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THE MAKING OF A GOOD IMPRESSION: INFORMATION HIDING IN AD EXCHANGES

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Appendix A

Proof of Theorem 1

Proof. Since the bids for impressions from publisher θ are less than or equal to u_θ, $\sum_\theta p_\theta u_\theta$ is an upper bound of the expected revenue. We will show that the expected revenue from the policy of complete revealing converges to this upper bound as $|C|$ approaches infinity. To establish this, we will show that, as $|C| \to \infty$,

$$\mathbb{E}[\mathrm{SH}_c\{w^c(\theta)\}] \to u_\theta$$

To establish this, we will show that $\mathrm{SH}_c\{w^c(\theta)\} \to u_\theta$ in probability. That is, for any $\delta_\theta > 0$,

$$\lim_{|C|\to\infty} P\left(\left|u_\theta - \mathrm{SH}_c\left\{w^c(\theta)\right\}\right| \geq \delta_\theta\right) = 0$$

Since $\mathrm{SH}_c\{w^c(\theta)\} \leq u_\theta$, we will show that $\lim_{|C|\to\infty} P\left(\mathrm{SH}_c\left\{w^c(\theta)\right\} \leq u_\theta - \delta_\theta\right) = 0$.

Let $F_\theta(x)$ and $F_\theta^{\mathrm{SH}}(x)$ denote the cumulative density function of $w^c(\theta)$ and $\mathrm{SH}_c\{w^c(\theta)\}$, respectively. Then, we have

$$\begin{aligned} P\left(\mathrm{SH}_c\left\{w^c(\theta)\right\} \leq u_\theta - \delta_\theta\right) &= F_\theta^{SH}\left(u_\theta - \delta_\theta\right) \\ &= \left[F_\theta\left(u_\theta - \delta_\theta\right)\right]^{|C|} + |C|\left[F_\theta\left(u_\theta - \delta_\theta\right)\right]^{|C|-1}\left[1 - F_\theta\left(u_\theta - \delta_\theta\right)\right] \end{aligned}$$

Let $h = F_\theta(u_\theta - \delta_\theta) < 1$, then

$$
\begin{aligned}
&\lim_{|C|\to\infty} P\left(\mathrm{SH}_c\left\{w^c(\theta)\right\} \le u_\theta - \delta_\theta\right) \\
&= \lim_{|C|\to\infty}\left[h^{|C|} + |C| h^{|C|-1}(1-h)\right] \\
&= \lim_{|C|\to\infty} h^{|C|} + \lim_{|C|\to\infty}\left[|C| h^{|C|-1}(1-h)\right] \\
&= 0 + \lim_{|C|\to\infty}\left[|C| h^{|C|-1}(1-h)\right] \\
&= (1-h)\lim_{|C|\to\infty}\left(|C| h^{|C|-1}\right) \\
&= (1-h)\lim_{|C|\to\infty}\left\{\frac{1}{\left(h^{-1}\right)^{|C|-1}\ln\left(h^{-1}\right)}\right\} \\
&= 0
\end{aligned}
$$

where the penultimate equality follows from L'Hôpital's rule. This completes the proof of the claim.

Proof of Theorem 2

Proof. It suffices to demonstrate a specific instance (of the optimization problem under the one-call design) for which the stated claim holds. Let $C = \{1, 2\}$, and $\Theta = \{1, 2\}$. Let $|A^c| = 1$ for all $c \in C$. Thus, the advertiser label a can be suppressed. In this instance, let bidder 1 hold a valuation of 10 if $\theta = 1$, and 0 if $\theta = 2$. That is, for $c = 1$, $v^1(1) = 10$, $v^1(2) = 0$. Let bidder 2 hold a valuation of 10 if $\theta = 2$, and 0 if $\theta = 1$. That is, for $c = 2$, $v^2(1) = 0$, $v^2(2) = 10$. Let $p_1 = p_2 = \frac{1}{2}$, that is, both publisher identities are equally likely.

Consider the policy of always revealing θ. Then, for any realization of θ, one bid is 10 and the other bid is 0. Since the second-highest bid is 0, then any impression generates a revenue of 0. Therefore, the expected revenue generated from the auction is 0. Next, consider the policy of always hiding θ. In this case, each bidder submits a bid equal to $\frac{10+0}{2} = 5$ and, therefore, the second-highest bid is 5. Thus, the expected revenue generated from the auction is 5. The result follows.

Proof of Theorem 3

Proof. In this case, we have $|C| = 2$ and $|A^c| = 1$ for all $c \in C$. Therefore, we can simplify the objective function by replacing $\mathrm{SH}_c\{.\}$ with $\min_c\{.\}$ and suppressing the advertiser label a. Then the optimization problem becomes

$$
\max_{\vec{q}_R}\left\{\sum_\theta p_\theta q_R^\theta \mathrm{E}\left[\min_c\left\{v^c(\theta)\right\}\right] + \mathrm{E}\left[\min_c\left\{\sum_\theta p_\theta q_H^\theta v^c(\theta)\right\}\right]\right\}
$$

where $\vec{q}_R \in [0,1]^{|\Theta|}$. Observe that

$$
\begin{aligned}
&Rev^{\mathrm{I}}(\vec{q}_{\mathrm{R}}) \\
&= \textstyle\sum_\theta p_\theta q_R^\theta \mathbb{E}\left[\min_c\{v^c(\theta)\}\right] + \mathbb{E}\left[\min_c\{\sum_\theta p_\theta q_H^\theta v^c(\theta)\}\right] \\
&= \textstyle\sum_\theta \mathbb{E}\left[\min_c\{p_\theta q_R^\theta v^c(\theta)\}\right] + \mathbb{E}\left[\min_c\{\sum_\theta p_\theta q_H^\theta v^c(\theta)\}\right] \\
&= \mathbb{E}\left[\textstyle\sum_\theta \min_c\{p_\theta q_R^\theta v^c(\theta)\}\right] + \mathbb{E}\left[\min_c\{\sum_\theta p_\theta (1-q_R^\theta) v^c(\theta)\}\right] \\
&\leq \mathbb{E}\left[\min_c\{\textstyle\sum_\theta p_\theta q_R^\theta v^c(\theta)\}\right] + \mathbb{E}\left[\min_c\{\sum_\theta p_\theta (1-q_R^\theta) v^c(\theta)\}\right] \\
&= \mathbb{E}\left[\min_c\{\textstyle\sum_\theta p_\theta q_R^\theta v^c(\theta)\} + \min_c\{\sum_\theta p_\theta (1-q_R^\theta) v^c(\theta)\}\right] \\
&\leq \mathbb{E}\left[\min_c\{\textstyle\sum_\theta p_\theta q_R^\theta v^c(\theta) + \sum_\theta p_\theta (1-q_R^\theta) v^c(\theta)\}\right] \\
&= \mathbb{E}\left[\min_c\{\textstyle\sum_\theta p_\theta v^c(\theta)(q_R^\theta + 1 - q_R^\theta)\}\right] \\
&= \mathbb{E}\left[\min_c\{\textstyle\sum_\theta p_\theta v^c(\theta)\}\right] \\
&= Rev^{\mathrm{I}}(\vec{0}).
\end{aligned}
$$

Thus, the expected revenue under any feasible policy is smaller than $Rev^{\mathrm{I}}(\vec{0})$, which is the expected revenue from the complete hiding policy. This implies the optimality of the complete hiding policy.

Proof of Theorem 4

Proof. Consider the following instance of the optimization problem under the one-call design. For an integer $k \geq 2$, let $C = \{1,2,\dots,2k-1,2k\}$ and $\Theta = \{1,2,\dots,k\}$. Let $|A^c| = 1$ for all $c \in C$. Therefore, we suppress the advertiser label a. In this instance, let bidders 1 and 2 hold a valuation of 1 when $\theta = 1$, and 0 otherwise. That is, for $c = 1,2$, $v^1(1) = v^2(1) = 1, v^1(2) = v^2(2) = 0, \dots, v^1(k) = v^2(k) = 0$. Similarly, let bidders 3 and 4 hold a valuation of 1 when $\theta = 2$, and 0 otherwise. That is, for $c = 3,4$, $v^3(1) = v^4(1) = 0, v^3(2) = v^4(2) = 1, v^3(3) = v^4(3) = 0, \dots, v^3(k) = v^4(k) = 0$. In general, for $j = 1,2,\dots,k$, let bidders $2j-1$ and $2j$ hold a valuation of 1 when $\theta = j$, and 0 otherwise. That is, for $c = 2j-1, 2j$, $v^{2j-1}(j) = v^{2j}(j) = 1, \text{and} v^c(\theta) = 0, \forall c \notin \{2j-1, 2j\} \text{and} \theta \neq j$. Let $p_\theta = \frac{1}{k}$ for all $\theta \in \Theta$, i.e., all publisher identities are equally likely.

Consider the policy of always revealing θ. Then, for any realization of θ, the second-highest bid is 1. Thus, every impression generates a revenue of 1. Therefore, the expected revenue generated from an auction is 1. Next, consider the policy of always hiding θ. In this case, each bidder submits a bid equal to $\frac{1}{k}$ and, therefore, the second-highest bid is $\frac{1}{k}$. Thus, the expected revenue generated from an auction is $\frac{1}{k}$. The result follows.

Proof of Theorem 5

Proof. From inequality (5), we have $Rev^{\mathrm{I}}(\vec{q}_{\mathrm{R}}) \leq Rev^{\mathrm{I}}(\vec{1}) + Rev^{\mathrm{I}}(\vec{0})$. Thus,

$$\mathrm{OPT}_1 \leq Rev^{\mathrm{I}}(\vec{1}) + Rev^{\mathrm{I}}(\vec{0})$$

Since the maximum of two numbers is at least as large as their average, we have

$$\max\{Rev^{\mathrm{I}}(\vec{1}), Rev^{\mathrm{I}}(\vec{0})\} \geq \frac{1}{2}(Rev^{\mathrm{I}}(\vec{1}) + Rev^{\mathrm{I}}(\vec{0}))$$

The two inequalities above imply that

$$\max\{Rev^{\mathrm{I}}(\vec{1}), Rev^{\mathrm{I}}(\vec{0})\} \geq \frac{1}{2}\mathrm{OPT}_1$$

To establish the tightness of the bound, consider the following example. Let $C = \{1, 2, 3, 4\}$ and $\Theta = \{1, 2, 3, 4\}$. Let $|A^c| = 1$ for all $c \in C$. Therefore, we suppress the advertiser label a. The matrix of valuations, where the rows correspond to c and the columns correspond to θ, is as follows:

$$\begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 \end{pmatrix}$$

Let $p_\theta = \frac{1}{4}$ for all $\theta \in \Theta$.

Consider the policy of complete revealing. For the first two realizations of θ, the second-highest bid is 1, while, for the last two realizations of θ, the second-highest bid is 0. Thus, $Rev^{\mathrm{I}}(\vec{1}) = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$. Also, $Rev^{\mathrm{I}}(\vec{0}) = \mathrm{SH}\left\{\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right\} = \frac{1}{2}$. Finally, $\max\{Rev^{\mathrm{I}}(\vec{1}), Rev^{\mathrm{I}}(\vec{0})\} = \frac{1}{2} \geq \frac{1}{2}\mathrm{OPT}_1$. This implies that $\mathrm{OPT}_1 \leq 1$. The policy of revealing the first two publisher identities and hiding the last two, has expected revenue $\frac{1}{4} + \frac{1}{4} + \frac{1}{2}\min\{1,1\} = 1$, and is therefore optimal. The tightness follows.

Proof of Theorem 6

Proof. For a uniform policy, we have $q_{\mathrm{R}}^{\theta} = q$ for every $\theta \in \Theta$. Then, the expected revenue for such a policy is

$$\begin{aligned} Rev^{\mathrm{I}}(\vec{q}_{\mathrm{R}}) &= \textstyle\sum_\theta p_\theta q \mathbb{E}[\mathrm{SH}_c\{w^c(\theta)\}] + \mathbb{E}\left[SH_c\left\{\max_a \textstyle\sum_\theta p_\theta (1-q) v^c(\theta, a)\right\}\right] \quad \text{(from(1))} \\ &= q \sum_\theta p_\theta \mathbb{E}[\mathrm{SH}_c\{w^c(\theta)\}] + (1-q)\mathbb{E}\left[SH_c\left\{\max_a \sum_\theta p_\theta v^c(\theta, a)\right\}\right] \\ &= qRev^{\mathrm{I}}(\vec{1}) + (1-q)Rev^{\mathrm{I}}(\vec{0}) \quad \text{(from (3) and (4))} \end{aligned} \qquad (11)$$

We know from inequality (5) that

$$\mathrm{OPT}_1 \leq Rev^{\mathrm{I}}(\vec{1}) + Rev^{\mathrm{I}}(\vec{0})$$

Thus, for a uniform policy with $q_{\mathrm{R}}^{\theta} = \frac{1}{2}$, we have

$$\begin{aligned} Rev^{\mathrm{I}}\left(\frac{1}{2}\right) &= \frac{1}{2} Rev^{\mathrm{I}}(\vec{1}) + \frac{1}{2} Rev^{\mathrm{I}}(\vec{0}) \\ &\geq \frac{1}{2}\mathrm{OPT}_1 \end{aligned}$$

The tightness of this bound can be shown by the example in the proof of Theorem 5.

Proof of Theorem 8

Proof. Consider the following example. Let $C = \{1,2\}$ and $\Theta = \{1,2,\ldots,2n\}$ where $n \geq 2$. Let each bidder $c \in C$ work with exactly n advertisers. More specifically, let $A^1 = \{1,2,\ldots,n\}$ and $A^2 = \{n+1,\ldots,2n\}$. Let $v^1(\theta, a) = 1$ if $\theta = a$ and $v^1(\theta, a) = 0$ otherwise. Let $v^2(\theta, a) = 1$ if $\theta = a + n$ and $v^2(\theta, a) = 0$ otherwise. An explanation follows.

For the first bidder ($c = 1$), an impression from any of the first n publishers ($\theta \in \{1,2,\ldots,n\}$) generates a value of 1 when the specific ad $a = \theta$ is served. All other publisher-ad pairs result in zero value. Similarly, for the second bidder, an impression from any publisher $\theta \in \{n+1,\ldots,2n\}$ generates a value of 1 when the specific ad $a = \theta$ is served, whereas all other (a, θ) pairs provide zero value. The matrices of valuations for the two bidders are provided below:

$$\text{Valuations for Bidder 1}(\mathrm{c}=1) \qquad\qquad \text{Valuations for Bidder 2}(c=2)$$

$$a \begin{array}{c} \\ 1 \\ 2 \\ \vdots \\ n \end{array} \begin{array}{c} \theta \\ \begin{array}{ccccccc} 1 & 2 & \cdots & n & n+1 & \cdots & 2n \end{array} \\ \begin{bmatrix} 1 & 0 & \cdots & 0 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 & 0 & \cdots & 0 \end{bmatrix} \end{array} \qquad a \begin{array}{c} \\ n+1 \\ n+2 \\ \vdots \\ 2n \end{array} \begin{array}{c} \theta \\ \begin{array}{ccccccc} 1 & \cdots & n & n+1 & n+2 & \cdots & 2n \end{array} \\ \begin{bmatrix} 0 & \cdots & 0 & 1 & 0 & \cdots & 0 \\ 0 & \cdots & 0 & 0 & 1 & \cdots & 0 \\ \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & 0 & 0 & \cdots & 1 \end{bmatrix} \end{array}$$

Let $p_\theta = \frac{1}{2n}$ for all $\theta \in \Theta$, that is, all publisher identities are equally likely. This completes the specification of the example.

We will prove our result by showing the claims that the optimal expected revenue under one-call design (OPT_1) is $\frac{1}{2n}$ and that the optimal expected revenue under the two-call design (OPT_2) is at least $\frac{1}{2}$. Thus, $\frac{\mathrm{OPT}_2}{\mathrm{OPT}_1}$ is larger than n, which can be made arbitrarily large. We now proceed to prove the two claims made above.

In the one-call design, notice that the expected revenue of the complete revealing policy, that is, $Rev^{\mathrm{I}}(\vec{1})$, is 0 since the second-highest bid is 0 for every impression. Similarly, the expected revenue of the complete hiding policy, i.e., $Rev^{\mathrm{I}}(\vec{0})$, is $\frac{1}{2n}$ since both bidders have an expected valuation of $\frac{1}{2n}$ for every impression. Next, consider the policy of revealing every impression with probability $\frac{1}{2}$ (i.e., $\vec{q}_{\mathrm{R}} = \frac{1}{2}$). We know from (11) that the expected revenue under this policy is $Rev^{\mathrm{I}}(\frac{1}{2}) = \frac{1}{2}Rev^{\mathrm{I}}(\vec{0}) + \frac{1}{2}Rev^{\mathrm{I}}(\vec{1}) = \frac{1}{4n}$. We also know from Theorem 6 that this policy ($\vec{q}_{\mathrm{R}} = \frac{1}{2}$) generates a revenue of at least half of OPT_1. This implies that $\mathrm{OPT}_1 \le \frac{1}{2n}$, which as we have seen above, is $Rev^{\mathrm{I}}(\vec{0})$. Therefore, $\mathrm{OPT}_1 = \frac{1}{2n}$.

In the two-call design, consider the policy of choosing $\vec{q}_{\mathrm{R}} = \vec{0}$. Since θ is known to the winning bidder before choosing the ad, we see from the matrices of valuations above that each bidder obtains a value of 1 from 50% of the publishers and a value of 0 from the remaining publishers. Thus, under the $\vec{q}_{\mathrm{R}} = \vec{0}$ policy, each of the two bidders submits a bid equal to $\frac{1}{2}$ for every impression. Therefore, the expected revenue under this policy is $Rev^{\mathrm{II}}(\vec{0}) = \frac{1}{2}$. This implies that $\mathrm{OPT}_2 \ge \frac{1}{2}$.

Proof of Theorem 9

Proof. The arguments used in the proofs of Statements 1 and 2 are similar to those in the proofs of Theorems 5, and 6. Therefore, for brevity, we avoid providing the proofs of these statements here. Note that the policy of complete revealing is the same under the one-call and the two-call designs. Therefore, Statement 3 follows from Theorem 1. Recall that the instances in the proofs of Theorems 2 and 4 use $|A|^c = 1$ for all $c \in C$. Since the expected revenue of any policy stays the same under the one-call and the two-call designs when $|A|^c = 1$ for all $c \in C$, the proof of Statement 5 follows from those of Theorems 2 and 4.

We now proceed to prove Statement 4. Consider a policy of $\vec{q}_{\mathrm{R}}$. Then the expected revenue of this policy is

$$
\begin{aligned}
Rev^{\mathrm{II}}(\vec{q}_{\mathrm{R}}) &= \sum_{\theta} p_\theta q_{\mathrm{R}}^{\theta} \mathbb{E}\left[\min_c \{w^c(\theta)\}\right] + \mathbb{E}\left[\min_c \left\{\sum_{\theta} p_\theta (1 - q_{\mathrm{R}}^{\theta}) w^c(\theta)\right\}\right] \\
&= \mathbb{E}\left[\textstyle\sum_\theta \min_c \{p_\theta q_{\mathrm{R}}^{\theta} w^c(\theta)\}\right] + \mathbb{E}\left[\min_c \{\textstyle\sum_\theta p_\theta (1 - q_{\mathrm{R}}^{\theta}) w^c(\theta)\}\right] \\
&\le \mathbb{E}\left[\min_c \{\textstyle\sum_\theta p_\theta q_{\mathrm{R}}^{\theta} w^c(\theta)\}\right] + \mathbb{E}\left[\min_c \{\textstyle\sum_\theta p_\theta (1 - q_{\mathrm{R}}^{\theta}) w^c(\theta)\}\right] \\
&\le \mathbb{E}\left[\min_c \{\textstyle\sum_\theta p_\theta q_{\mathrm{R}}^{\theta} w^c(\theta) + \textstyle\sum_\theta p_\theta (1 - q_{\mathrm{R}}^{\theta}) w^c(\theta)\}\right] \\
&= \mathbb{E}\left[\min_c \{\textstyle\sum_\theta p_\theta w^c(\theta)\}\right] \\
&= Rev^{\mathrm{II}}(\vec{0}).
\end{aligned}
$$

The result follows.

Appendix B

Derivations of the Closed-Form Expressions in Simplification under Uniform Bidder Valuations

Assume that the valuation of bidder $c \in C$ toward publisher $\theta \in \Theta$ (i.e., $w_c(\theta)$) follows a uniform distribution with the support $[0, b_{c\theta}]$. For a given θ, let $b_{(c),\theta}$ be the c^{th} smallest $b_{c\theta}$ and let $b_{(0),\theta} = 0$. For a given publisher identity θ, let $F_\theta^{(c)}(x)$ and $f_\theta^{(c)}(x)$ denote the cumulative density function and the probability density function, respectively, of bidder (c)'s valuation towards publisher θ (i.e., $w_{(c)}(\theta)$), where $w_{(c)}(\theta)$ follows a uniform distribution with support $[0, b_{(c),\theta}]$. Then,

$$\mathbb{E}\left[\max_{c}\{w^{c}(\theta)\}\right] = \sum_{c=1}^{|C|}\left\{\sum_{i=1}^{c}\int_{b_{(i-1),\theta}}^{b_{(i),\theta}} x\left[\prod_{j\neq c,j>i-1} F_{\theta}^{(j)}(x)\right] f_{\theta}^{(c)}(x)dx\right\}$$
$$= \sum_{c=1}^{|C|}\left\{\sum_{i=1}^{c}\int_{b_{(i-1),\theta}}^{b_{(i),\theta}} x\left[\prod_{j\neq c,j>i-1}\frac{x}{b_{(j),\theta}}\right]\frac{1}{b_{(c),\theta}}dx\right\}$$
$$= \sum_{c=1}^{|C|}\left\{\sum_{i=1}^{c}\int_{b_{(i-1),\theta}}^{b_{(i),\theta}}\left[\prod_{j>i-1}\frac{x}{b_{(j),\theta}}\right]dx\right\}$$
$$= \sum_{c=1}^{|C|}\left\{\sum_{i=1}^{c}\int_{b_{(i-1),\theta}}^{b_{(i),\theta}}\frac{x^{|C|-i+1}}{\prod_{j>i-1} b_{(j),\theta}}dx\right\}$$
$$= \sum_{c=1}^{|C|}\left\{\sum_{i=1}^{c}\frac{1}{\prod_{j>i-1} b_{(j),\theta}}\int_{b_{(i-1),\theta}}^{b_{(i),\theta}} x^{|C|-i+1}dx\right\}$$
$$= \sum_{c=1}^{|C|}\left\{\sum_{i=1}^{c}\frac{1}{\prod_{j>i-1} b_{(j),\theta}}\left[\frac{b_{(i),\theta}^{|C|-i+2}-b_{(i-1),\theta}^{|C|-i+2}}{|C|-i+2}\right]\right\}$$
$$= \sum_{c=1}^{|C|}\left\{\sum_{i=1}^{c}\frac{b_{(i),\theta}^{|C|+2-i}-b_{(i-1),\theta}^{|C|+2-i}}{(|C|+2-i)\prod_{j>i-1} b_{(j),\theta}}\right\}$$

The last expression above is the one specified in the "Simplification under Uniform Bidder Valuations" in the article.

Next, we derive a closed-form expression for $\mathbb{E}[\mathrm{SH}_c\{w^c(\theta)\}]$.

$$\mathbb{E}[\mathrm{SH}_c\{w^c(\theta)\}]$$

$$= \sum_{i=1}^{|C|-1}\int_{b_{(i-1),\theta}}^{b_{(i),\theta}} x\left[\sum_{k\neq|C|,k>i-1}\left(1-F_{\theta}^{(k)}(x)\right)\prod_{l\neq|C|,k,l>i-1} F_{\theta}^{(l)}(x)\right] f_{\theta}^{(|C|)}(x)dx +$$
$$\sum_{c=1}^{|C|-1}\sum_{i=1}^{c}\int_{b_{(i-1),\theta}}^{b_{(i),\theta}} x\left[\sum_{k\neq c,k>i-1}\left(1-F_{\theta}^{(k)}(x)\right)\prod_{l\neq c,k,l>i-1} F_{\theta}^{(l)}(x)\right] f_{\theta}^{(c)}(x)dx$$

(the first term captures the case in which the bidder with bids from $\mathrm{Uniform}[0, \mathrm{b}_{(|C|),\theta}]$ has the second highest bid, and the second term captures all the other possibilities.)

$$= \sum_{i=1}^{|C|-1}\int_{b_{(i-1),\theta}}^{b_{(i),\theta}} x\left[\sum_{k\neq|C|,k>i-1}\left(1-\frac{x}{b_{(k),\theta}}\right)\prod_{l\neq|C|,k,l>i-1}\frac{x}{b_{(l),\theta}}\right]\frac{1}{b_{(|C|),\theta}}dx +$$

$$\sum_{c=1}^{|C|-1}\sum_{i=1}^{c}\int_{b_{(i-1),\theta}}^{b_{(i),\theta}} x\left[\sum_{k\neq c,k>i-1}\left(1-\frac{x}{b_{(k),\theta}}\right)\prod_{l\neq c,k,l>i-1}\frac{x}{b_{(l),\theta}}\right]\frac{1}{b_{(c),\theta}}dx$$

$$= \sum_{i=1}^{|C|-1}\int_{b_{(i-1),\theta}}^{b_{(i),\theta}} x\left[\sum_{k\neq|C|,k>i-1}\left(1-\frac{x}{b_{(k),\theta}}\right)\frac{x^{|C|-i-1}}{\prod_{l\neq k,l>i-1} b_{(l),\theta}}\right]dx +$$

$$\sum_{c=1}^{|C|-1}\sum_{i=1}^{c}\int_{b_{(i-1),\theta}}^{b_{(i),\theta}} x\left[\sum_{k\neq c,k>i-1}\left(1-\frac{x}{b_{(k),\theta}}\right)\frac{x^{|C|-i-1}}{\prod_{l\neq k,l>i-1} b_{(l),\theta}}\right]dx$$

$$= \sum_{i=1}^{|C|-1}\int_{b_{(i-1),\theta}}^{b_{(i),\theta}} x\left[\sum_{k\neq|C|,k>i-1}\left(\frac{x^{|C|-i-1}}{\prod_{l\neq k,l>i-1} b_{(l),\theta}}-\frac{x^{|C|-i}}{\prod_{l>i-1} b_{(l),\theta}}\right)\right]dx +$$

$$\sum_{c=1}^{|C|-1}\sum_{i=1}^{c}\int_{b_{(i-1),\theta}}^{b_{(i),\theta}} x\left[\sum_{k\neq c,k>i-1}\left(\frac{x^{|C|-i-1}}{\prod_{l\neq k,l>i-1} b_{(l),\theta}}-\frac{x^{|C|-i}}{\prod_{l>i-1} b_{(l),\theta}}\right)\right]dx$$

$$=\sum_{i=1}^{|C|-1}\int_{b_{(i-1),\theta}}^{b_{(i),\theta}} x\left[\sum_{k\neq |C|,k>i-1}\frac{x^{|C|-i-1}}{\prod_{l\neq k,l>i-1} b_{(l),\theta}}-\sum_{k\neq |C|,k>i-1}\frac{x^{|C|-i}}{\prod_{l>i-1} b_{(l),\theta}}\right]dx+$$

$$\sum_{c=1}^{|C|-1}\sum_{i=1}^{c}\int_{b_{(i-1),\theta}}^{b_{(i),\theta}} x\left[\sum_{k\neq c,k>i-1}\frac{x^{|C|-i-1}}{\prod_{l\neq k,l>i-1} b_{(l),\theta}}-\sum_{k\neq c,k>i-1}\frac{x^{|C|-i}}{\prod_{l>i-1} b_{(l),\theta}}\right]dx$$

$$=\sum_{i=1}^{|C|-1}\int_{b_{(i-1),\theta}}^{b_{(i),\theta}} x\left[x^{|C|-i-1}\sum_{k\neq |C|,k>i-1}\frac{1}{\prod_{l\neq k,l>i-1} b_{(l),\theta}}-x^{|C|-i}\sum_{k\neq |C|,k>i-1}\frac{1}{\prod_{l>i-1} b_{(l),\theta}}\right]dx+$$

$$\sum_{c=1}^{|C|-1}\sum_{i=1}^{c}\int_{b_{(i-1),\theta}}^{b_{(i),\theta}} x\left[x^{|C|-i-1}\sum_{k\neq c,k>i-1}\frac{1}{\prod_{l\neq k,l>i-1} b_{(l),\theta}}-x^{|C|-i}\sum_{k\neq c,k>i-1}\frac{1}{\prod_{l>i-1} b_{(l),\theta}}\right]dx$$

$$=\sum_{i=1}^{|C|-1}\int_{b_{(i-1),\theta}}^{b_{(i),\theta}}\left[x^{|C|-i}\sum_{k\neq |C|,k>i-1}\frac{1}{\prod_{l\neq k,l>i-1} b_{(l),\theta}}-x^{|C|-i+1}\sum_{k\neq |C|,k>i-1}\frac{1}{\prod_{l>i-1} b_{(l),\theta}}\right]dx+$$

$$\sum_{c=1}^{|C|-1}\sum_{i=1}^{c}\int_{b_{(i-1),\theta}}^{b_{(i),\theta}}\left[x^{|C|-i}\sum_{k\neq c,k>i-1}\frac{1}{\prod_{l\neq k,l>i-1} b_{(l),\theta}}-x^{|C|-i+1}\sum_{k\neq c,k>i-1}\frac{1}{\prod_{l>i-1} b_{(l),\theta}}\right]dx$$

$$=\sum_{i=1}^{|C|-1}\left\{\int_{b_{(i-1),\theta}}^{b_{(i),\theta}} x^{|C|-i}\sum_{k\neq |C|,k>i-1}\frac{1}{\prod_{l\neq k,l>i-1} b_{(l),\theta}}dx-\int_{b_{(i-1),\theta}}^{b_{(i),\theta}} x^{|C|-i+1}\sum_{k\neq |C|,k>i-1}\frac{1}{\prod_{l>i-1} b_{(l),\theta}}dx\right\}+$$

$$\sum_{c=1}^{|C|-1}\sum_{i=1}^{c}\left\{\int_{b_{(i-1),\theta}}^{b_{(i),\theta}} x^{|C|-i}\sum_{k\neq c,k>i-1}\frac{1}{\prod_{l\neq k,l>i-1} b_{(l),\theta}}dx-\int_{b_{(i-1),\theta}}^{b_{(i),\theta}} x^{|C|-i+1}\sum_{k\neq c,k>i-1}\frac{1}{\prod_{l>i-1} b_{(l),\theta}}dx\right\}$$

$$=\sum_{i=1}^{|C|-1}\left\{\sum_{k\neq |C|,k>i-1}\frac{1}{\prod_{l\neq k,l>i-1} b_{(l),\theta}}\int_{b_{(i-1),\theta}}^{b_{(i),\theta}} x^{|C|-i}dx-\sum_{k\neq |C|,k>i-1}\frac{1}{\prod_{l>i-1} b_{(l),\theta}}\int_{b_{(i-1),\theta}}^{b_{(i),\theta}} x^{|C|-i+1}dx\right\}+$$

$$\sum_{c=1}^{|C|-1}\sum_{i=1}^{c}\left\{\sum_{k\neq c,k>i-1}\frac{1}{\prod_{l\neq k,l>i-1} b_{(l),\theta}}\int_{b_{(i-1),\theta}}^{b_{(i),\theta}} x^{|C|-i}dx-\sum_{k\neq c,k>i-1}\frac{1}{\prod_{l>i-1} b_{(l),\theta}}\int_{b_{(i-1),\theta}}^{b_{(i),\theta}} x^{|C|-i+1}dx\right\}$$

$$=\sum_{i=1}^{|C|-1}\left\{\sum_{k\neq |C|,k>i-1}\frac{b_{(i),\theta}^{|C|-i+1}-b_{(i-1),\theta}^{|C|-i+1}}{(|C|-i+1)\prod_{l\neq k,l>i-1} b_{(l),\theta}}-\sum_{k\neq |C|,k>i-1}\frac{b_{(i),\theta}^{|C|-i+2}-b_{(i-1),\theta}^{|C|-i+2}}{(|C|-i+2)\prod_{l>i-1} b_{(l),\theta}}\right\}+$$

$$\sum_{c=1}^{|C|-1}\sum_{i=1}^{c}\left\{\sum_{k\neq c,k>i-1}\frac{b_{(i),\theta}^{|C|-i+1}-b_{(i-1),\theta}^{|C|-i+1}}{(|C|-i+1)\prod_{l\neq k,l>i-1} b_{(l),\theta}}-\sum_{k\neq c,k>i-1}\frac{b_{(i),\theta}^{|C|-i+2}-b_{(i-1),\theta}^{|C|-i+2}}{(|C|-i+2)\prod_{l>i-1} b_{(l),\theta}}\right\}.$$

The last expression above is the one specified in the "Simplification under Uniform Bidder Valuations" in the article.

Appendix C

Viability of Information Hiding by Competing Ad Exchanges

Consider two ad exchanges (X and Y), four bidders (B1, B2, B3, and B4), and four publishers (P1, P2, P3, and P4). Ad exchange X sells impressions from publishers P1 and P2, and ad exchange Y sells impressions from publishers P3 and P4. The valuations of the bidders toward the impressions from different publishers are given in Table C1. For example, the value 1 in row B1 and column P1 indicates that bidder B1 has valuation 1 towards an impression from publisher P1. This bidder has a valuation of 0, 0.8, and 0, for the impressions from publishers P2, P3, and P4, respectively.

Table C1. Bidder Valuations Toward Publishers

	X		Y	
	P1	P2	P3	P4
B1	1	0	0.8	0
B2	0	1	0	0.8
B3	0	0.8	1	0
B4	0.8	0	0	1

The game has two stages:

- First, ad exchanges X and Y simultaneously decide on their information revelation policies. For an ad exchange, there are two options: (1) Reveal the publisher identity to bidders in all auctions (CR policy). (2) Hide the publisher identity to bidders in all auctions (CH policy).
- Second, bidders B1-B4 choose one ad exchange to join after knowing the information revelation policies chosen by the two ad exchanges.

Using backward induction, we first analyze how bidders choose between the two ad exchanges in the second stage. Since there are two options for an ad exchange, there are four information revelation scenarios that the four bidders face:

Scenario 1: Both the ad exchanges choose the CR policy.
Scenario 2: Ad exchange X chooses the CR policy and ad exchange Y chooses the CH policy.
Scenario 3: Ad exchange X chooses the CH policy and ad exchange Y chooses the CR policy.
Scenario 4: Both ad exchanges choose the CH policy.

Table C2 describes the game between the four bidders and reports the possible equilibria under scenario 1 above. Each cell in Table C2 reports the payoffs corresponding to a set of joining decisions made by the four bidders. For example, the cell in the last row and last column (i.e., in row B2-Y-B1-Y and column B4-Y-B3-Y) reports that if bidders B1−B4 all choose to join ad exchange Y, then the payoffs of bidders B1−B4 are 0, 0, 0.1, and 0.1, respectively. These four payoffs are computed as follows: Since all four bidders join ad exchange Y and this ad exchange chooses the CR policy, the four bidders bid their true valuations listed in Table C1. If the impression is from publisher P3, then bidders B1--B4 bid 0.8, 0, 1, 0, respectively. Thus, bidder B3 wins the auction and pays 0.8. Therefore, the payoff of bidder B3 is $1 - 0.8 = 0.2$. The payoff of each of the other three bidders is 0 because they do not win. Similarly, if the impression is from publisher P4, then the payoff of bidder B4 is 0.2 and the payoff of each of the other three bidders is 0. Since the probability that an impression is from either publisher P3 or P4 is 0.5, the expected payoffs of bidders B3 and B4 are both $0.5 \times 0.2 + 0.5 \times 0 = 0.1$, and the expected payoffs of bidders B1 and B2 are 0.

Table C2. Game Between the Bidders in Scenario 1 (The possible equilibria are highlighted in red)

Scenario 1: X chooses CR Policy Y chooses CR Policy				B4: X, B3: X	B4: X, B3: Y	B4: Y, B3: X	B4: Y, B3: Y
B2	X	B1	X	(0.1, 0.1, 0, 0)	($\underline{0.1}$, $\underline{0.5}$, $\underline{0.5}$, 0)	($\underline{0.5}$, $\underline{0.1}$, 0, $\underline{0.5}$)	**($\underline{0.5}$, $\underline{0.5}$, $\underline{0.5}$, $\underline{0.5}$)** **Rev X = 0** **Rev Y = 0**
B2	X	B1	Y	($\underline{0.4}$, 0.1, 0, 0.4)	(0, $\underline{0.5}$, $\underline{0.1}$, 0.4)	(0.4, $\underline{0.1}$, 0, $\underline{0.5}$)	(0, $\underline{0.5}$, $\underline{0.1}$, $\underline{0.5}$)
B2	Y	B1	X	(0.1, $\underline{0.4}$, 0.4, 0)	($\underline{0.1}$, 0.4, $\underline{0.5}$, $\underline{0.4}$)	($\underline{0.5}$, 0, 0.4,$\underline{0.1}$)	($\underline{0.5}$, 0, $\underline{0.5}$, 0.1)
B2	Y	B1	Y	**($\underline{0.4}$, $\underline{0.4}$, $\underline{0.4}$, $\underline{0.4}$)** **Rev X = 0** **Rev Y = 0**	(0, 0.4, 0.1, $\underline{0.4}$)	(0.4, 0, $\underline{0.4}$, 0.1)	(0, 0, 0.1, 0.1)

An entry in this payoff vector is underlined in Table C2 if it is the best response of the corresponding bidder, given the choices of which ad exchange to join by the other bidders. For example, consider the payoff vector ($\underline{0.5}$, 0.1,0, $\underline{0.5}$) corresponding to the choice of bidders B1, B2, and B3 to join ad exchange X and bidder B4 to join ad exchange Y. Here, given the choice by bidders B2 and B3 to join exchange X and bidder B4 to join Y, the best response of Bidder B1 is to join exchange X and her corresponding payoff is 0.5. Clearly, for a cell to correspond to an equilibrium, all four elements in its payoff vector should be underlined. There are two possible equilibria of the bidders under scenario 1: One is that bidders B1 and B2 join ad exchange Y and bidders B3 and B4 join ad exchange X. The other is that bidders B1 and B2 join ad exchange X and bidders B3 and B4 join ad exchange Y. The two equilibria are highlighted in red in Table C2.

Based on the valuations of bidders in Table C1, it is easy to verify that in both these equilibria, the revenue of each ad exchange will be zero. For example, for the case in which bidders B1 and B2 join ad exchange Y while bidders B3 and B4 join ad exchange X, ad exchange X gets zero revenue because the second-highest bid in an auction of an impression from either publisher P1 or P2 is zero. The same calculation can be done for ad exchange Y. The revenues of ad exchanges X and Y are reported in the table as Rev X and Rev Y, respectively.

The games between the bidders in scenarios 2 through 4 and the corresponding equilibria are reported in a similar fashion in TablesC3 through C5, respectively. Note that we assume that the probability that an impression sold by ad exchange X (resp., ad exchange Y) is from P1 or P2 (resp., P3 or P4) is 0.5. Thus, when an ad exchange hides the publisher identity from bidders, each bidder submits her average valuation as her bid in the auction. For example, from Table 3, Bidder B3 has valuation 1 toward the impressions from P3 and valuation 0 toward the impressions from P4. If ad exchange Y chooses CH policy, then Bidder B3 will submit 0.5 as her bid. Similarly, under the condition that ad exchange Y chooses the CH policy, the valuations from bidders B1, B2, and B4 are 0.4, 0.4, and 0.5, respectively. As an example, consider scenario 2 where ad exchange X chooses the CR policy and ad exchange Y chooses the CH policy. If (say) all the bidders join ad exchange Y, then the bids from bidders B1−B4 are 0.4, 0.4, 0.5, and 0.5, respectively. Therefore, bidders B3 and B4 have an equal chance (i.e., 50%) to win the auction and pay 0.5. The corresponding payoff of bidder B3 and of bidder B4 is $50\% \times [0.5 \times (1 - 0.5) + 0.5 \times (0 - 0.5)] = 0$. The payoff of each of the other two bidders is 0, since they do not win. Therefore, the payoff vector in the last row and last column of Table C3 is (0,0,0,0).

All potential equilibria for the bidders are highlighted in red in Tables C3 through C5; the corresponding revenues of the ad exchanges are also reported. For example, in Scenario 2, one possible equilibrium is that bidders B1 and B2 choose ad exchange X while bidders B3 and B4 choose ad exchange Y. The corresponding revenues of exchanges X and Y are 0 and 0.5, respectively.

Next, we analyze the first stage of the game, namely the game between the ad exchanges. Since there exist multiple equilibria under each scenario, we consider a few possibilities by choosing one possible equilibrium for each scenario.

Possibility 1: In scenario 1, bidders B1 and B2 join ad exchange X while bidders B3 and B4 join ad exchange Y; the corresponding revenues of ad exchanges X and Y are 0 and 0, respectively. In scenario 2, bidders B1 and B2 join ad exchange X while bidders B3 and B4 join ad exchange Y; the corresponding revenues of ad exchanges X and Y are 0 and 0.5, respectively. In scenario 3, bidders B1 and B2 join ad exchange X while bidders B3 and B4 join ad exchange Y; the corresponding revenues of ad exchanges X and Y are 0.5 and 0, respectively. In scenario 4, bidders B1 and B4 join ad exchange Y while bidders B2 and B3 join ad exchange X; the corresponding revenues of ad exchanges X and Y are 0.4 and 0.4, respectively. The game between the two ad exchanges is described in Table C6. *The unique equilibrium for the ad exchanges under this possibility is that both of them choose the CH policy.*

Table C3. Game Between the Bidders in Scenario 2 (The possible equilibria are highlighted in red)

Scenario 2: X chooses CR Policy Y chooses CH Policy				B4			
				X		Y	
				B3		B3	
				X	Y	X	Y
B2	X	B1	X	(0.1, 0.1, 0, 0)	**(0.1, 0.5, 0.5, 0)** **Rev X = 0.4** **Rev Y = 0**	**(0.5, 0.1, 0, 0.5)** **Rev X = 0.4** **Rev Y = 0**	**(0.5, 0.5, 0, 0)** **Rev X = 0** **Rev Y = 0**
			Y	(0.4, 0.1, 0, 0.4)	(0, 0.5, 0.1, 0.4)	(0, 0.1, 0, 0.1)	(0, 0.5, 0, 0)
	Y	B1	X	(0.1, 0.4, 0.4, 0)	(0.1, 0, 0.1, 0)	(0.5, 0, 0.4, 0.1)	(0.5, 0, 0, 0)
			Y	(0, 0, 0.4, 0.4)	(0, 0, 0.1, 0.4)	(0, 0, 0.4, 0.1)	(0, 0, 0, 0)

Table C4. Game Between the Bidders in Scenario 3 (The possible equilibria are highlighted in red)

Scenario 3: X chooses CH Policy Y chooses CR Policy				B4			
				X		Y	
				B3		B3	
				X	Y	X	Y
B2	X	B1	X	(0, 0, 0, 0)	(0, 0, 0.5, 0)	(0, 0, 0, 0.5)	**(0, 0, 0.5, 0.5)** **Rev X = 0.5** **Rev Y = 0**
			Y	(0.4, 0.1, 0, 0)	(0, 0.1, 0.1, 0)	(0.4, 0.1, 0, 0.5)	**(0, 0.5, 0.1, 0.5)** **Rev X = 0** **Rev Y = 0.4**
	Y	B1	X	(0.1, 0.4, 0, 0)	(0.1, 0.4, 0.5, 0)	(0.1, 0, 0, 0.1)	**(0.5, 0, 0.5, 0.1)** **Rev X = 0** **Rev Y = 0.4**
			Y	(0.4, 0.4, 0, 0)	(0, 0.4, 0.1, 0.4)	(0.4, 0, 0.4, 0.1)	(0, 0, 0.1, 0.1)

Table C5 Game Between the Bidders in Scenario 4 (The possible equilibria are highlighted in red)

Scenario 4: X chooses CH Policy Y chooses CH Policy				B4			
				X		Y	
				B3		B3	
				X	Y	X	Y
B2	X	B1	X	(0, 0, 0, 0)	**(0, 0, 0.5, 0)** **Rev X = 0.5** **Rev Y = 0**	**(0, 0, 0, 0.5)** **Rev X = 0.5** **Rev Y = 0**	**(0, 0, 0, 0)** **Rev X = 0.5** **Rev Y = 0.5**
			Y	(0.4, 0.1, 0, 0)	**(0, 0.1, 0.1, 0)** **Rev X = 0.4** **Rev Y = 0.4**	**(0.1, 0, 0, 0.1)** **Rev X = 0.4** **Rev Y = 0.4**	**(0, 0.5, 0, 0)** **Rev X = 0** **Rev Y = 0.5**
	Y	B1	X	(0.1, 0.4, 0, 0)	**(0.1, 0, 0.1, 0)** **Rev X = 0.4** **Rev Y = 0.4**	**(0.1, 0, 0, 0.1)** **Rev X = 0.4** **Rev Y = 0.4**	**(0.5, 0, 0, 0)** **Rev X = 0** **Rev Y = 0.5**
			Y	(0, 0, 0, 0)	(0, 0, 0.1, 0.4)	(0, 0, 0.4, 0.1)	(0, 0, 0, 0)

Table C6. Game Between Ad Exchanges under Possibility 1 (The equilibrium is highlighted in red)

		Y	
		CR Policy	*CH Policy*
X	*CR Policy*	(0,0)	(0,0.5)
	CH Policy	(0.5,0)	**(0.4, 0.4)**

Possibility 2: In scenario 1, bidders B1 and B2 join ad exchange X while bidders B3 and B4 join ad exchange Y; the corresponding revenues of ad exchanges X and Y are 0 and 0, respectively. In scenario 2, bidders B1 and B2 join ad exchange X while bidders B3 and B4 join ad exchange Y; the corresponding revenues of ad exchanges X and Y are 0 and 0.5, respectively. In scenario 3, bidders B1 join ad exchange X while the other three bidders join ad exchange Y; the corresponding revenues of ad exchanges X and Y are 0 and 0.4, respectively. In scenario 4, bidders B1 join ad exchange X while the other three bidders join ad exchange Y; the corresponding revenues of ad exchanges X and Y are 0 and 0.5, respectively. The game between the two ad exchanges is described in Table C7. *There are two equilibria for the ad exchanges under this possibility: One is that both the ad exchanges choose the CH policy while the other is that ad exchange X chooses the CR policy and ad exchange Y chooses the CH policy.*

Table C7. Game Between the Ad Exchanges under Possibility 2 (The possible equilibria are highlighted in red)

		Y	
		CR Policy	*CH Policy*
X	*CR Policy*	(0,0)	**(0,0.5)**
	CH Policy	(0,0.4)	**(0,0.5)**

Possibility 3: In scenario 1, bidders B1 and B2 join ad exchange X while bidders B3 and B4 join ad exchange Y; the corresponding revenues of ad exchanges X and Y are 0 and 0, respectively. In scenario 2, bidders B4 joins ad exchange Y while the other three bidders join ad exchange X; the corresponding revenues of ad exchanges X and Y are 0.4 and 0, respectively. In scenario 3, bidders B1 join ad exchange X while the other three bidders join ad exchange Y; the corresponding revenues of ad exchanges X and Y are 0 and 0.4, respectively. In scenario 4, bidders B1 join ad exchange X while the other three bidders join ad exchange Y; the corresponding revenues of ad exchanges X and Y are 0 and 0.5, respectively. The game between the two ad exchanges is described in Table C8. *There are two equilibria of the ad exchanges under this possibility: One is that both the ad exchanges choose the CR policy while the other is that ad exchange X chooses the CR policy and ad exchange Y chooses the CH policy.*

Table C8. Game Between the Ad Exchanges under Possibility 3 (The possible equilibria are highlighted in red)

		Y	
		CR Policy	*CH Policy*
X	*CR Policy*	**(0,0)**	**(0.4,0)**
	CH Policy	(0,0.4)	(0,0.5)

As the above analysis shows, it is possible that either one of the ad exchanges or both of them choose to hide in equilibrium. This analysis also gives us a glimpse of the complexity involved in studying a more-general setting with an arbitrary number of competing ad exchanges and bidders, and continuous hide/reveal decisions.

Note: *With a more-careful choice of the valuations in Table C1, it should be possible to ensure that each of the second-stage and first-stage games in the analysis above have unique equilibria. Since our goal is only to demonstrate the possibility of ad exchanges choosing to hide in equilibrium, the present analysis is sufficient for our purpose.*