{j-1}\right)\left(W T P-p_{j}\right)\right] \\
& =\frac{1}{U B-L B} \cdot\left[(\sigma-L B)(W T P-p)+\sum_{j=1}^{\infty} \delta_{B}^{j} \cdot\left(\sigma-W T P_{\text {shade }}\right) \cdot(d-1) \cdot\left[\left(W T P-W T P_{\text {shade }}\right) \cdot d^{j-1}-c \cdot\left(\sigma-W T P_{\text {shade }}\right) \cdot\left(d^{2}\right)^{j-1}\right]\right] \\
& =\frac{1}{U B-L B} \cdot\left[(\sigma-L B)(W T P-p)+\left(\sigma-W T P_{\text {shade }}\right) \cdot(d-1) \cdot \sum_{j=1}^{\infty} \delta_{B}^{j}\left[\left(W T P-W T P_{\text {shade }}\right) \cdot d^{j-1}-c \cdot\left(\sigma-W T P_{\text {shade }}\right) \cdot\left(d^{2}\right)^{j-1}\right]\right]
\end{aligned}
$$

Performing the summation for the geometric progression yields
(Remember: $\sum_{j=0}^{\infty} q^{j}=\frac{1}{1-q}$ for $\mathrm{q}<1$ )

$$
\begin{aligned}
& \sum_{j=1}^{\infty} \delta_{B}^{j}\left[\left(W T P-W T P_{\text {shade }}\right) \cdot d^{j-1}-c \cdot\left(\sigma-W T P_{\text {shade }}\right) \cdot\left(d^{2}\right)^{j-1}\right] \\
& =\delta_{B} \cdot \sum_{j=1}^{\infty}\left[\left(W T P-W T P_{\text {shade }}\right) \cdot\left(\delta_{B} d\right)^{j-1}-c \cdot\left(\sigma-W T P_{\text {shade }}\right) \cdot\left(d^{2} \delta_{B}\right)^{j-1}\right] \\
& =\delta_{B} \cdot \sum_{j=0}^{\infty}\left[\left(W T P-W T P_{\text {shade }}\right) \cdot\left(\delta_{B} d\right)^{j}-c \cdot\left(\sigma-W T P_{\text {shade }}\right) \cdot\left(d^{2} \delta_{B}\right)^{j}\right] \\
& =\delta_{B} \cdot\left[\left(W T P-W T P_{\text {shade }}\right) \cdot \frac{1}{1-\delta_{B} d}-c \cdot\left(\sigma-W T P_{\text {shade }}\right) \cdot \frac{1}{1-d^{2} \delta_{B}}\right]
\end{aligned}
$$

and thus

$$
\begin{aligned}
& u_{B}\left(W T P_{\text {shade }}, p\right) \\
& =\frac{1}{U B-L B} \cdot\left[(\sigma-L B)(W T P-p)+\left(\sigma-W T P_{\text {shade }}\right) \cdot(d-1) \cdot \delta_{B} \cdot\left[\left(W T P-W T P_{\text {shade }}\right) \cdot \frac{1}{1-\delta_{B} d}-c \cdot\left(\sigma-W T P_{\text {shade }}\right) \cdot \frac{1}{1-d^{2} \delta_{B}}\right]\right]
\end{aligned}
$$

It can be shown that $(d-1) /\left(1-\delta_{B} d^{2}\right)=-0.5$ and $(d-1) /\left(1-\delta_{B} d\right)=-d$, so that we can simplify to

$$
\begin{aligned}
& u_{B}\left(W T P_{\text {shade }}, p\right) \\
& =\frac{1}{U B-L B} \cdot\left[(\sigma-L B)(W T P-p)+\delta_{B} \cdot\left[\frac{1}{2} c \cdot\left(\sigma-W T P_{\text {shade }}\right)^{2}-d \cdot\left(W T P-W T P_{\text {shade }}\right) \cdot\left(\sigma-W T P_{\text {shade }}\right)\right]\right]
\end{aligned}
$$

Differentiating the consumer surplus $u_{B}$ with respect to $p$ yields the first order condition

$$
\begin{aligned}
& 0=L B-\sigma+\frac{d \sigma}{d p} \cdot(W T P-p)+ \\
& \delta_{B} \cdot\left[c \cdot\left(\sigma-W T P_{\text {shade }}\right)\left(\frac{d \sigma}{d p}-\frac{d W T P_{\text {shade }}}{d p}\right)-d \cdot\left[\left(W T P-W T P_{\text {shade }}\right)\left(\frac{d \sigma}{d p}-\frac{d W T P_{\text {shade }}}{d p}\right)-\frac{d W T P_{\text {shade }}}{d p} \cdot\left(\sigma-W T P_{\text {shade }}\right)\right]\right]
\end{aligned}
$$

At this point, the buyer can calculate her optimal offer $p^{*}$ by substituting $W T P_{\text {shade }}=W T P$. This means the buyer has the same incentives to offer $p^{*}$ as to shade her offers. This also implies

$$
\frac{d W T P_{\text {shade }}}{d p}=\frac{d W T P}{d p}, \frac{d \sigma}{d p}=\frac{1-\frac{d W T P}{d p}}{w}+\frac{d W T P}{d p} \text { and } \sigma=\frac{p-W T P}{1-\delta_{S}+\delta_{S} c}+W T P=\frac{p-W T P}{w}+W T P
$$

and results in the first order differential equation

$$
\begin{aligned}
& 0!=L B-\sigma+\frac{d \sigma}{d p} \cdot(W T P-p)+\delta_{B} \cdot\left[c \cdot(\sigma-W T P)\left(\frac{d \sigma}{d p}-\frac{d W T P}{d p}\right)+d \cdot\left(\frac{d W T P}{d p}\right) \cdot(\sigma-W T P)\right] \\
& =L B-\frac{p-W T P}{w}-W T P+\left(\frac{1-\frac{d W T P}{d p}}{w}+\frac{d W T P}{d p}\right) \cdot(W T P-p)+\delta_{B} \cdot\left[c \cdot\left(\frac{p-W T P}{w}\right)\left(\frac{1-\frac{d W T P}{d p}}{w}\right)+d \cdot\left(\frac{d W T P}{d p}\right) \cdot\left(\frac{p-W T P}{w}\right)\right] \\
& =w^{2}(L B-W T P)-w(p-W T P)+\left[w\left(1-\frac{d W T P}{d p}\right)+w^{2}\left(\frac{d W T P}{d p}\right)\right] \cdot(W T P-p)+\delta_{B} \cdot\left[c \cdot(p-W T P)\left(1-\frac{d W T P}{d p}\right)+d \cdot w\left(\frac{d W T P}{d p}\right) \cdot(p-W T P)\right] \\
& =w^{2}(L B-W T P)+\left[2 w+w^{2}\left(\frac{d W T P}{d p}\right)-w\left(\frac{d W T P}{d p}\right)\right] \cdot(W T P-p)+\delta_{B} \cdot\left[c \cdot(p-W T P)\left(1-\frac{d W T P}{d p}\right)+d \cdot w\left(\frac{d W T P}{d p}\right) \cdot(p-W T P)\right] \cdot \frac{d p}{d W T P} \\
& =w^{2}(L B-W T P) \cdot \frac{d p}{d W T P}+\left(2 w \cdot \frac{d p}{d W T P}+w^{2}-w\right) \cdot(W T P-p)+\delta_{B} \cdot\left[c \cdot(p-W T P)\left(\frac{d p}{d W T P}-1\right)+d \cdot w \cdot(p-W T P)\right] \\
& =w^{2}(L B-W T P) \cdot \frac{d p}{d W T P}+(p-W T P)\left[w-w^{2}-2 w \cdot \frac{d p}{d W T P}+\delta_{B} \cdot c \cdot\left(\frac{d p}{d W T P}-1\right)+\delta_{B} \cdot d \cdot w\right]
\end{aligned}
$$

which can be solved to yield $\mathrm{p}(W T P)$, which is typically lower than her optimal offer in the case where her valuation is known.

## Appendix E

## Experimental Instructions

## Information Given to Participants

Surf to the following URL: ... Please do not use the Back-Button on the navigation bar. It is not allowed to open any other program or window during the experimental session.

Once the website is loaded, you can log into the market platform using username and password from the dispensed cards. You are a prospective buyer who is offering for 12 hypothetical products. For these products, you obtain information about the interval of the seller's costs given by a lower and an upper bound. Moreover, you suffer from bargaining costs in the form of a percentage discount of your payoff. The seller faces opportunity costs as well and is therefore interested in a fast agreement. However, the seller will not sell below his costs and thus the information on the distribution of the seller's cost is important.

Furthermore, you see the number of offers you already submitted on the product and the amount of your last offer. For all products you have a resale value. Other buyers can have different resale values for the same product. Hereby, other buyers can value the same product higher or lower than you do. Overall, there are two different resale values for every product and you are randomly assigned to one of these. You also obtain information on the second resale value of the given product. However, the seller is aware of these two segments by thorough marketing research and tries to maximize his profit. The sequence of products you are trying to buy is different for all buyers.

## Explaining the Mechanism

The seller applies a mechanism called NYOP. This means that you as buyer make an offer indicating your willingness-to-pay. If your offer surpasses a secret threshold price set by the seller, you buy the product for the price denoted by your offer. If your offer is below the threshold price, it will be rejected. You can place another offer after a rejection. You can repeat offering until you are successful or you are not interested anymore. Note that you do not compete with other buyers; you solely have to surpass the secret threshold price with your offer.

## How to Earn Money

Your payoff in the game is the difference between your resale value and a successful offer. If you stop offering for a product, your payoff for this product is zero. If you offer more than your resale value, you realize negative profit. The payoff depends additionally on the number of offers you placed to get the product. Depending on the level of a discount factor, your payoff is discounted for every offer. There are two different discount factors: Either your payoff is diminished by 5 percent per offer or by 25 percent per offer. If you surpass the threshold price with your first offer, your payoff is always 100 percent. The seller suffers from bargaining costs as well. The discount factor is the same for both seller and buyer for a given product. The closer you offer to the threshold price, the higher your profit. The less offers you need, the higher your profit.

Market 1 and Market 3 do not obtain any further information here.
Market 2: For every product, the threshold price is static and already set and thus does not change. Remember that the seller does not sell below his costs.

[^1]Try to maximize your profit!

We will draw lots for winners ( $1 / 3$ ) from the participants who we will remunerate with their virtual profit multiplied with some factor. The remuneration will take place in approximately two weeks. We will inform the winners about time and location of the remuneration. Subsequent to the offering process you will have to answer a questionnaire. Please answer in all conscience.

## References

Cramton, P. C. 1984. "Bargaining with Incomplete Information: An Infinite-Horizon Model with Two-Sided Uncertainty," Review of Economic Studies (51:4), pp. 579-593.
Fudenberg, D., Levine, D., and Tirole, J. 1985. "Infinite-Horizon Models of Bargaining with One-Sided Incomplete Information," in Game Theoretic Models of Bargaining, A. Roth (ed.), Cambridge, UK: University of Cambridge Press.


[^0]:    ${ }^{1}$ A player could leave the equilibrium path with updating of beliefs not being possible. Cramton (1984) shows how such conjectures should be constructed to support a very similar equilibrium.
    ${ }^{2}$ We can safely assume that $W T P \geq L B$, since any buyer with $L B>W T P$ would not enter negotiations.

[^1]:    Market 4: The secret threshold price is not static but changes due to your offering behavior. You actually haggle with the seller. It is therefore possible that an offer that was rejected in early offer rounds gets accepted later because the seller realized that you really have a low willingness-to-pay. The seller does not learn anything across products. The game starts anew with a new product. Do not forget that the seller does not sell below his costs.

