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EFFECTS OF COMPETITION AMONG INTERNET SERVICE PROVIDERS AND CONTENT PROVIDERS ON THE NET NEUTRALITY DEBATE

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Appendix A

Delays under Packet Discrimination

Under outcome *NN*, both CPs receive the same priority on both ISPs. Thus, $w_{CY\{NN\}} = w_{CG\{NN\}} = \frac{1}{\mu - N_{CY\{NN\}}\lambda - N_{CG\{NN\}}\lambda}$ and $w_{DY\{NN\}} = w_{DG\{NN\}} = \frac{1}{\mu - N_{DY\{NN\}}\lambda - N_{DG\{NN\}}\lambda}$.

Under outcome *NY*, both CPs receive the same priority on *C* and *Y* receives higher priority on *D*. Thus, $w_{CY\{NY\}} = w_{CG\{NY\}} = \frac{1}{\mu - N_{CY\{NY\}}\lambda - N_{CG\{NY\}}\lambda}$, $w_{DY\{NY\}} = \frac{1}{\mu - N_{DY\{NY\}}\lambda}$, and $w_{DG\{NY\}} = \frac{\mu}{(\mu - N_{DY\{NY\}}\lambda)(\mu - N_{DY\{NY\}}\lambda - N_{DG\{NY\}}\lambda)}$.

Under outcome NG, both CPs receive the same priority on C and G receives higher priority on D. Thus, $w_{CY\{NG\}} = w_{CG\{NG\}} = \frac{1}{\mu - N_{CY\{NG\}}\lambda - N_{CG\{NG\}}\lambda}$, $w_{DY\{NG\}} = \frac{\mu}{(\mu - N_{DG\{NG\}}\lambda)(\mu - N_{DY\{NG\}}\lambda - N_{DG\{NG\}}\lambda)}$, and $w_{DG\{NG\}} = \frac{1}{\mu - N_{DG\{NG\}}\lambda}$.

Under outcome NB, both CPs receive the same priority on both ISPs. Thus, $w_{CY\{NB\}} = w_{CG\{NB\}} = \frac{1}{\mu - N_{CY\{NB\}}\lambda - N_{CG\{NB\}}\lambda}$ and $w_{DY\{NB\}} = w_{DG\{NB\}} = \frac{1}{\mu - N_{DY\{NB\}}\lambda - N_{DG\{NB\}}\lambda}$.

Under outcome YN, Y receives higher priority on C and both CPs receive the same priority on D. Thus, $w_{CY\{YN\}} = \frac{1}{\mu - N_{CY\{YN\}}\lambda}$, $w_{CG\{YN\}} = \frac{\mu}{(\mu - N_{CY\{YN\}}\lambda)(\mu - N_{CY\{YN\}}\lambda - N_{CG\{YN\}}\lambda)}$, and $w_{DY\{YN\}} = w_{DG\{YN\}} = \frac{1}{\mu - N_{DY\{YN\}}\lambda - N_{DG\{YN\}}\lambda}$.

Under outcome YY, Y receives higher priority on both ISPs. Thus, $w_{CY\{YY\}} = \frac{1}{\mu - N_{CY\{YY\}}\lambda}$, $w_{CG\{YY\}} = \frac{\mu}{(\mu - N_{CY\{YY\}}\lambda)(\mu - N_{CY\{YY\}}\lambda - N_{CG\{YY\}}\lambda)}$, $w_{DY\{YY\}} = \frac{1}{\mu - N_{DY\{YY\}}\lambda}$, and $w_{DG\{YY\}} = \frac{\mu}{(\mu - N_{DY\{YY\}}\lambda)(\mu - N_{DY\{YY\}}\lambda - N_{DG\{YY\}}\lambda)}$.

Under outcome YG, Y receives higher priority on C and G receives higher priority on D. Thus, $w_{CY\{YG\}} = \frac{1}{\mu - N_{CY\{YG\}}\lambda}$, $w_{CG\{YG\}} = \frac{\mu}{(\mu - N_{CY\{YG\}}\lambda)(\mu - N_{CY\{YG\}}\lambda - N_{CG\{YG\}}\lambda)}$, $w_{DY\{YG\}} = \frac{\mu}{(\mu - N_{DG\{YG\}}\lambda)(\mu - N_{DY\{YG\}}\lambda - N_{DG\{YG\}}\lambda)}$, and $w_{DG\{YG\}} = \frac{1}{\mu - N_{DG\{YG\}}\lambda}$.

Under outcome YB, Y receives higher priority on C and both CPs receive the same priority on D. Thus, $w_{CY\{YB\}} = \frac{1}{\mu - N_{CY\{YB\}}\lambda}$, $w_{CG\{YB\}} = \frac{\mu}{(\mu - N_{CY\{YB\}}\lambda)(\mu - N_{CY\{YB\}}\lambda - N_{CG\{YB\}}\lambda)}$, and $w_{DY\{YB\}} = w_{DG\{YB\}} = \frac{1}{\mu - N_{DY\{YB\}}\lambda - N_{DG\{YB\}}\lambda}$.

Under outcome GN, G receives higher priority on C and both CPs receive the same priority on D. Thus, $w_{CY\{GN\}} = \frac{\mu}{(\mu - N_{CG\{GN\}}\lambda)(\mu - N_{CY\{GN\}}\lambda - N_{CG\{GN\}}\lambda)}$, $w_{CG\{GN\}} = \frac{1}{\mu - N_{CG\{GN\}}\lambda}$, and $w_{DY\{GN\}} = w_{DG\{GN\}} = \frac{1}{\mu - N_{DY\{GN\}}\lambda - N_{DG\{GN\}}\lambda}$.

Under outcome GY, G receives higher priority on C and Y receives higher priority on D. Thus, $w_{CY\{GY\}} = \frac{\mu}{(\mu - N_{CG\{GY\}}\lambda)(\mu - N_{CY\{GY\}}\lambda - N_{CG\{GY\}}\lambda)}$, $w_{CG\{GY\}} = \frac{1}{\mu - N_{CG\{GY\}}\lambda}$, $w_{DY\{GY\}} = \frac{1}{\mu - N_{DY\{GY\}}\lambda}$, and $w_{DG\{GY\}} = \frac{\mu}{(\mu - N_{DY\{GY\}}\lambda)(\mu - N_{DY\{GY\}}\lambda - N_{DG\{GY\}}\lambda)}$.

Under outcome GG, G receives higher priority on both ISPs. Thus, $w_{CY\{GG\}} = \frac{\mu}{(\mu - N_{CG\{GG\}}\lambda)(\mu - N_{CY\{GG\}}\lambda - N_{CG\{GG\}}\lambda)}$, $w_{CG\{GG\}} = \frac{1}{\mu - N_{CG\{GG\}}\lambda}$, $w_{DY\{GG\}} = \frac{\mu}{(\mu - N_{DG\{GG\}}\lambda)(\mu - N_{DY\{GG\}}\lambda - N_{DG\{GG\}}\lambda)}$, and $w_{DG\{GG\}} = \frac{1}{\mu - N_{DG\{GG\}}\lambda}$.

Under outcome GB, G receives higher priority on C and both CPs receive the same priority on D. Thus, $w_{CY\{GB\}} = \frac{\mu}{(\mu - N_{CG\{GB\}}\lambda)(\mu - N_{CY\{GB\}}\lambda - N_{CG\{GB\}}\lambda)}$, $w_{CG\{GB\}} = \frac{1}{\mu - N_{CG\{GB\}}\lambda}$, and $w_{DY\{GB\}} = w_{DG\{GB\}} = \frac{1}{\mu - N_{DY\{GB\}}\lambda - N_{DG\{GB\}}\lambda}$.

Under outcome BN, both CPs receive the same priority for both ISPs. Thus, $w_{CY\{BN\}} = w_{CG\{BN\}} = \frac{1}{\mu - N_{CY\{BN\}}\lambda - N_{CG\{BN\}}\lambda}$ and $w_{DY\{BN\}} = w_{DG\{BN\}} = \frac{1}{\mu - N_{DY\{BN\}}\lambda - N_{DG\{BN\}}\lambda}$.

Under outcome BY, both CPs receive the same priority on C and Y receives higher priority on D. Thus, $w_{CY\{BY\}} = w_{CG\{BY\}} = \frac{1}{\mu - N_{CY\{BY\}}\lambda - N_{CG\{BY\}}\lambda}$, $w_{DY\{BY\}} = \frac{1}{\mu - N_{DY\{BY\}}\lambda}$, and $w_{DG\{BY\}} = \frac{\mu}{(\mu - N_{DY\{BY\}}\lambda)(\mu - N_{DY\{BY\}}\lambda - N_{DG\{BY\}}\lambda)}$.

Under outcome BG, both CPs receive the same priority on C and G receives higher priority on D. Thus, $w_{CY\{BG\}} = w_{CG\{BG\}} = \frac{1}{\mu - N_{CY\{BG\}}\lambda - N_{CG\{BG\}}\lambda}$, $w_{DY\{BG\}} = \frac{\mu}{(\mu - N_{DG\{BG\}}\lambda)(\mu - N_{DY\{BG\}}\lambda - N_{DG\{BG\}}\lambda)}$, and $w_{DG\{BG\}} = \frac{1}{\mu - N_{DG\{BG\}}\lambda}$.

Under outcome BB, both CPs receive the same priority for both ISPs. Thus, $w_{CY\{BB\}} = w_{CG\{BB\}} = \frac{1}{\mu - N_{CY\{BB\}}\lambda - N_{CG\{BB\}}\lambda}$ and $w_{DY\{BB\}} = w_{DG\{BB\}} = \frac{1}{\mu - N_{DY\{BB\}}\lambda - N_{DG\{BB\}}\lambda}$.

Appendix B

CPs' Incentive Compatibility Constraints

Under outcome NN, CPs' incentive compatibility constraints are $\pi_{Y\{NN\}} \geq \pi_{Y\{YN\}}, \pi_{Y\{NY\}}, \pi_{Y\{YY\}}$ and $\pi_{G\{NN\}} \geq \pi_{G\{GN\}}, \pi_{G\{NG\}}, \pi_{G\{GG\}}$.

Under outcome NY, CPs' incentive compatibility constraints are $\pi_{Y\{NY\}} \geq \pi_{Y\{YY\}}, \pi_{Y\{NN\}}, \pi_{Y\{YN\}}$ and $\pi_{G\{NY\}} \geq \pi_{G\{GY\}}, \pi_{G\{NB\}}, \pi_{G\{GB\}}$.

Under outcome NG, CPs' incentive compatibility constraints are $\pi_{Y\{NG\}} \geq \pi_{Y\{YG\}}, \pi_{Y\{NB\}}, \pi_{Y\{YB\}}$ and $\pi_{G\{NG\}} \geq \pi_{G\{GG\}}, \pi_{G\{NN\}}, \pi_{G\{GN\}}$.

Under outcome NB, CPs' incentive compatibility constraints are $\pi_{Y\{NB\}} \geq \pi_{Y\{YB\}}, \pi_{Y\{NG\}}, \pi_{Y\{YG\}}$ and $\pi_{G\{NB\}} \geq \pi_{G\{GB\}}, \pi_{G\{NY\}}, \pi_{G\{GY\}}$.

Under outcome YN, CPs' incentive compatibility constraints are $\pi_{Y\{YN\}} \geq \pi_{Y\{NN\}}, \pi_{Y\{YY\}}, \pi_{Y\{NY\}}$ and $\pi_{G\{YN\}} \geq \pi_{G\{BN\}}, \pi_{G\{YG\}}, \pi_{G\{BG\}}$.

Under outcome YY, CPs' incentive compatibility constraints are $\pi_{Y\{YY\}} \geq \pi_{Y\{NY\}}, \pi_{Y\{YN\}}, \pi_{Y\{NN\}}$ and $\pi_{G\{YY\}} \geq \pi_{G\{BY\}}, \pi_{G\{YB\}}, \pi_{G\{BB\}}$.

Under outcome YG, CPs' incentive compatibility constraints are $\pi_{Y\{YG\}} \geq \pi_{Y\{NG\}}, \pi_{Y\{YB\}}, \pi_{Y\{NB\}}$ and $\pi_{G\{YG\}} \geq \pi_{G\{BG\}}, \pi_{G\{YN\}}, \pi_{G\{BN\}}$.

Under outcome YB, CPs' incentive compatibility constraints are $\pi_{Y\{YB\}} \geq \pi_{Y\{NB\}}, \pi_{Y\{YG\}}, \pi_{Y\{NG\}}$ and $\pi_{G\{YB\}} \geq \pi_{G\{BB\}}, \pi_{G\{YY\}}, \pi_{G\{BY\}}$.

Under outcome GN, CPs' incentive compatibility constraints are $\pi_{Y\{GN\}} \geq \pi_{Y\{BN\}}, \pi_{Y\{GY\}}, \pi_{Y\{BY\}}$ and $\pi_{G\{GN\}} \geq \pi_{G\{NN\}}, \pi_{G\{GG\}}, \pi_{G\{NG\}}$.

Under outcome GY, CPs' incentive compatibility constraints are $\pi_{Y\{GY\}} \geq \pi_{Y\{BY\}}, \pi_{Y\{GN\}}, \pi_{Y\{BN\}}$ and $\pi_{G\{GY\}} \geq \pi_{G\{NY\}}, \pi_{G\{GB\}}, \pi_{G\{NB\}}$.

Under outcome GG, CPs' incentive compatibility constraints are $\pi_{Y\{GG\}} \geq \pi_{Y\{BG\}}, \pi_{Y\{GB\}}, \pi_{Y\{BB\}}$ and $\pi_{G\{GG\}} \geq \pi_{G\{NG\}}, \pi_{G\{GN\}}, \pi_{G\{NN\}}$.

Under outcome GB, CPs' incentive compatibility constraints are $\pi_{Y\{GB\}} \geq \pi_{Y\{BB\}}, \pi_{Y\{GG\}}, \pi_{Y\{BG\}}$ and $\pi_{G\{GB\}} \geq \pi_{G\{NB\}}, \pi_{G\{GY\}}, \pi_{G\{NY\}}$.

Under outcome BN, CPs' incentive compatibility constraints are $\pi_{Y\{BN\}} \geq \pi_{Y\{GN\}}, \pi_{Y\{BY\}}, \pi_{Y\{GY\}}$ and $\pi_{G\{BN\}} \geq \pi_{G\{YN\}}, \pi_{G\{BG\}}, \pi_{G\{YG\}}$.

Under outcome BY, CPs' incentive compatibility constraints are $\pi_{Y\{BY\}} \geq \pi_{Y\{GY\}}, \pi_{Y\{BN\}}, \pi_{Y\{GN\}}$ and $\pi_{G\{BY\}} \geq \pi_{G\{YY\}}, \pi_{G\{BB\}}, \pi_{G\{YB\}}$.

Under outcome BG, CPs' incentive compatibility constraints are $\pi_{Y\{BG\}} \geq \pi_{Y\{GG\}}, \pi_{Y\{BB\}}, \pi_{Y\{GB\}}$ and $\pi_{G\{BG\}} \geq \pi_{G\{YG\}}, \pi_{G\{BN\}}, \pi_{G\{YN\}}$.

Under outcome BB, CPs' incentive compatibility constraints are $\pi_{Y\{BB\}} \geq \pi_{Y\{GB\}}, \pi_{Y\{BG\}}, \pi_{Y\{GG\}}$ and $\pi_{G\{BB\}} \geq \pi_{G\{YB\}}, \pi_{G\{BY\}}, \pi_{G\{YY\}}$.

Appendix C

Proof of Lemma 1: The Symmetric Equilibrium Case

Consumers have four choices of ISP-CP combinations: CY, CG, DY, and DG. Consumer demands for these four ISP-CP combinations can be derived by analyzing the curves of indifferent consumers. There are six curves of indifferent consumers based on the pairwise comparisons among the four ISP-CP combinations. For a given outcome ij, where $i, j = N$ (Neither CP pays), Y (Only Y pays), G (Only G pays), and B (Both CPs pay), these six curves of indifferent consumers can be characterized by four points $x_{C\{ij\}}$, $x_{D\{ij\}}$, $z_{Y\{ij\}}$, and $z_{G\{ij\}}$: consumers located on $x = x_{C\{ij\}}$ are indifferent between CY and CG; consumers located on $x = x_{D\{ij\}}$ are indifferent between DY and DG; consumers located on $z = z_{Y\{ij\}}$ are indifferent between CY and DY; consumers located on $z = z_{G\{ij\}}$ are indifferent between CG and DG; consumers located on the line that goes through points $\left(x_{C\{ij\}}, z_{Y\{ij\}}\right)$ and $\left(x_{D\{ij\}}, z_{G\{ij\}}\right)$ are indifferent between CY and DG; and consumers located on the line that goes through points $\left(x_{C\{ij\}}, z_{G\{ij\}}\right)$ and $\left(x_{D\{ij\}}, z_{Y\{ij\}}\right)$ are indifferent between CG and DY.

Comparing consumers' utility functions for the corresponding pairs of ISP-CP combinations yields $x_{C\{ij\}} = \frac{1}{2} + \frac{d\lambda\left(w_{CG\{ij\}} - w_{CY\{ij\}}\right)}{2t}$, $x_{D\{ij\}} = \frac{1}{2} + \frac{d\lambda\left(w_{DG\{ij\}} - w_{DY\{ij\}}\right)}{2t}$, $z_{Y\{ij\}} = \frac{1}{2} + \frac{F_D - F_C}{2k} + \frac{d\lambda\left(w_{DY\{ij\}} - w_{CY\{ij\}}\right)}{2k}$, and $z_{G\{ij\}} = \frac{1}{2} + \frac{F_D - F_C}{2k} + \frac{d\lambda\left(w_{DG\{ij\}} - w_{CG\{ij\}}\right)}{2k}$. Considering symmetric equilibrium with $F_C = F_D$, we have $z_{Y\{ij\}} = \frac{1}{2} + \frac{d\lambda(w_{DY\{ij\}} - w_{CY\{ij\}})}{2k}$ and $z_{G\{ij\}} = \frac{1}{2} + \frac{d\lambda(w_{DG\{ij\}} - w_{CG\{ij\}})}{2k}$. We observe that the sign of $x_{C\{ij\}} - x_{D\{ij\}}$ is the same as the sign of $z_{Y\{ij\}} - z_{G\{ij\}}$. In particular, $x_{C\{ij\}} = x_{D\{ij\}}$ if and only if $z_{Y\{ij\}} = z_{G\{ij\}}$.

Each outcome ij is determined by the ISPs' pricing decisions and the corresponding content providers' delivery service choices. We use indicator functions $I_{CY\{ij\}}$, $I_{CG\{ij\}}$, $I_{DY\{ij\}}$ and $I_{DG\{ij\}}$, which take values of 0 or 1, to represent whether content providers Y and G would pay for preferential delivery on ISPs C and D. To be consistent with the four ISP-CP combinations on the unit square, we denote outcome ij by the matrix $\begin{bmatrix} I_{DY\{ij\}} & I_{DG\{ij\}} \\ I_{CY\{ij\}} & I_{CG\{ij\}} \end{bmatrix}$. We introduce two types of actions (horizontal and vertical flips) to explore the connections among the 16 outcomes:

Horizontal Flip: Decisions of Y and G are simultaneously interchanged on ISPs C and D. Specifically, horizontal flip changes outcome ij dictated by $\begin{bmatrix} I_{DY\{ij\}} & I_{DG\{ij\}} \\ I_{CY\{ij\}} & I_{CG\{ij\}} \end{bmatrix}$ to outcome $i'j'$ dictated by $\begin{bmatrix} I_{DG\{ij\}} & I_{DY\{ij\}} \\ I_{CG\{ij\}} & I_{CY\{ij\}} \end{bmatrix}$, where $i' = \begin{cases} N, & \text{if } i = N \\ G, & \text{if } i = Y \\ Y, & \text{if } i = G \\ B, & \text{if } i = B \end{cases}$ and $j' = \begin{cases} N, & \text{if } j = N \\ G, & \text{if } j = Y \\ Y, & \text{if } j = G \\ B, & \text{if } j = B \end{cases}$.

Vertical Flip: Decisions of Y and G are simultaneously interchanged across ISPs C and D. Specifically, vertical flip changes outcome ij dictated by $\begin{bmatrix} I_{DY\{ij\}} & I_{DG\{ij\}} \\ I_{CY\{ij\}} & I_{CG\{ij\}} \end{bmatrix}$ to outcome ji dictated by $\begin{bmatrix} I_{CY\{ij\}} & I_{CG\{ij\}} \\ I_{DY\{ij\}} & I_{DG\{ij\}} \end{bmatrix}$.

Among the 16 outcomes, some outcomes permute amongst themselves when horizontal flip or vertical flip is applied and therefore can be grouped together into four invariant classes: (a) outcomes NN, NB, BN, and BB; (b) outcomes YY and GG; (c) outcomes YG and GY; (d) outcomes NY, NG, YN, GN, BY, BG, YB, and GB. In the following discussion, we give precise description of the changes to the indifferent customers when horizontal flip or vertical flip is applied to an outcome.

Applying Horizontal Flip: The decisions of Y on the two ISPs are interchanged with the decisions of G in a given outcome. Horizontal flip changes outcome ij dictated by $\begin{bmatrix} I_{DY\{ij\}} & I_{DG\{ij\}} \\ I_{CY\{ij\}} & I_{CG\{ij\}} \end{bmatrix}$ to outcome $i'j'$ dictated by $\begin{bmatrix} I_{DG\{ij\}} & I_{DY\{ij\}} \\ I_{CG\{ij\}} & I_{CY\{ij\}} \end{bmatrix}$. That is we have $I_{CY\{i'j'\}} = I_{CG\{ij\}}$, $I_{CG\{i'j'\}} = I_{CY\{ij\}}$, $I_{DY\{i'j'\}} = I_{DG\{ij\}}$, and $I_{DG\{i'j'\}} = I_{DY\{ij\}}$. When the decisions in outcome ij are changed to $i'j'$, the decisions of Y on C and D and the decisions of G on C and D are interchanged. The queuing priorities are interchanged on ISPs C and D. This simultaneously interchanges the waiting times and market demand on C and D according to the new queuing priorities. We note that fees for all customers are equal so the redistribution is dependent solely on waiting times. Interchanging waiting times on ISPs C and D yields $w_{CY\{i'j'\}} = w_{CG\{ij\}}$, $w_{CG\{i'j'\}} = w_{CY\{ij\}}$, $w_{DY\{i'j'\}} = w_{DG\{ij\}}$, and $w_{DG\{i'j'\}} = w_{DY\{ij\}}$. This gives $x_{C\{ij\}} + x_{C\{i'j'\}} = \frac{1}{2} + \frac{d\lambda(w_{CG\{ij\}} - w_{CY\{ij\}})}{2t} + \frac{1}{2} + \frac{d\lambda(w_{CG\{i'j'\}} - w_{CY\{i'j'\}})}{2t} = \frac{1}{2} + \frac{d\lambda(w_{CG\{ij\}} - w_{CY\{ij\}})}{2t} + \frac{1}{2} - \frac{d\lambda(w_{CG\{ij\}} - w_{CY\{ij\}})}{2t} = 1$, which implies $x_{C\{i'j'\}} = 1 - x_{C\{ij\}}$. Similarly, we have $x_{D\{i'j'\}} = 1 - x_{D\{ij\}}$, $z_{Y\{ij\}} = z_{G\{i'j'\}}$, and $z_{G\{ij\}} = z_{Y\{i'j'\}}$. We note that the positions of these curves of indifferent consumers relative to the line of $x = \frac{1}{2}$ or $z = \frac{1}{2}$ remain the same according to the decisions of Y and G.

Applying Vertical Flip: The decisions of Y and G on C are interchanged with their decisions on D in a given outcome. Vertical flip changes outcome ij dictated by $\begin{bmatrix} I_{DY\{ij\}} & I_{DG\{ij\}} \\ I_{CY\{ij\}} & I_{CG\{ij\}} \end{bmatrix}$ to outcome ji dictated by $\begin{bmatrix} I_{CY\{ij\}} & I_{CG\{ij\}} \\ I_{DY\{ij\}} & I_{DG\{ij\}} \end{bmatrix}$. That is we have $I_{CY\{ji\}} = I_{DY\{ij\}}$, $I_{CG\{ji\}} = I_{DG\{ij\}}$, $I_{DY\{ji\}} = I_{CY\{ij\}}$, and $I_{DG\{ji\}} = I_{CG\{ij\}}$. When the decisions in outcome ij are changed to ji, the decisions of Y and G on C are swapped with the decisions of Y and G on D. The queuing priorities are interchanged on ISPs C and D. This simultaneously interchanges the waiting times and market demand on ISPs C and D according to the new queuing priorities. We note that fees for all customers are equal so the redistribution is dependent solely on waiting times. Interchanging waiting times on ISPs C and D yields $w_{CY\{ji\}} = w_{DY\{ij\}}$, $w_{CG\{ji\}} = w_{DG\{ij\}}$, $w_{DY\{ji\}} = w_{CY\{ij\}}$, and $w_{DG\{ji\}} = w_{CG\{ij\}}$. This gives $x_{C\{ij\}} = \frac{1}{2} + \frac{d\lambda(w_{CG\{ij\}} - w_{CY\{ij\}})}{2k} = \frac{1}{2} + \frac{d\lambda(w_{DG\{ji\}} - w_{DY\{ji\}})}{2k} = x_{D\{ji\}}$. Similarly, $x_{D\{ij\}} = x_{C\{ji\}}$. We also have $z_{Y\{ij\}} + z_{Y\{ji\}} = \frac{1}{2} + \frac{d\lambda(w_{DY\{ij\}} - w_{CY\{ij\}})}{2t} + \frac{1}{2} + \frac{d\lambda(w_{DY\{ji\}} - w_{CY\{ji\}})}{2t} = \frac{1}{2} - \frac{d\lambda(w_{DY\{ji\}} - w_{CY\{ji\}})}{2t} + \frac{1}{2} + \frac{d\lambda(w_{DY\{ji\}} - w_{CY\{ji\}})}{2t} = 1$, which implies $z_{Y\{ji\}} = 1 - z_{Y\{ij\}}$. Similarly, $z_{G\{ji\}} = 1 - z_{G\{ij\}}$.

Next we apply the above results of horizontal and vertical flips to each of the classes (a) through (d) to characterize the demand distribution under each outcome.

(Class a) Outcomes NN, NB, BN, and BB

Under outcomes NN, NB, BN, and BB, all customers have equal queuing priorities. Therefore applying horizontal flip or vertical flip to these outcomes will not change the queuing priorities. Hence the indifferent consumers remain unchanged when horizontal flip or vertical flip is applied.

From horizontal flip relations, we have $x_{C\{NN\}} = 1 - x_{C\{NN\}}$, $x_{D\{NN\}} = 1 - x_{D\{NN\}}$, $x_{C\{NB\}} = 1 - x_{C\{NB\}}$, $x_{D\{NB\}} = 1 - x_{D\{NB\}}$, $x_{C\{BN\}} = 1 - x_{C\{BN\}}$, $x_{D\{BN\}} = 1 - x_{D\{BN\}}$, $x_{C\{BB\}} = 1 - x_{C\{BB\}}$, and $x_{D\{BB\}} = 1 - x_{D\{BB\}}$. That is $x_{C\{NN\}} = x_{C\{NB\}} = x_{C\{BN\}} = x_{C\{BB\}} = \frac{1}{2}$ and $x_{D\{NN\}} = x_{D\{NB\}} = x_{D\{BN\}} = x_{D\{BB\}} = \frac{1}{2}$. From vertical flip relations, we have $z_{G\{NN\}} = 1 - z_{G\{NN\}}$, $z_{Y\{NN\}} = 1 - z_{Y\{NN\}}$, $z_{G\{NB\}} = 1 - z_{G\{NB\}}$, $z_{Y\{NB\}} = 1 - z_{Y\{NB\}}$, $z_{G\{BN\}} = 1 - z_{G\{BN\}}$, $z_{Y\{BN\}} = 1 - z_{Y\{BN\}}$, $z_{G\{BB\}} = 1 - z_{G\{BB\}}$, $z_{Y\{BB\}} = 1 - z_{Y\{BB\}}$. That is $z_{G\{NN\}} = z_{G\{NB\}} = z_{G\{BN\}} = z_{G\{BB\}} = \frac{1}{2}$ and $z_{Y\{NN\}} = z_{Y\{NB\}} = z_{Y\{BN\}} = z_{Y\{BB\}} = \frac{1}{2}$.

Therefore, as shown in Figure C1, the market demand for CY, DY, CG, and DG are equal under outcomes NN, NB, BN, and BB. That is $N_{CY\{NN\}} = N_{DY\{NN\}} = N_{CG\{NN\}} = N_{DG\{NN\}} = \frac{1}{4}$, $N_{CY\{NB\}} = N_{DY\{NB\}} = N_{CG\{NB\}} = N_{DG\{NB\}} = \frac{1}{4}$, $N_{CY\{BN\}} = N_{DY\{BN\}} = N_{CG\{BN\}} = N_{DG\{BN\}} = \frac{1}{4}$, and $N_{CY\{BB\}} = N_{DY\{BB\}} = N_{CG\{BB\}} = N_{DG\{BB\}} = \frac{1}{4}$.

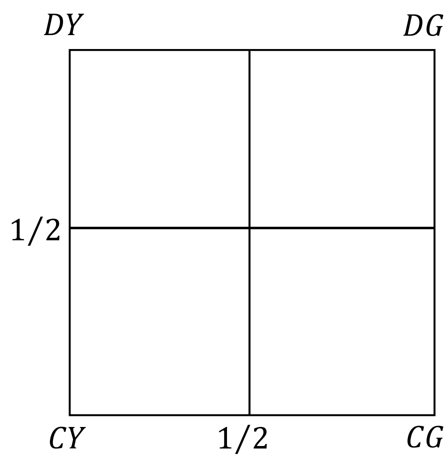


Figure C1. Demand Distribution of Class a (outcomes NN, NB, BN, and BB)

(Class b) Outcomes YY and GG

Under outcome YY, only Y pays for preferential delivery on both ISPs. Under outcome GG, only G pays for preferential delivery on both ISPs. Thus, $w_{CG\{YY\}} - w_{CY\{YY\}} > 0$, $w_{DG\{YY\}} - w_{DY\{YY\}} > 0$, $w_{CG\{GG\}} - w_{CY\{GG\}} < 0$, and $w_{DG\{GG\}} - w_{DY\{GG\}} < 0$.

Vertical flip does not change the decisions of Y and G on C and D in outcomes YY and GG. Therefore we have $x_{C\{YY\}} = x_{D\{YY\}} > \frac{1}{2}$, $z_{Y\{YY\}} = 1 - z_{Y\{YY\}} \implies z_{Y\{YY\}} = \frac{1}{2}$, $z_{G\{YY\}} = 1 - z_{G\{YY\}} \implies z_{G\{YY\}} = \frac{1}{2}$, $x_{C\{GG\}} = x_{D\{GG\}} < \frac{1}{2}$, $z_{Y\{GG\}} = 1 - z_{Y\{GG\}} \implies z_{Y\{GG\}} = \frac{1}{2}$, and $z_{G\{GG\}} = 1 - z_{G\{GG\}} \implies z_{G\{GG\}} = \frac{1}{2}$. Moreover, horizontal flip applied to outcome YY gives outcome GG and vice versa. This gives $x_{C\{YY\}} = 1 - x_{C\{GG\}} = x_{D\{YY\}} = 1 - x_{D\{GG\}}$. Thus, we simplify the notations to $x_{C\{YY\}} = x_{D\{YY\}} = x_{\{YY\}} > \frac{1}{2}$ and $x_{C\{GG\}} = x_{D\{GG\}} = x_{\{GG\}} < \frac{1}{2}$. Therefore, as shown in Figure C2, the demands for CY, DY, CG, and DG in outcomes YY and GG are related such that $N_{CY\{YY\}} = N_{DY\{YY\}} = N_{CG\{GG\}} = N_{DG\{GG\}} = \frac{1-x_{\{GG\}}}{2}$ and $N_{CY\{GG\}} = N_{DY\{GG\}} = N_{CG\{YY\}} = N_{DG\{YY\}} = \frac{x_{\{GG\}}}{2}$. In other words, ISPs C and D have the same market share, i.e., $N_{C\{YY\}} = N_{D\{YY\}} = N_{C\{GG\}} = N_{D\{GG\}} = \frac{1}{2}$. Within each ISP, the paying CP gets more customers than the non-paying CP, i.e., $N_{CY\{YY\}} = N_{DY\{YY\}} = N_{CG\{GG\}} = N_{DG\{GG\}} > \frac{1}{4} > N_{CY\{GG\}} = N_{DY\{GG\}} = N_{CG\{YY\}} = N_{DG\{YY\}}$.

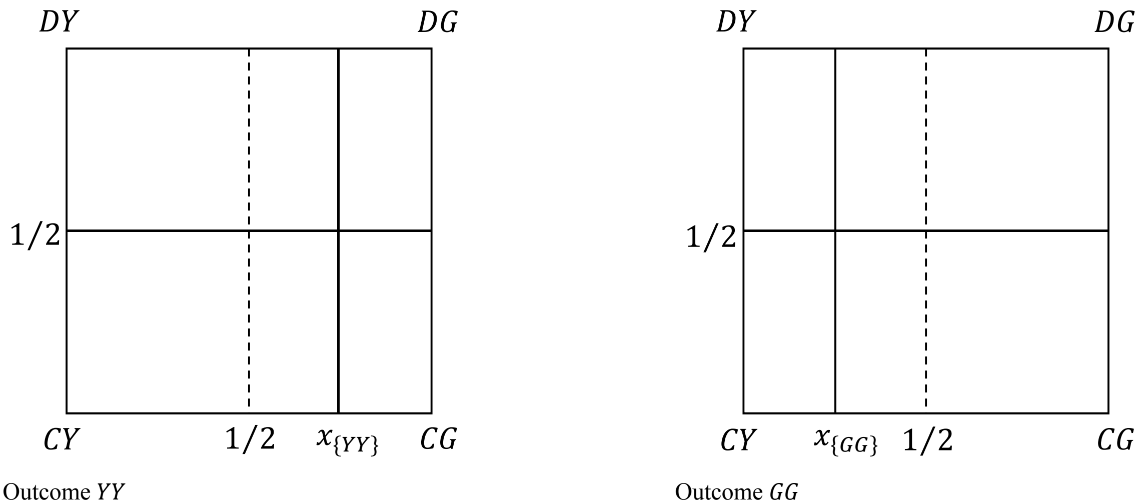


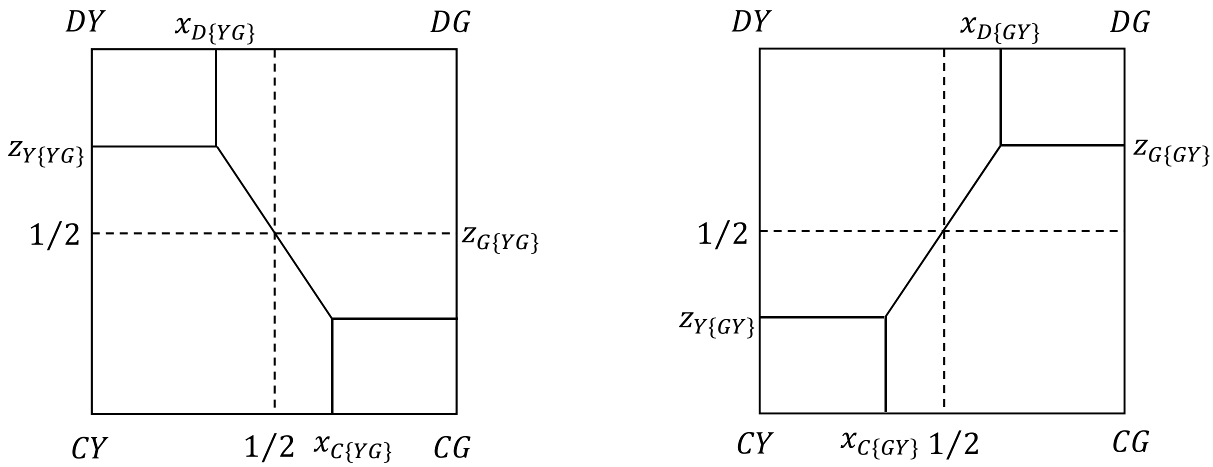
Figure C2. Demand Distribution of Class b (outcomes YY and GG)

(Class c) Outcomes YG and GY

Under outcome YG, only Y pays for preferential delivery on C and only G pays for preferential delivery on D. Under outcome GY, only G pays for preferential delivery on C and only Y pays for preferential delivery on D. Thus $w_{CG\{YG\}} - w_{CY\{YG\}} > 0$, $w_{DG\{YG\}} - w_{DY\{YG\}} < 0$, $w_{CG\{GY\}} - w_{CY\{GY\}} < 0$, and $w_{DG\{GY\}} - w_{DY\{GY\}} > 0$. Therefore we have $x_{C\{YG\}} > \frac{1}{2} > x_{D\{YG\}}$ and $x_{C\{GY\}} < \frac{1}{2} < x_{D\{GY\}}$. Since the sign of $x_{C\{ij\}} - x_{D\{ij\}}$ is the same as the sign of $z_{Y\{ij\}} - z_{G\{ij\}}$ for any outcome ij, we have $z_{Y\{YG\}} > z_{G\{YG\}}$, and $z_{Y\{GY\}} < z_{G\{GY\}}$.

Observe that both horizontal flip and vertical flip applied to outcome YG gives outcome GY and vice versa. Through the connection of horizontal flip, we have $x_{C\{YG\}} = 1 - x_{C\{GY\}}$, $x_{D\{YG\}} = 1 - x_{D\{GY\}}$, $z_{Y\{YG\}} = z_{G\{GY\}}$, and $z_{G\{YG\}} = z_{Y\{GY\}}$. Through the connection of vertical flip, we have $x_{C\{YG\}} = x_{D\{GY\}}$, $x_{D\{YG\}} = x_{C\{GY\}}$, $z_{Y\{YG\}} = 1 - z_{Y\{GY\}}$, and $z_{G\{YG\}} = 1 - z_{G\{GY\}}$. Combining the two set of equalities gives $x_{D\{YG\}} = 1 - x_{C\{YG\}}$, $x_{D\{GY\}} = 1 - x_{C\{GY\}}$, $z_{G\{YG\}} = 1 - z_{Y\{YG\}}$, and $z_{G\{GY\}} = 1 - z_{Y\{GY\}}$. Since $z_{Y\{YG\}} > z_{G\{YG\}}$ and $z_{Y\{GY\}} < z_{G\{GY\}}$, the last set of equalities says that $z_{Y\{YG\}} > \frac{1}{2} > z_{G\{YG\}}$ and $z_{Y\{GY\}} < \frac{1}{2} < z_{G\{GY\}}$. This says that the indifferent consumers $x_{C\{YG\}}$ and $x_{D\{YG\}}$ (as well as $x_{C\{GY\}}$ and $x_{D\{GY\}}$) are symmetrically positioned on either side of $x = \frac{1}{2}$. Likewise, $z_{Y\{YG\}}$ and $z_{G\{YG\}}$ (as well as $z_{Y\{GY\}}$ and $z_{G\{GY\}}$) are symmetrically positioned on either side of $z = \frac{1}{2}$. Therefore the demands for CY, DY, CG, and DG in outcomes YG and GY are related such that $N_{CY\{YG\}} = N_{DG\{YG\}} = N_{CG\{GY\}} = N_{DY\{GY\}}$ and $N_{CG\{YG\}} = N_{DY\{YG\}} = N_{CY\{GY\}} = N_{DG\{GY\}}$.

As shown in Figure C3, ISPs C and D have the same market share, i.e., $N_{C\{YG\}} = N_{D\{YG\}} = N_{C\{GY\}} = N_{D\{GY\}} = \frac{1}{2}$. Within each ISP, the paying CP gets more customers than the non-paying CP, i.e., $N_{CY\{YG\}} = N_{DG\{YG\}} = N_{CG\{GY\}} = N_{DY\{GY\}} > \frac{1}{4} > N_{CY\{GY\}} = N_{DG\{GY\}} = N_{CG\{YG\}} = N_{DY\{YG\}}$.



Outcome YG Outcome GY

Figure C3. Demand Distribution of Class c (outcomes YG and GY)

(Class d) Outcomes NY, NG, YN, GN, BY, BG, YB, and GB

Based on CPs' delivery service choices in outcomes NY, NG, YN, GN, BY, BG, YB, and GB, we know that $w_{CG\{NY\}} - w_{CY\{NY\}} = w_{CG\{NG\}} - w_{CY\{NG\}} = 0$, $w_{DG\{YN\}} - w_{DY\{YN\}} = w_{DG\{GN\}} - w_{DY\{GN\}} = 0$, $w_{DG\{NY\}} - w_{DY\{NY\}} > 0$, $w_{DG\{NG\}} - w_{DY\{NG\}} < 0$, $w_{CG\{YN\}} - w_{CY\{YN\}} > 0$, and $w_{CG\{GN\}} - w_{CY\{GN\}} < 0$. Therefore we have $x_{C\{NY\}} = x_{C\{NG\}} = \frac{1}{2}$, $x_{D\{YN\}} = x_{D\{GN\}} = \frac{1}{2}$, $x_{D\{NY\}} > \frac{1}{2} > x_{D\{NG\}}$, and $x_{C\{YN\}} > \frac{1}{2} > x_{C\{GN\}}$. Since the sign of $x_{C\{ij\}} - x_{D\{ij\}}$ is the same as the sign of $z_{Y\{ij\}} - z_{G\{ij\}}$ for any outcome ij, we have $z_{Y\{NY\}} < z_{G\{NY\}}$ and $z_{Y\{YN\}} > z_{G\{YN\}}$. Likewise, we have $z_{Y\{NG\}} > z_{G\{NG\}}$ and $z_{Y\{GN\}} < z_{G\{GN\}}$.

Successive applications of horizontal flip and vertical flip connect outcomes NY, NG, YN, and GN as follows:

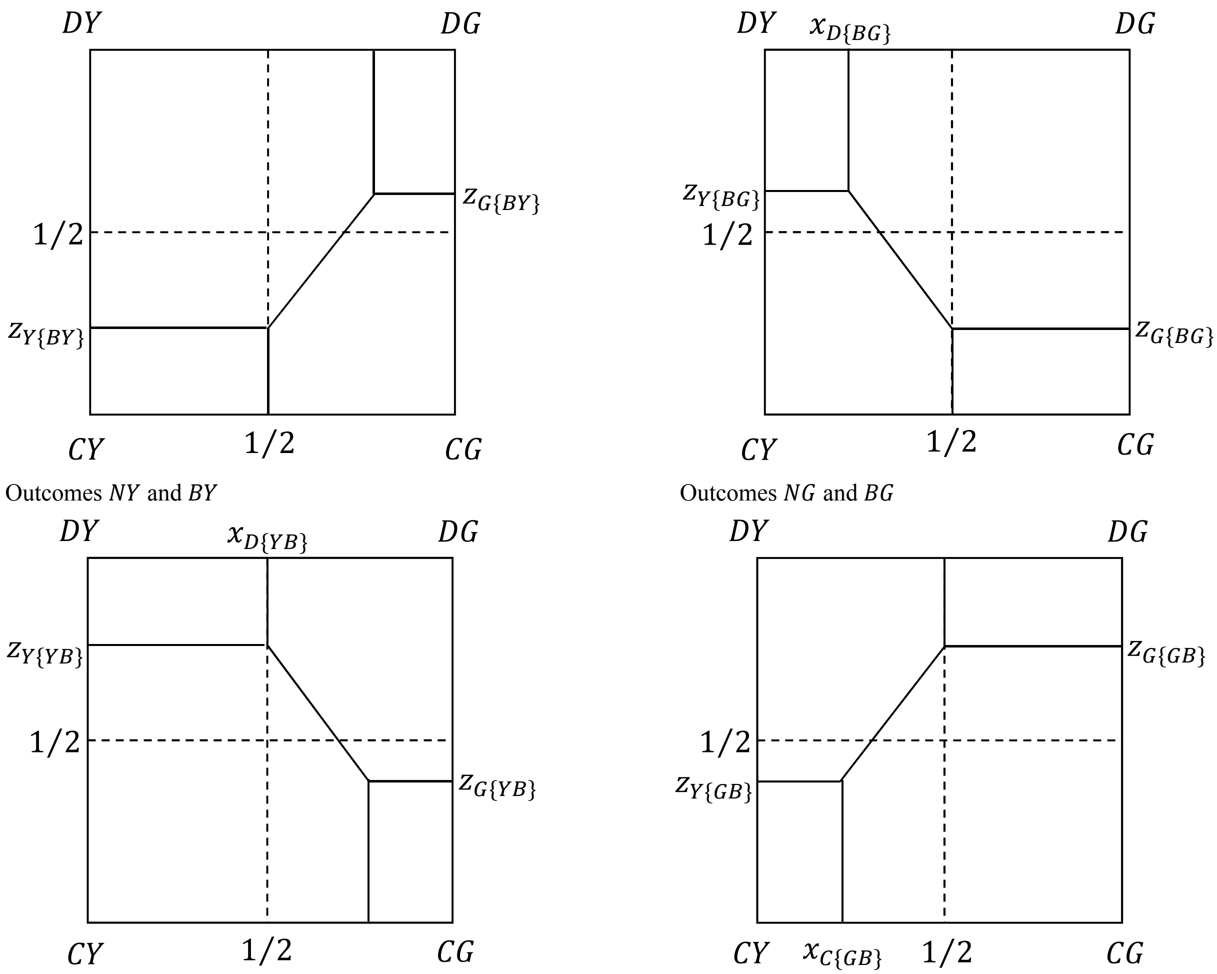
$$\begin{array}{ccc} \text{Outcome } NY & \xleftrightarrow{\text{Horizontal Flip}} & \text{Outcome } NG \\ \text{Vertical Flip} \updownarrow & & \updownarrow \text{Vertical Flip} \\ \text{Outcome } YN & \xleftrightarrow{\text{Horizontal Flip}} & \text{Outcome } GN \end{array}$$

Through horizontal flip, we have $x_{C\{NG\}} = 1 - x_{C\{NY\}} = \frac{1}{2}$, $x_{D\{NG\}} = 1 - x_{D\{NY\}} < \frac{1}{2}$, $z_{Y\{NG\}} = z_{G\{NY\}}$, $z_{G\{NG\}} = z_{Y\{NY\}}$, $x_{C\{GN\}} = 1 - x_{C\{YN\}} < \frac{1}{2}$, $x_{D\{GN\}} = 1 - x_{D\{YN\}} = \frac{1}{2}$, $z_{Y\{GN\}} = z_{G\{YN\}}$, and $z_{G\{GN\}} = z_{Y\{YN\}}$. Through vertical flip, we have $x_{C\{YN\}} = x_{D\{NY\}} > \frac{1}{2}$, $x_{D\{YN\}} = x_{C\{NY\}} = \frac{1}{2}$, $z_{Y\{YN\}} = 1 - z_{Y\{NY\}}$, $z_{G\{YN\}} = 1 - z_{G\{NY\}}$, $x_{C\{GN\}} = x_{D\{NG\}} < \frac{1}{2}$, $x_{D\{GN\}} = x_{C\{NG\}} = \frac{1}{2}$, $z_{Y\{GN\}} = 1 - z_{Y\{NG\}}$, and $z_{G\{GN\}} = 1 - z_{G\{NG\}}$. Therefore the demand for CY, DY, CG, and DG in outcomes NY, NG, YN, and GN are related such that $N_{DY\{NY\}} = N_{DG\{NG\}} = N_{CG\{GN\}} = N_{CY\{YN\}}$, $N_{DG\{NY\}} = N_{DY\{NG\}} = N_{CY\{GN\}} = N_{CG\{YN\}}$, $N_{CG\{NY\}} = N_{CY\{NG\}} = N_{DY\{GN\}} = N_{DG\{YN\}}$, and $N_{CY\{NY\}} = N_{CG\{NG\}} = N_{DG\{GN\}} = N_{DY\{YN\}}$.

The demand analysis for outcomes BY, BG, YB, and GB is the same as that in outcomes NY, NG, YN, and GN since both CPs receive the same queuing priority when they both pay for preferential delivery. Therefore, the demand for CY, DY, CG, and DG in outcomes BY, BG, YB, and GB are related such that $N_{DY\{BY\}} = N_{DG\{BG\}} = N_{CG\{GB\}} = N_{CY\{YB\}}$, $N_{DG\{BY\}} = N_{DY\{BG\}} = N_{CY\{GB\}} = N_{CG\{YB\}}$, $N_{CG\{BY\}} = N_{CY\{BG\}} = N_{DY\{GB\}} = N_{DG\{YB\}}$, and $N_{CY\{BY\}} = N_{CG\{BG\}} = N_{DG\{GB\}} = N_{DY\{YB\}}$.

If Y and G make identical decisions on any ISP (C or D), consumers on that ISP will receive the same queuing priority. For example, under outcomes NG and BG, indifferent consumers of all four ISP-CP combinations are the same, which leads to identical demand distribution for CY, DY, CG, and DG. That is $N_{CY\{NG\}} = N_{CY\{BG\}}$, $N_{DY\{NG\}} = N_{DY\{BG\}}$, $N_{CG\{NG\}} = N_{CG\{BG\}}$, and $N_{DG\{NG\}} = N_{DG\{BG\}}$. By the same arguments above, we obtain the pairings with identical demand distribution for CY, DY, CG, and DG: outcomes NY and BY, outcomes YN and YB, and outcomes GN and GB.

As shown in Figure C4, outcomes in class d reveal particularly interesting demand patterns. For example, in outcome BG, although both CPs pay for preferential delivery on ISP C, Y gets fewer consumers than G from ISP C, i.e., $N_{CY\{BG\}} > N_{CG\{BG\}}$.



Outcomes YN and YB | Outcomes GN and GB

Figure C4. Demand Distribution of Class d (outcomes NY, NG, YN, GN, BY, BG, YB, and GB)

Summarizing the above analysis for the symmetric equilibrium case, we conclude that the 16 outcomes can be grouped into four classes, within which all outcomes are invariant under horizontal and vertical flips with similar consumer demand patterns.

Appendix D

Proof of Lemma 2: The Symmetric Equilibrium Case

We derive the possible symmetric equilibria in the packet discrimination regime by the following steps: step 1, prove that all outcomes involving only Y pays for priority delivery are infeasible; step 2, derive properties of the equilibrium fixed fee F; step 3, eliminate dominated outcomes.

Step 1: Prove that all outcomes involving only Y pays for priority delivery are infeasible

In step 1, we show that there is no feasible p for any outcome involving only Y pays. Therefore such outcomes (NY, YB, BY, YB, YG, GY, and YY) cannot be an equilibrium. Since some outcomes are infeasible for similar reasons, we group them together.

Outcomes NY and YN

Here we focus on showing that there is no feasible p for outcome NY, as the analysis for outcome YN is similar. For outcomes NY to be feasible, all the CPs' incentive compatibility constraints need to be satisfied: (1) $\pi_{Y\{NY\}} - \pi_{Y\{YY\}} \geq 0$; (2) $\pi_{Y\{NY\}} - \pi_{Y\{NN\}} \geq 0$; (3) $\pi_{Y\{NY\}} - \pi_{Y\{YB\}} \geq 0$; (4) $\pi_{G\{NY\}} - \pi_{G\{GY\}} \geq 0$; (5) $\pi_{G\{NY\}} - \pi_{G\{NB\}} \geq 0$; and (6) $\pi_{G\{NY\}} - \pi_{G\{GB\}} \geq 0$.

Inequality (2) is $-N_{DY\{NY\}}p + \left(N_{CY\{NY\}} - N_{CY\{NN\}} - N_{DY\{NN\}} + N_{DY\{NY\}}\right)r_Y \geq 0$. Since $N_{CY\{NY\}} + N_{DY\{NY\}} > \frac{1}{2}$ and $N_{CY\{NN\}} + N_{DY\{NN\}} = \frac{1}{2}$, inequality (2) can be reduced to $p \leq \frac{(N_{CY\{NY\}}+N_{DY\{NY\}}-1/2)r_Y}{N_{DY\{NY\}}}$.

Inequality (5) is $N_{DG\{NB\}}p + \left(N_{CG\{NY\}} - N_{CG\{NB\}} + N_{DG\{NY\}} - N_{DG\{NB\}}\right)r_G \geq 0$. Since $N_{DG\{NB\}} = \frac{1}{4}$, $N_{CG\{NB\}} + N_{DG\{NB\}} = \frac{1}{2}$, and $N_{CG\{NY\}} + N_{DG\{NY\}} < \frac{1}{2}$, inequality (5) can be reduced to $p \geq \frac{(1/2-N_{CG\{NY\}}-N_{DG\{NY\}})r_G}{1/4}$.

We know that $\frac{1}{2} - N_{CG\{NY\}} - N_{DG\{NY\}} = N_{CY\{NY\}} + N_{DY\{NY\}} - \frac{1}{2}$, $N_{DY\{NY\}} > \frac{1}{4}$, and $r_G \geq r_Y$. Thus we have $\frac{(N_{CY\{NY\}}+N_{DY\{NY\}}-1/2)r_Y}{N_{DY\{NY\}}} < \frac{(1/2-N_{CG\{NY\}}-N_{DG\{NY\}})r_G}{1/4}$. Therefore (2) and (5) are inconsistent. Hence there is no feasible p for outcome NY.

Outcomes YB and BY

Outcomes YB and BY are infeasible for similar reasons. Outcome YB is not feasible since the following incentive compatibility constraints are inconsistent: (1) $\pi_{Y\{YB\}} - \pi_{Y\{NG\}} \geq 0$ and (2) $\pi_{G\{YB\}} - \pi_{G\{BB\}} \geq 0$.

Inequality (1) can be reduced to $p \leq \frac{(N_{CY\{YB\}}+N_{DY\{YB\}}-N_{CY\{NG\}}-N_{DY\{NG\}})r_Y}{(N_{CY\{YB\}}+N_{DY\{YB\}})}$. Note that we have $N_{CY\{YB\}} + N_{DY\{YB\}} - N_{CY\{NG\}} - N_{DY\{NG\}} = N_{CY\{YB\}} + N_{DY\{YB\}} - \frac{1}{2} + \frac{1}{2} - N_{CY\{NG\}} - N_{DY\{NG\}}$. Since $N_{CY\{YB\}} + N_{DY\{YB\}} - \frac{1}{2} = \frac{1}{2} - N_{CY\{NG\}} - N_{DY\{NG\}}$, we have $N_{CY\{YB\}} + N_{DY\{YB\}} - N_{CY\{NG\}} - N_{DY\{NG\}} = 2\left(N_{CY\{YB\}} + N_{DY\{YB\}} - \frac{1}{2}\right) = 2\left(\frac{1}{2} - N_{CG\{YB\}} - N_{DG\{YB\}}\right)$. Thus inequality (1) can be simplified to $p \leq \frac{(1/2-N_{CG\{YB\}}-N_{DG\{YB\}})r_Y}{(N_{CY\{YB\}}+N_{DY\{YB\}})/2}$.

Inequality (2) can be reduced to $p \geq \frac{(1/2-N_{CG\{YB\}}-N_{DG\{YB\}})r_G}{(1/2-N_{DG\{YB\}})}$. Note that we have $N_{CY\{YB\}} + N_{DY\{YB\}} + N_{CG\{YB\}} + N_{DG\{YB\}} = 1$. But $N_{CG\{YB\}} < N_{DG\{YB\}}$. Thus we have $\frac{N_{CY\{YB\}}+N_{DY\{YB\}}}{2} > \frac{1}{2} - N_{DG\{YB\}}$. Therefore $p \geq \frac{(1/2-N_{CG\{YB\}}-N_{DG\{YB\}})r_G}{(1/2-N_{DG\{YB\}})} > \frac{(1/2-N_{CG\{YB\}}-N_{DG\{YB\}})r_G}{(N_{CY\{YB\}}+N_{DY\{YB\}})/2}$.

In addition, we know $r_G \geq r_Y$. Thus we have $\frac{(1/2-N_{CG\{YB\}}-N_{DG\{YB\}})r_G}{(1/2-N_{DG\{YB\}})} > \frac{(1/2-N_{CG\{YB\}}-N_{DG\{YB\}})r_G}{(N_{CY\{YB\}}+N_{DY\{YB\}})/2} \geq \frac{(1/2-N_{CG\{YB\}}-N_{DG\{YB\}})r_Y}{(N_{CY\{YB\}}+N_{DY\{YB\}})/2}$. Therefore inequalities (1) and (2) are inconsistent. Hence outcome YB is infeasible.

Outcomes YG and GY

Outcomes YG and GY are infeasible for similar reasons. Outcome YG is feasible provided $\pi_{Y\{YG\}} - \pi_{Y\{NB\}} \geq 0$, i.e., $\left(\frac{1}{4} - N_{CY\{YG\}}\right)p + \left(N_{CY\{YG\}} + N_{DY\{YG\}} - \frac{1}{2}\right)r_Y \geq 0$. Note that $N_{CY\{YG\}} + N_{DY\{YG\}} = \frac{1}{2}$. This gives $\left(\frac{1}{4} - N_{CY\{YG\}}\right)p \geq 0$. Since $N_{CY\{YG\}} > \frac{1}{4}$, we have $p \leq 0$. Hence there is no feasible p for outcome YG.

Outcome YY

Outcome YY is not feasible since the following incentive compatibility constraints are inconsistent: (1) $\pi_{Y\{YY\}} - \pi_{Y\{NN\}} \geq 0$ and (2) $\pi_{G\{YY\}} - \pi_{G\{BB\}} \geq 0$.

Inequality (1) is $\left(N_{CG\{YY\}} - N_{CG\{GG\}} - \frac{1}{2}\right)p + \left(N_{CG\{GG\}} - N_{CG\{YY\}}\right)r_Y \geq 0$. Note that $N_{CG\{YY\}} + N_{CY\{YY\}} = \frac{1}{2}$. Thus we have $N_{CG\{YY\}} - N_{CG\{GG\}} - \frac{1}{2} = -N_{CG\{GG\}} - N_{CY\{YY\}} = -2N_{CG\{GG\}} < 0$. Therefore inequality (1) can be reduced to $p \leq \left(\frac{1}{2} - \frac{N_{CG\{YY\}}}{2N_{CG\{GG\}}}\right)r_Y$. Inequality (2) can be reduced to $p \geq \left(1 - 4N_{CG\{YY\}}\right)r_G$.

Recall that $N_{CG\{YY\}} = N_{CY\{GG\}}$, $N_{CY\{YY\}} = N_{CG\{GG\}}$, and $N_{CY\{GG\}} + N_{CG\{GG\}} = \frac{1}{2}$. Thus inequality (1) may be re-written as $p \leq \left(1 - \frac{1}{4N_{CG\{GG\}}}\right) r_Y$ and inequality (2) may be re-written as $p \geq 4N_{CG\{GG\}} \left(1 - \frac{1}{4N_{CG\{GG\}}}\right) r_G$. Since $r_Y \leq r_G$ and $4N_{CG\{GG\}} > 1$, inequality (1) implies that $p \leq \left(1 - \frac{1}{4N_{CG\{GG\}}}\right) r_Y < r_G$ but inequality (2) implies that $p \geq 4N_{CG\{GG\}} \left(1 - \frac{1}{4N_{CG\{GG\}}}\right) r_G > r_G$. Therefore inequalities (1) and (2) are inconsistent and there is no feasible p for outcome YY.

To summarize the above, there is no feasible p for outcomes NY, YN, BY, YB, YG, GY, and YY, and therefore, they cannot be an equilibrium.

Step 2: Derive properties of the equilibrium fixed fee *F*

In step 2, we derive properties of the equilibrium fixed fee F. Here we first discuss some properties for all 16 outcomes and thus the subscript ij is omitted in this discussion. Under the assumption of full market coverage, the profit maximizing fixed fee F is such that the consumers of all four ISP-CP combinations (CY, DY, CG, and DG) with the lowest net utility will get zero net utility.

We now define the global utility function $U(x,z)$ for the entire market $[0,1] \times [0,1]$. First recall the definition of the demand distribution of each ISP-CP combinations characterized by the utility functions.

$$
\begin{aligned}
R_{CY} &= \{(x,z) \in [0,1] \times [0,1];\ u_{CY}(x,z) \geq \max\{u_{CG}(x,z), u_{DY}(x,z), u_{DG}(x,z)\}\} \\
R_{DY} &= \{(x,z) \in [0,1] \times [0,1];\ u_{DY}(x,z) \geq \max\{u_{DG}(x,z), u_{CY}(x,z), u_{CG}(x,z)\}\} \\
R_{CG} &= \{(x,z) \in [0,1] \times [0,1];\ u_{CG}(x,z) \geq \max\{u_{CY}(x,z), u_{DG}(x,z), u_{DY}(x,z)\}\} \\
R_{DG} &= \{(x,z) \in [0,1] \times [0,1];\ u_{DG}(x,z) \geq \max\{u_{DY}(x,z), u_{CG}(x,z), u_{CY}(x,z)\}\}
\end{aligned}
$$

Note that each of the following inequalities reduces to regions on $[0,1] \times [0,1]$ dictated by the indifference customers between mutual pairs of ISP-CP combinations:

$$
\begin{aligned}
u_{CG}(x,z) - u_{CY}(x,z) \geq 0 &\Leftrightarrow x \geq x_C \\
u_{DG}(x,z) - u_{DY}(x,z) \geq 0 &\Leftrightarrow x \geq x_D \\
u_{DY}(x,z) - u_{CY}(x,z) \geq 0 &\Leftrightarrow z \geq z_Y \\
u_{DG}(x,z) - u_{CG}(x,z) \geq 0 &\Leftrightarrow z \geq z_G \\
u_{DG}(x,z) - u_{CY}(x,z) \geq 0 &\Leftrightarrow z \geq L_-(x) \\
u_{DY}(x,z) - u_{CG}(x,z) \geq 0 &\Leftrightarrow z \geq L_+(x)
\end{aligned}
$$

Then the demand distributions can be written in terms of the indifference customers as follows:

$$
\begin{aligned}
R_{CY} &= \{(x,z) \in [0,1] \times [0,1];\ x \leq x_C, z \leq z_Y, z \leq L_-(x)\} \\
R_{DY} &= \{(x,z) \in [0,1] \times [0,1];\ x \leq x_D, z \geq z_Y, z \geq L_+(x)\} \\
R_{CG} &= \{(x,z) \in [0,1] \times [0,1];\ x \geq x_C, z \leq z_G, z \leq L_+(x)\} \\
R_{DG} &= \{(x,z) \in [0,1] \times [0,1];\ x \geq x_D, z \geq z_G, z \geq L_-(x)\}
\end{aligned}
$$

Define the global utility function $U(x,z)$ over the entire market $[0,1] \times [0,1]$:

$$
U(x,z) = \begin{cases} u_{CY}(x,z), & \text{if } (x,z) \in R_{CY} \\ u_{DY}(x,z), & \text{if } (x,z) \in R_{DY} \\ u_{CG}(x,z), & \text{if } (x,z) \in R_{CG} \\ u_{DG}(x,z), & \text{if } (x,z) \in R_{DG} \end{cases}
$$

By definition of the demand regions R_{CY}, R_{DY}, R_{CG}, and R_{DG}, the global utility function gives the maximal utility value for the consumer (x,z) according to its choice of ISP-CP combination. We also note that $U(x,z)$ is a continuous function over the set $[0,1] \times [0,1]$. Indeed, first note that the functions $u_{CY}(x,z)$, $u_{DY}(x,z)$, $u_{CG}(x,z)$, and $u_{DG}(x,z)$ are linear functions in (x,z) and thus are all continuous. Since $U(x,z)$ is piecewise defined over demand regions R_{CY}, R_{DY}, R_{CG}, and R_{DG}, we only need to check that $U(x,z)$ is continuous at each point on the boundaries between mutual pairs of the demand regions R_{CY}, R_{DY}, R_{CG}, and R_{DG}. We check each boundary:

- Between R_{CY} and R_{DY}, the boundary is along the line $z = z_Y$ on which $u_{CY} = u_{DY}$.
- Between R_{CY} and R_{CG}, the boundary is along the line $x = x_C$ on which $u_{CY} = u_{CG}$.
- Between R_{CY} and R_{DG}, the boundary is along the line $z = L_-(x)$ on which $u_{CY} = u_{DG}$.
- Between R_{DG} and R_{CG}, the boundary is along the line $z = z_G$ on which $u_{DG} = u_{CG}$.
- Between R_{DG} and R_{DY}, the boundary is along the line $x = x_D$ on which $u_{DG} = u_{DY}$.
- Between R_{DY} and R_{CG}, the boundary is along the line $z = L_+(x)$ on which $u_{DY} = u_{CG}$.

Since corresponding utility functions all matches along the boundaries between mutual pairs of the demand regions R_{CY}, R_{DY}, R_{CG}, and R_{DG}, the global utility function $U(x,z)$ is continuous over the entire set $[0,1] \times [0,1]$.

The global utility function $U(x,z)$ is a continuous function over the closed and bounded set $[0,1] \times [0,1]$. Therefore $U(x,z)$ attains its maximum and minimum at some points in the set $[0,1] \times [0,1]$. Under the assumption of full market coverage, the optimal fixed fees the ISPs charge consumers are such that the minimum of $U(x,z)$ equal to zero. In other words, the optimal fixed fee is the maximum fee such that all consumers get nonnegative utility.

Since $U(x,z)$ is piecewise defined by linear functions, it has no critical points in the interior of each demand regions R_{CY}, R_{DY}, R_{CG}, and R_{DG}. Therefore we only need to analyze the value of $U(x,z)$ along each mutual boundaries to capture the minimum of $U(x,z)$. Before we analyze the boundaries between R_{CY}, R_{DY}, R_{CG}, and R_{DG}, we recall that the demand distributions split into the three geometric types (i) $x_C = x_D$ and $z_Y = z_G$; (ii) $x_C < x_D$ and $z_Y < z_G$; and (iii) $x_C > x_D$ and $z_Y > z_G$.

The feasible outcomes NN, NB, BN, BB, and GG are of type (i), where the demand regions are all rectangular in shape. The feasible outcomes GN and GB are of type (ii), which have exactly two rectangles, and two pentagonal regions sharing a boundary along $z = L_+(x)$. And finally, the feasible outcomes NG and BG are of type (iii), which have exactly two rectangles, and two pentagonal regions sharing a boundary along $z = L_-(x)$.

We organize the analysis into two cases (A): $x_C \le x_D$and $z_Y \le z_G$ and (B): $x_C \ge x_D$ and $z_Y \ge z_G$. Cases (A) and (B) overlaps in those of type (i) here the diagonal boundary on $z = L_+(x)$ or $z = L_-(x)$ collapses to the point of intersection of these lines.

Case (A): $x_C \le x_D$ and $z_Y \le z_G$

There are five boundaries including a segment on $z = L_+(x)$.

(A1) Boundary between R_{CY} and R_{DY}. This boundary is along the horizontal line $z = z_Y$ and is the line segment joining $(0, z_Y)$ and the point (x_C, z_Y). Since $u_{CY} = u_{DY}$ on this boundary, along the boundary we may write for $0 \le x \le x_C$, $U(x, z_Y) = u_{CY}(x, z_Y) = V - tx - kz_Y - d\lambda w_{CY} - F_C$, or $U(x, z_Y) = u_{DY}(x, z_Y) = V - tx - k(1 - z_Y) - d\lambda w_{DY} - F_D$. In either formula, we see that on this boundary $U(x,z)$ is a decreasing function of x. Therefore $U(x,z)$ minimizes at (x_C, z_Y) on the boundary between R_{CY} and R_{DY}.

(A2) Boundary between R_{DG} and R_{CG}. This boundary is along the horizontal line $z = z_G$ and is the line segment joining (x_D, z_G) and the point $(1, z_G)$. Since $u_{DG} = u_{CG}$ on this boundary, along the boundary we may write for $x_D \le x \le 1$, $U(x, z_G) = u_{CG}(x, z_G) = V - t(1 - x) - kz_G - d\lambda w_{CG} - F_C$, or $U(x, z_G) = u_{DG}(x, z_G) = V - t(1 - x) - k(1 - z_G) - d\lambda w_{DG} - F_D$. In either formula, we see that on this boundary $U(x,z)$ is a increasing function of x. Therefore $U(x,z)$ minimizes at (x_D, z_G) on the boundary between R_{DG} and R_{CG}.

(A3) Boundary between R_{CY} and R_{CG}. This boundary is along the vertical line $x = x_C$ and is the line segment joining $(x_C, 0)$ and the point (x_C, z_Y). Since $u_{CY} = u_{CG}$ on this boundary, along the boundary we may write for $0 \le z \le z_Y$, $U(x_C, z) = u_{CY}(x_C, z) = V - tx_C - kz - d\lambda w_{CY} - F_C$, or $U(x_C, z) = u_{CG}(x_C, z) = V - t(1 - x_C) - kz - d\lambda w_{CG} - F_C$. In either formula, we see that on this boundary $U(x,z)$ is a decreasing function of z. Therefore $U(x,z)$ minimizes at (x_C, z_Y) on the boundary between R_{CY} and R_{CG}.

(A4) Boundary between R_{DG} and R_{DY}. This boundary is along the vertical line $x = x_D$ and is the line segment joining (x_D, z_G) and the point $(x_D, 1)$. Since $u_{CY} = u_{CG}$ on this boundary, along the boundary we may write for $z_G \le z \le 1$, $U(x_D, z) = u_{DY}(x_D, z) = V - tx_D - k(1 - z) - d\lambda w_{DY} - F_D$, or $U(x_D, z) = u_{DG}(x_D, z) = V - t(1 - x_D) - k(1 - z) - d\lambda w_{DG} - F_D$. In either formula, we see that on this boundary $U(x,z)$ is a increasing function of z. Therefore $U(x,z)$ minimizes at (x_D, z_G) on the boundary between R_{DG} and R_{DY}.

(A5) Boundary between R_{CG} and R_{DY}. This boundary is along the line $z = L_+(x)$ and is the line segment joining (x_C, z_Y) and the point (x_D, z_G). We parameterize the directed line segment as follows: For $0 \le s \le 1$, $x = (1 - s)x_C + sx_D$ and $z = (1 - s)z_Y + sz_G$. On this boundary the utility function U is a function of the parameter s. Since $u_{CG} = u_{DY}$ on this boundary, along the boundary we may write for $0 \le s \le 1$, $U(s) = u_{DY}((1 - s)x_C + sx_D, (1 - s)z_Y + sz_G) = V - t[(1 - s)x_C + sx_D] - k[1 - (1 - s)z_Y - sz_G] - d\lambda w_{DY} - F_D = V + st(x_C - x_D) - tx_C - k(1 - z_Y) + sk(z_G - z_Y) - d\lambda w_{DY} - F_D$, or $U(s) = u_{CG}((1 - s)x_C + sx_D, (1 - s)z_Y + sz_G) = V(\lambda) - t[1 - (1 - s)x_C - sx_D] - k[(1 - s)z_Y + sz_G] - d\lambda w_{CG} - F_C = V(\lambda) - t(1 - x_C) + st(x_D - x_C) - kz_Y + sk(z_Y - z_G) - d\lambda w_{CG} - F_C$. If $x_D = x_C$ and $z_Y = z_G$ then $U(s)$ is a constant not dependent on s. However, in general we note that the slope of $z = L_+(x)$ is given by $\frac{t}{k} = \frac{z_G - z_Y}{x_D - x_C}$, i.e., $k(z_G - z_Y) = t(x_D - x_C)$. Thus the values of $U(s)$ reduces to the constant: $U(s) = V - tx_C - k(1 - z_Y) - d\lambda w_{DY} - F_D$ or $U(s) = V - t(1 - x_C) - kz_Y - d\lambda w_{CG} - F_C$. From the analysis above, we could see that $U(x,z)$ minimizes on the points along the boundary on the line $z = L_+(x)$. In particular, $U(x,z)$ minimizes at (x_C, z_Y) or (x_D, z_G) with the same value.

Case (B): $x_C \ge x_D$and $z_Y \ge z_G$

There are five boundaries including a segment on $z = L_-(x)$.

(B1) Boundary between R_{CY} and R_{DY}. This boundary is along the horizontal line $z = z_Y$ and is the line segment joining $(0, z_Y)$ and the point (x_D, z_Y). Since $u_{CY} = u_{DY}$ on this boundary, along the boundary we may write for $0 \leq x \leq x_D$, $U(x, z_Y) = u_{CY}(x, z_Y) = V - tx - kz_Y - d\lambda w_{CY} - F_C$, or $U(x, z_Y) = u_{DY}(x, z_Y) = V - tx - k(1 - z_Y) - d\lambda w_{DY} - F_D$. In either formula, we see that on this boundary $U(x, z)$ is a decreasing function of x. Therefore $U(x, z)$ minimizes at (x_D, z_Y) on the boundary between R_{CY} and R_{DY}.

(B2) Boundary between R_{DG} and R_{CG}. This boundary is along the horizontal line $z = z_G$ and is the line segment joining (x_C, z_G) and the point $(1, z_G)$. Since $u_{DG} = u_{CG}$ on this boundary, along the boundary we may write for $x_C \leq x \leq 1$, $U(x, z_G) = u_{CG}(x, z_G) = V - t(1 - x) - kz_G - d\lambda w_{CG} - F_C$, or $U(x, z_G) = u_{DG}(x, z_G) = V - t(1 - x) - k(1 - z_G) - d\lambda w_{DG} - F_D$. In either formula, we see that on this boundary $U(x, z)$ is a increasing function of x. Therefore $U(x, z)$ minimizes at (x_C, z_G) on the boundary between R_{DG} and R_{CG}.

(B3) Boundary between R_{CY} and R_{CG}. This boundary is along the vertical line $x = x_C$ and is the line segment joining $(x_C, 0)$ and the point (x_C, z_G). Since $u_{CY} = u_{CG}$ on this boundary, along the boundary we may write for $0 \leq z \leq z_G$, $U(x_C, z) = u_{CY}(x_C, z) = V - tx_C - kz - d\lambda w_{CY} - F_C$, or $U(x_C, z) = u_{CG}(x_C, z) = V - t(1 - x_C) - kz - d\lambda w_{CG} - F_C$. In either formula, we see that on this boundary $U(x, z)$ is a decreasing function of z. Therefore $U(x, z)$ minimizes at (x_C, z_G) on the boundary between R_{CY} and R_{CG}.

(B4) Boundary between R_{DG} and R_{DY}. This boundary is along the vertical line $x = x_D$ and is the line segment joining (x_D, z_Y) and the point $(x_D, 1)$. Since $u_{CY} = u_{CG}$ on this boundary, along the boundary we may write for $z_Y \leq z \leq 1$, $U(x_D, z) = u_{DY}(x_D, z) = V - tx_D - k(1 - z) - d\lambda w_{DY} - F_D$, or $U(x_D, z) = u_{DG}(x_D, z) = V - t(1 - x_D) - k(1 - z) - d\lambda w_{DG} - F_D$. In either formula, we see that on this boundary $U(x, z)$ is a increasing function of z. Therefore $U(x, z)$ minimizes at (x_D, z_Y) on the boundary between R_{DG} and R_{DY}.

(B5) Boundary between R_{CY} and R_{DG}. This boundary is along the line $z = L_-(x)$ and is the line segment joining (x_D, z_Y) and the point (x_C, z_G). We parameterize the directed line segment as follows: For $0 \leq s \leq 1$, $x = (1 - s)x_D + sx_C$ and $z = (1 - s)z_Y + sz_G$. On this boundary the utility function U is a function of the parameter s. Since $u_{CY} = u_{DG}$ on this boundary, along the boundary we may write for $0 \leq s \leq 1$, $U(s) = u_{DG}((1 - s)x_D + sx_C, (1 - s)z_Y + sz_G) = V - t[1 - (1 - s)x_D - sx_C] - k[1 - (1 - s)z_Y - sz_G] - d\lambda w_{DG} - F_D = V + st(x_C - x_D) - t(1 - x_D) - k(1 - z_Y) + sk(z_G - z_Y) - d\lambda w_{DG} - F_D$, or $U(s) = u_{CY}((1 - s)x_D + sx_C, (1 - s)z_Y + sz_G) = V - t[(1 - s)x_D + sx_C] - k[(1 - s)z_Y + sz_G] - d\lambda w_{CY} - F_C = V - tx_D + st(x_D - x_C) - kz_Y + sk(z_Y - z_G) - d\lambda w_{CY} - F_C$. If $x_D = x_C$ and $z_Y = z_G$ then $U(s)$ is a constant not dependent on s. However, in general we note that the slope of $z = L_-(x)$ is given by: $-\frac{t}{k} = \frac{z_G - z_Y}{x_C - x_D}$, i.e., $k(z_G - z_Y) = t(x_D - x_C)$. Thus the values of $U(s)$ reduces to the constant: $U(s) = V - t(1 - x_D) - k(1 - z_Y) - d\lambda w_{DG} - F_D$ or $U(s) = V - tx_D - kz_Y - d\lambda w_{CY} - F_C$. From the analysis above, we could see that $U(x, z)$ minimizes on the points along the boundary on the line $z = L_-(x)$. In particular, $U(x, z)$ minimizes at (x_D, z_Y) or (x_C, z_G) with the same value.

Maximum Fees for Case A: The maximum fees occur when the minimum of the global utility function is zero. Therefore from the formulas in (A5), the maximum fees are given by: $V - tx_C - k(1 - z_Y) - d\lambda w_{DY} - F_D = 0$ and $V - t(1 - x_C) - kz_Y - d\lambda w_{CG} - F_C = 0$. This gives the maximum fees: $F_C = V - t\left(1 - \frac{x_C + x_D}{2}\right) - k\left(\frac{z_Y + z_G}{2}\right) - d\lambda w_{CG}$ and $F_D = V - t\left(\frac{x_C + x_D}{2}\right) - k\left(1 - \frac{z_Y + z_G}{2}\right) - d\lambda w_{DY}$.

Maximum Fees for Case B: The maximum fees occur when the minimum of the global utility function is zero. Therefore from the formulas in (B5), the maximum fees are given by: $V - t(1 - x_D) - k(1 - z_Y) - d\lambda w_{DG} - F_D = 0$ and $V - tx_D - kz_Y - d\lambda w_{CY} - F_C = 0$. This gives the maximum fees: $F_C = V - t\left(\frac{x_C + x_D}{2}\right) - k\left(\frac{z_Y + z_G}{2}\right) - d\lambda w_{CY}$ and $F_D = V - t\left(1 - \frac{x_C + x_D}{2}\right) - k\left(1 - \frac{z_Y + z_G}{2}\right) - d\lambda w_{DG}$.

We next solve for the optimal fixed fee for feasible outcomes in symmetric equilibrium when $F_C = F_D = F$.

Optimal F for outcomes NN, NB, BN, and BB: All waiting times are the same and $x_C = x_D = z_Y = z_G = \frac{1}{2}$. Using the formulas for maximum fees above, we get $F_{\{NN\}} = F_{\{NB\}} = F_{\{BN\}} = F_{\{BB\}} = V - \frac{t}{2} - \frac{k}{2} - \frac{d\lambda}{\mu - \lambda/2}$.

Optimal F for outcome GG: In this outcome, $z_{Y\{GG\}} = z_{G\{GG\}} = \frac{1}{2}$ and $x_{D\{GG\}} = x_{C\{GG\}} < \frac{1}{2}$. We have four formulas for F which must be consistent. We verify that those in Case A and Case B both reduces to the following formula for F: $F_{\{GG\}} = V - t\left(1 - x_{D\{GG\}}\right) - \frac{k}{2} - \frac{2d\lambda}{2\mu - (1 - x_{D\{GG\}})\lambda}$.

Optimal F for outcomes NG and BG: These outcomes have the same demand distributions and so the same indifferent customers and waiting times. We use the formulas for Case B for these outcomes: $F_{\{BG\}} = F_{\{NG\}} = V - t\left(1 - x_{D\{BG\}}\right) - k\left(1 - z_{Y\{BG\}}\right) - \frac{d\lambda}{\mu - N_{DG\{BG\}}\lambda}$.

Optimal F for outcomes GN and GB: These outcomes have the same demand distributions and so the same indifferent customers and waiting times. We use the formulas for Case A for these outcomes: $F_{\{GB\}} = F_{\{GN\}} = V - t\left(1 - x_{C\{GB\}}\right) - kz_{Y\{GB\}} - \frac{d\lambda}{\mu - N_{CG\{GB\}}\lambda}$.

Step 3: Eliminate Dominated Outcomes

From step 1, we know that outcomes NY, YN, YY, YG, YB, GY, and BY can be eliminated from the equilibrium analysis due to no feasible p.

Next, we further eliminate other dominated outcomes by comparing CPs' profits. Recall that: $\pi_{C\{NN\}} = \pi_{D\{NN\}} = \frac{F_{\{NN\}}}{2}$; $\pi_{C\{NB\}} = \frac{F_{\{NB\}}}{2}$ and $\pi_{D\{NB\}} = \frac{F_{\{NB\}}}{2} + \frac{\lambda p_{\{NB\}}}{2}$; $\pi_{C\{BN\}} = \frac{F_{\{BN\}}}{2} + \frac{\lambda p_{\{BN\}}}{2}$ and $\pi_{D\{BN\}} = \frac{F_{\{BN\}}}{2}$; and $\pi_{C\{BB\}} = \pi_{D\{BB\}} = \frac{F_{\{BB\}}}{2} + \frac{\lambda p_{\{BB\}}}{2}$. For outcomes NN, NB, BN, and BB, we have $F_{\{NN\}} = F_{\{NB\}} = F_{\{BN\}} = F_{\{BB\}}$ and equal demand distributions amongst all ISP-CP combinations. Comparing pairs of these outcomes yields $\pi_{D\{NN\}} < \pi_{D\{NB\}}$, $\pi_{D\{BN\}} < \pi_{D\{BB\}}$, and $\pi_{C\{NB\}} < \pi_{C\{BB\}}$. Therefore, outcomes NN, BN, and NB are dominated and can be eliminated.

Next, we compare outcomes NG, GN, BG, and GB. We know $F_{\{NG\}} = F_{\{GN\}} = F_{\{BG\}} = F_{\{GB\}}$, and the following demand distributions amongst all ISP-CP combinations: $N_{CY\{NG\}} = N_{DY\{GN\}} = N_{CY\{BG\}} = N_{DY\{GB\}}$, $N_{DY\{NG\}} = N_{CY\{GN\}} = N_{DY\{BG\}} = N_{CY\{GB\}}$, $N_{CG\{NG\}} = N_{DG\{GN\}} = N_{CG\{BG\}} = N_{DG\{GB\}}$, and $N_{DG\{NG\}} = N_{CG\{GN\}} = N_{DG\{BG\}} = N_{CG\{GB\}}$. Recall that $\pi_{C\{NG\}} = (N_{CY\{NG\}} + N_{CG\{NG\}})F_{\{NG\}}$, $\pi_{D\{NG\}} = (N_{DY\{NG\}} + N_{DG\{NG\}})F_{\{NG\}} + \lambda p_{\{NG\}} N_{DG\{NG\}}$, $\pi_{C\{GN\}} = (N_{CY\{GN\}} + N_{CG\{GN\}})F_{\{GN\}} + \lambda p_{\{GN\}} N_{CG\{GN\}}$, $\pi_{D\{GN\}} = (N_{DY\{GN\}} + N_{DG\{GN\}})F_{\{GN\}}$, $\pi_{C\{BG\}} = (N_{CY\{BG\}} + N_{CG\{BG\}})(F_{\{BG\}} + \lambda p_{\{BG\}})$, $\pi_{D\{BG\}} = (N_{DY\{BG\}} + N_{DG\{BG\}})F_{\{BG\}} + \lambda p_{\{BG\}} N_{DG\{BG\}}$, $\pi_{C\{GB\}} = (N_{CY\{GB\}} + N_{CG\{GB\}})F_{\{GB\}} + \lambda p_{\{GB\}} N_{CG\{GB\}}$, and $\pi_{D\{GB\}} = (N_{DY\{GB\}} + N_{DG\{GB\}})(F_{\{GB\}} + \lambda p_{\{GB\}})$. Comparing pairs of these outcomes yields $\pi_{C\{NG\}} < \pi_{C\{BG\}}$ and $\pi_{D\{GN\}} < \pi_{D\{GB\}}$. Therefore, outcomes NG and GN are dominated and can be eliminated from the equilibrium analysis.

Therefore, after eliminating all the dominated outcomes, we conclude that outcomes GG, GB, BG, and BB as the only four possible symmetric equilibria.

Appendix E

Proofs of Lemma 1 and Lemma 2: The Asymmetric Equilibrium Case

We derive the possible asymmetric equilibria in the packet discrimination regime by the following steps: in step 1, we characterize consumers demand patterns; in step 2, we derive properties of the equilibrium fixed fees F_C and F_D; in step 3, we eliminate dominated outcomes and derive the only possible asymmetric equilibria. Without loss of generality, we assume $F_C = F_D + \Delta F$, where $\Delta F \geq 0$.

Step 1: Characterize Consumer Demand Patterns in Asymmetric Equilibrium

Similar to the analysis of symmetric equilibrium, we compare consumers' utility functions for the corresponding pairs of ISP-CP combinations and derive $x_{C\{ij\}} = \frac{1}{2} + \frac{d\lambda(w_{CG\{ij\}} - w_{CY\{ij\}})}{2t}$, $x_{D\{ij\}} = \frac{1}{2} + \frac{d\lambda(w_{DG\{ij\}} - w_{DY\{ij\}})}{2t}$, $z_{Y\{ij\}} = \frac{1}{2} - \frac{F_C - F_D}{2k} + \frac{d\lambda(w_{DY\{ij\}} - w_{CY\{ij\}})}{2k}$, and $z_{G\{ij\}} = \frac{1}{2} - \frac{F_C - F_D}{2k} + \frac{d\lambda(w_{DG\{ij\}} - w_{CG\{ij\}})}{2k}$. Note that the sign of $x_{C\{ij\}} - x_{D\{ij\}}$ is the same as the sign of $z_{Y\{ij\}} - z_{G\{ij\}}$.

Each outcome ij is determined by the ISPs' pricing decisions and the corresponding content providers' delivery service choices. As in the symmetric case, we denote outcome ij by the matrix $\begin{bmatrix} I_{DY\{ij\}} & I_{DG\{ij\}} \\ I_{CY\{ij\}} & I_{CG\{ij\}} \end{bmatrix}$. When considering asymmetric equilibrium, horizontal flip still applies to permuting the outcomes while vertical flip no longer applies since $F_C \geq F_D$.

Among the 16 outcomes, we still have four invariant classes under horizontal flip: (a) outcomes NN, NB, BN, and BB; (b) outcomes YY and GG; (c) outcomes YG and GY; (d) outcomes NY, NG, YN, GN, BY, BG, YB, and GB. Next we apply the horizontal flip to each of the classes (a) through (d) to characterize the demand distribution under each outcome.

(Class a) Outcomes NN, NB, BN, and BB

Under outcomes NN, NB, BN, and BB, all customers have equal queuing priorities. Therefore applying horizontal flip to these outcomes will not change the queuing priorities. Hence the indifferent customers remain unchanged when horizontal flip is applied.

From horizontal flip relations, we have $x_{C\{NN\}} = 1 - x_{C\{NN\}}$, $x_{D\{NN\}} = 1 - x_{D\{NN\}}$, $x_{C\{NB\}} = 1 - x_{C\{NB\}}$, $x_{D\{NB\}} = 1 - x_{D\{NB\}}$, $x_{C\{BN\}} = 1 - x_{C\{BN\}}$, $x_{D\{BN\}} = 1 - x_{D\{BN\}}$, $x_{C\{BB\}} = 1 - x_{C\{BB\}}$, and $x_{D\{BB\}} = 1 - x_{D\{BB\}}$. That is $x_{C\{NN\}} = x_{C\{NB\}} = x_{C\{BN\}} = x_{C\{BB\}} = \frac{1}{2}$ and $x_{D\{NN\}} = x_{D\{NB\}} = x_{D\{BN\}} = x_{D\{BB\}} = \frac{1}{2}$.

Since the sign of $x_{C\{ij\}} - x_{D\{ij\}}$ is the same as the sign of $z_{Y\{ij\}} - z_{G\{ij\}}$ for any outcome ij and $x_C = x_D$ under outcomes NN, NB, BN, and BB, we have $z_Y = z_G$. Next we prove that $z_Y = z_G \le \frac{1}{2}$ by contradiction. First, suppose $z_Y < \frac{1}{2} - \frac{F_C - F_D}{2k}$. Then $z_Y < \frac{1}{2}$ since $F_C \ge F_D$. This implies $N_{DY} > N_{CY}$ which gives $w_{DY} > w_{CY}$. But we also have $z_Y = \frac{1}{2} - \frac{F_C - F_D}{2k} + \frac{d\lambda(W_{DY} - W_{CY})}{2k}$. Therefore $\frac{1}{2} - \frac{F_C - F_D}{2k} + \frac{d\lambda(W_{DY} - W_{CY})}{2k} < \frac{1}{2} - \frac{F_C - F_D}{2k}$ which gives $w_{DY} - w_{CY} < 0$. A contradiction arises. Therefore, we must have $z_Y \ge \frac{1}{2} - \frac{F_C - F_D}{2k}$. Second, suppose $z_Y > \frac{1}{2}$. Then $N_{DY} < N_{CY}$ which gives $w_{DY} < w_{CY}$. But we have $F_C \ge F_D$. Therefore $z_Y = \frac{1}{2} + \frac{F_D - F_C}{2k} + \frac{d\lambda(W_{DY} - W_{CY})}{2k} < \frac{1}{2}$. A contradiction arises. Therefore we must have $z_Y \le \frac{1}{2}$.

Therefore, as shown in Figure E1, the market demands for CY and CG are equal, and the market demands of DY and DG are equal under outcomes NN, NB, BN, and BB. That is $N_{CY\{NN\}} = N_{CG\{NN\}} = N_{CY\{NB\}} = N_{CG\{NB\}} = N_{CY\{BN\}} = N_{CG\{BN\}} = N_{CY\{BB\}} = N_{CG\{BB\}} < \frac{1}{4}$ and $N_{DY\{NN\}} = N_{DG\{NN\}} = N_{DY\{NB\}} = N_{DG\{NB\}} = N_{DY\{BN\}} = N_{DG\{BN\}} = N_{DY\{BB\}} = N_{DG\{BB\}} > \frac{1}{4}$.

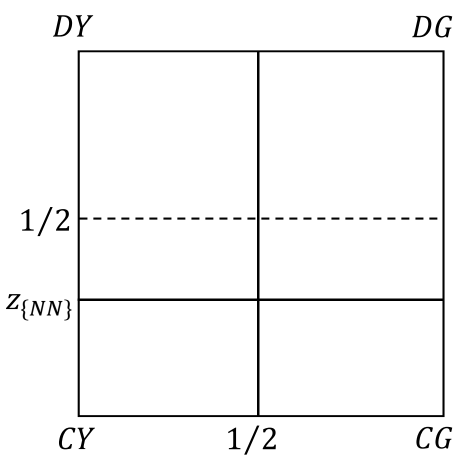


Figure E1. Demand Distribution of Class a (outcomes NN, NB, BN, and BB)

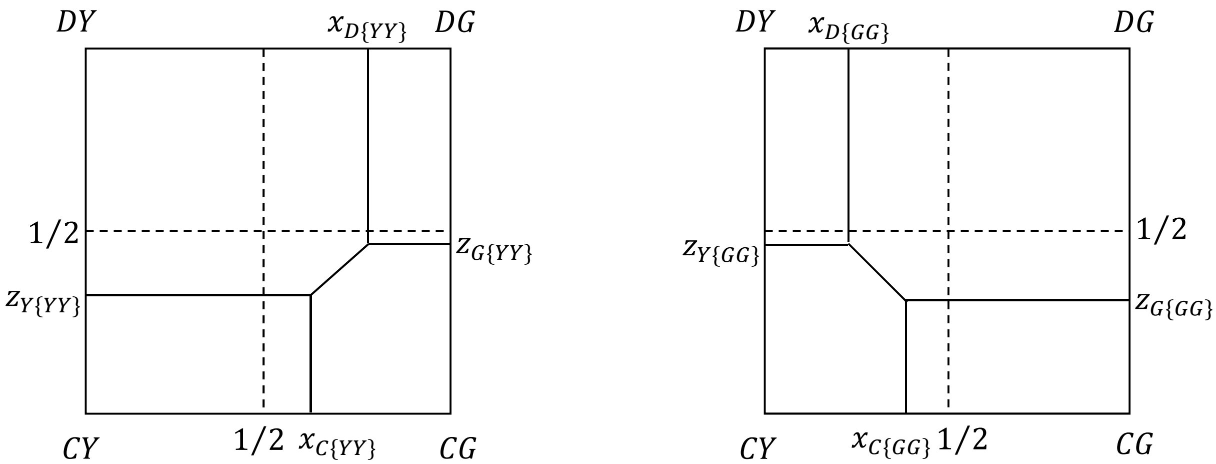
(Class b) Outcomes YY and GG

Based on symmetry under horizontal flip, we can obtain the demand distribution of YY by reflecting the demand distribution of outcome GG through the line $x = \frac{1}{2}$. Thus we may focus on deriving the demand distribution of outcome GG.

We know from the analysis of symmetric equilibrium that when $F_C = F_D$, $x_{D\{GG\}} = x_{C\{GG\}} < \frac{1}{2}$ and $z_{G\{GG\}} = z_{Y\{GG\}} = \frac{1}{2}$. When $F_C > F_D$, we know that $x_{C\{GG\}} = \frac{1}{2} + \frac{d\lambda(w_{CG\{GG\}} - w_{CY\{GG\}})}{2t}$, $x_{D\{GG\}} = \frac{1}{2} + \frac{d\lambda(w_{DG\{GG\}} - w_{DY\{GG\}})}{2t}$, $z_{Y\{GG\}} = \frac{1}{2} - \frac{F_C - F_D}{2k} + \frac{d\lambda(w_{DY\{GG\}} - w_{CY\{GG\}})}{2k}$, and $z_{G\{GG\}} = \frac{1}{2} - \frac{F_C - F_D}{2k} + \frac{d\lambda(w_{DG\{GG\}} - w_{CG\{GG\}})}{2k}$. Since only G pays for preferential delivery on both ISPs, $w_{CG\{GG\}} - w_{CY\{GG\}} < 0$, and $w_{DG\{GG\}} - w_{DY\{GG\}} < 0$. Thus, $x_{C\{GG\}} < \frac{1}{2}$ and $x_{D\{GG\}} < \frac{1}{2}$.

Furthermore, $x_{C\{GG\}} - x_{D\{GG\}} = \frac{d\lambda[(w_{DY\{GG\}} - w_{DG\{GG\}}) - (w_{CY\{GG\}} - w_{CG\{GG\}})]}{2t} = \frac{d\lambda}{2t}\left[\frac{N_{DG\{GG\}}\lambda + N_{DY\{GG\}}\lambda}{(\mu - N_{DG\{GG\}}\lambda)(\mu - N_{DG\{GG\}}\lambda - N_{DY\{GG\}}\lambda)} - \frac{N_{CG\{GG\}}\lambda + N_{CY\{GG\}}\lambda}{(\mu - N_{CG\{GG\}}\lambda)(\mu - N_{CG\{GG\}}\lambda - N_{CY\{GG\}}\lambda)}\right] > 0$ since $N_{DG\{GG\}} > N_{CG\{GG\}}$ and $N_{DG\{GG\}} + N_{DY\{GG\}} > N_{CG\{GG\}} + N_{CY\{GG\}}$. Therefore, we have $x_{D\{GG\}} < x_{C\{GG\}} < \frac{1}{2}$. Since the sign of $x_{C\{ij\}} - x_{D\{ij\}}$ is the same as the sign of $z_{Y\{ij\}} - z_{G\{ij\}}$ for any outcome ij, we have $z_{G\{GG\}} < z_{Y\{GG\}}$. Furthermore, we know that $z_{G\{GG\}} < \frac{1}{2}$ since $N_{DG\{GG\}} + N_{DY\{GG\}} > N_{CG\{GG\}} + N_{CY\{GG\}}$.

Therefore, as shown in Figure E2, $x_{D\{GG\}} < x_{C\{GG\}} < \frac{1}{2}$, $z_{G\{GG\}} < z_{Y\{GG\}}$, and $z_{G\{GG\}} < \frac{1}{2}$. Based on horizontal flip, we know that $x_{D\{YY\}} = 1 - x_{D\{GG\}}$, $x_{C\{YY\}} = 1 - x_{C\{GG\}}$, $z_{G\{YY\}} = z_{Y\{GG\}}$, and $z_{Y\{YY\}} = z_{G\{GG\}}$. Therefore, $x_{D\{YY\}} > x_{C\{YY\}} > \frac{1}{2}$, $z_{Y\{YY\}} < z_{G\{YY\}}$, and $z_{Y\{YY\}} < \frac{1}{2}$.



Outcome YY

Outcome GG

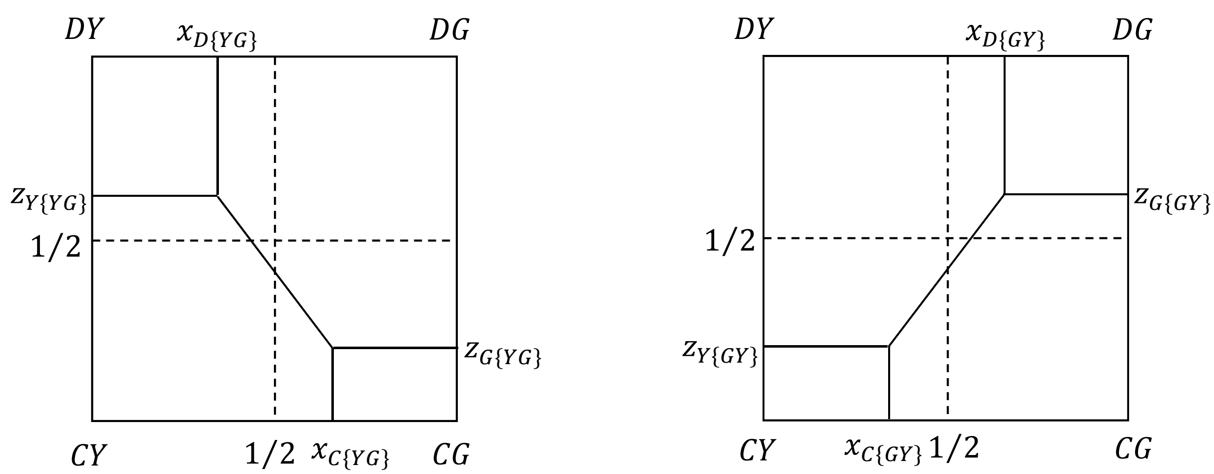
Figure E2. Demand Distribution of Class b (outcomes YY and GG)

(Class c) Outcomes YG and GY

Based on symmetry under horizontal flip, we can obtain the demand distribution of GY by reflecting the demand distribution of outcome YG through the line $x = \frac{1}{2}$. Thus we may focus on deriving the demand distribution of outcome YG.

Under outcome YG, only Y pays for preferential delivery on C and only G pays for preferential delivery on D. Thus, $w_{CG\{YG\}} - w_{CY\{YG\}} > 0$ and $w_{DG\{YG\}} - w_{DY\{YG\}} < 0$. Therefore we have $x_{C\{YG\}} > \frac{1}{2} > x_{D\{YG\}}$. Since the sign of $x_{C\{ij\}} - x_{D\{ij\}}$ is the same as the sign of $z_{Y\{ij\}} - z_{G\{ij\}}$ for any outcome ij, we have $z_{Y\{YG\}} > z_{G\{YG\}}$. When $F_C > F_D$, we know that $N_{DG\{YG\}} + N_{DY\{YG\}} > N_{CG\{YG\}} + N_{CY\{YG\}}$. Therefore, we have $z_{G\{YG\}} < \frac{1}{2}$.

Therefore, as shown in Figure E3, $x_{D\{YG\}} < \frac{1}{2} < x_{C\{YG\}}$, $z_{G\{YG\}} < z_{Y\{YG\}}$, and $z_{G\{YG\}} < \frac{1}{2}$. Based on horizontal flip, we know that $x_{D\{GY\}} = 1 - x_{D\{YG\}}$, $x_{C\{GY\}} = 1 - x_{C\{YG\}}$, $z_{G\{GY\}} = z_{Y\{YG\}}$, and $z_{Y\{GY\}} = z_{G\{YG\}}$. Therefore, $x_{C\{GY\}} < \frac{1}{2} < x_{D\{GY\}}$, $z_{Y\{GY\}} < z_{G\{GY\}}$, and $z_{Y\{GY\}} < \frac{1}{2}$.



Outcome YG

Outcome GY

Figure E3. Demand Distribution of Class c (outcomes YG and GY)

(Class d) Outcomes NY, NG, YN, GN, BY, BG, YB, and GB

The demand analysis for outcomes BY, BG, YB, and GB is the same as that in outcomes NY, NG, YN, and GN since both CPs receive the same queuing priority when they both pay for preferential delivery. Based on symmetry under horizontal flip, we can obtain the demand distribution of NY by reflecting the demand distribution of outcome NG through the line $x = \frac{1}{2}$. Similarly, we can obtain the demand distribution of YN by reflecting the demand distribution of outcome GN through the line $x = \frac{1}{2}$. Thus, we may focus on deriving the demand distribution of outcomes NG and GN.

In outcome NG, neither CP pays on C and only G pays on D. Thus, $w_{CG\{NG\}} - w_{CY\{NG\}} = 0$ and $w_{DG\{NG\}} - w_{DY\{NG\}} < 0$. Therefore, we have $x_{D\{NG\}} < x_{C\{NG\}} = \frac{1}{2}$. Since the sign of $x_{C\{ij\}} - x_{D\{ij\}}$ is the same as the sign of $z_{Y\{ij\}} - z_{G\{ij\}}$ for any outcome ij, we have $z_{G\{NG\}} < z_{Y\{NG\}}$. When $F_C > F_D$, we know that $N_{DG\{NG\}} + N_{DY\{NG\}} > N_{CG\{NG\}} + N_{CY\{NG\}}$. Therefore, we have $z_{G\{NG\}} < \frac{1}{2}$.

Therefore, as shown in Figure E4, $x_{D\{NG\}} < x_{C\{NG\}} = \frac{1}{2}$, $z_{G\{NG\}} < z_{Y\{NG\}}$, and $z_{G\{NG\}} < \frac{1}{2}$. Based on horizontal flip, we know that $x_{D\{NY\}} = 1 - x_{D\{NG\}}$, $x_{C\{NY\}} = 1 - x_{C\{NG\}}$, $z_{G\{NY\}} = z_{Y\{NG\}}$, and $z_{Y\{NY\}} = z_{G\{NG\}}$. Therefore, $x_{C\{NY\}} < x_{D\{NY\}} = \frac{1}{2}$, $z_{Y\{NY\}} < z_{G\{NY\}}$, and $z_{Y\{NY\}} < \frac{1}{2}$.

Similarly, in outcome GN, only G pays on C and neither CP pays on D. Thus, $w_{CG\{GN\}} - w_{CY\{GN\}} < 0$ and $w_{DG\{GN\}} - w_{DY\{GN\}} = 0$. Therefore, we have $x_{C\{GN\}} < x_{D\{GN\}} = \frac{1}{2}$. Since the sign of $x_{C\{ij\}} - x_{D\{ij\}}$ is the same as the sign of $z_{Y\{ij\}} - z_{G\{ij\}}$ for any outcome ij, we have $z_{Y\{GN\}} < z_{G\{GN\}}$. From the analysis of symmetric equilibria, we know that when $F_C = F_D$, $z_{Y\{GN\}} < \frac{1}{2}$. Thus, when $F_C > F_D$, have $z_{Y\{GN\}} < \frac{1}{2}$.

Therefore, as shown in Figure E4, $x_{C\{GN\}} < x_{D\{GN\}} = \frac{1}{2}$, $z_{Y\{GN\}} < z_{G\{GN\}}$, and $z_{Y\{GN\}} < \frac{1}{2}$. Based on horizontal flip, we know that $x_{D\{YN\}} = 1 - x_{D\{GN\}}$, $x_{C\{YN\}} = 1 - x_{C\{GN\}}$, $z_{G\{YN\}} = z_{Y\{GN\}}$, and $z_{Y\{YN\}} = z_{G\{GN\}}$. Therefore, $x_{D\{YN\}} = \frac{1}{2} < x_{C\{YN\}}$, $z_{G\{YN\}} < z_{Y\{YN\}}$, and $z_{G\{YN\}} < \frac{1}{2}$.

The demand patterns for outcomes BY, BG, YB, and GB are identical to that for outcomes NY, NG, YN, and GN respectively.

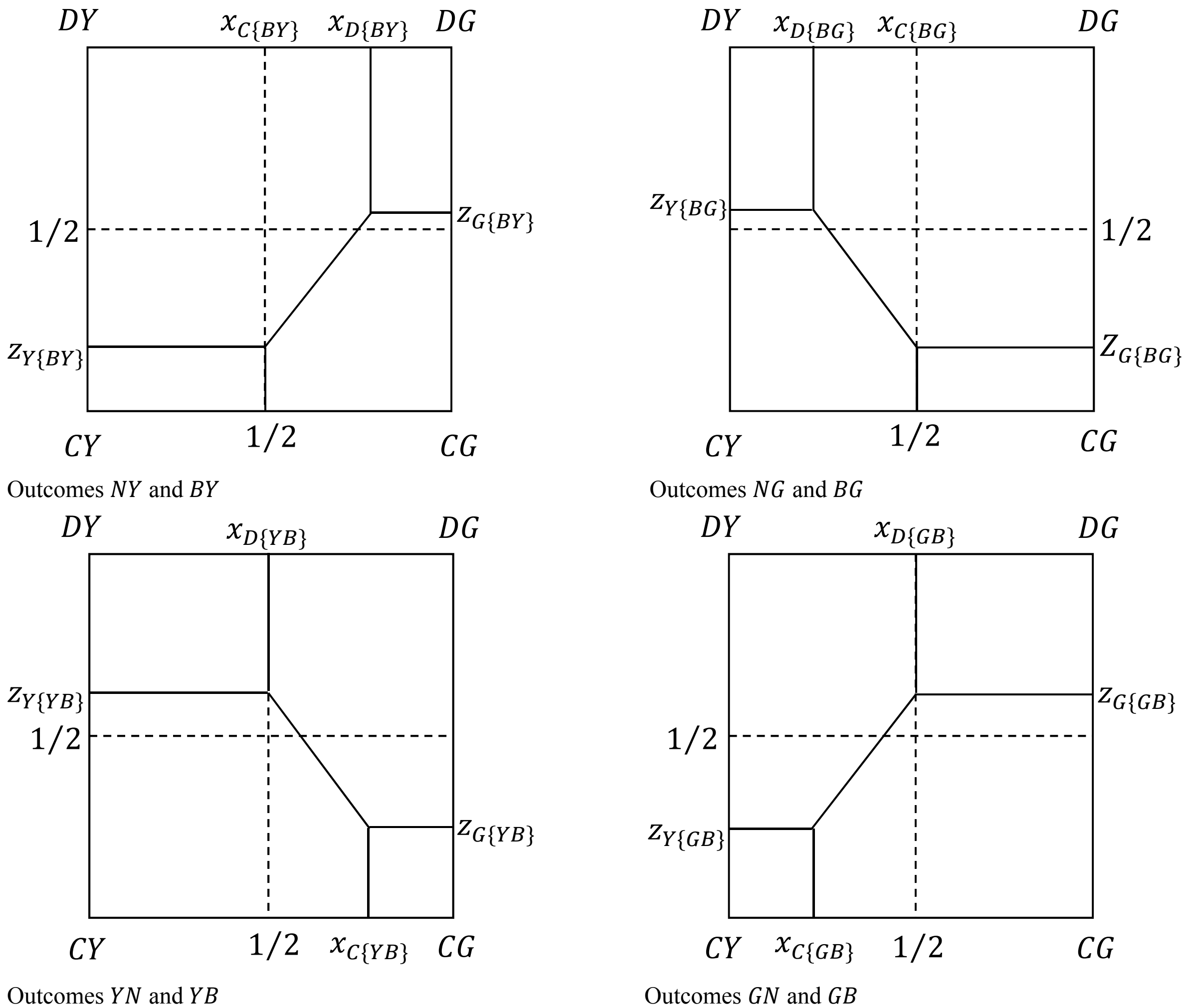


Figure E4. Demand Distribution of Class d (Outcomes NY, NG, YN, GN, BY, BG, YB, and GB)

Step 2: Derive Properties of the Equilibrium Fixed Fees F_C and F_D in Asymmetric Equilibrium

As shown in step 2 in Appendix D, the equilibrium fixed fees F_C and F_D take two different forms: in Case (A) when $x_C \leq x_D$ and $z_Y \leq z_G$, we have $F_C = V - t\left(1 - \frac{x_C + x_D}{2}\right) - k\left(\frac{z_Y + z_G}{2}\right) - d\lambda w_{CG}$ and $F_D = V - t\left(\frac{x_C + x_D}{2}\right) - k\left(1 - \frac{z_Y + z_G}{2}\right) - d\lambda w_{DY}$; in Case (B) when $x_C \geq x_D$ and $z_Y \geq z_G$, we have $F_C = V - t\left(\frac{x_C + x_D}{2}\right) - k\left(\frac{z_Y + z_G}{2}\right) - d\lambda w_{CY}$ and $F_D = V - t\left(1 - \frac{x_C + x_D}{2}\right) - k\left(1 - \frac{z_Y + z_G}{2}\right) - d\lambda w_{DG}$. Among the 16 outcomes, outcomes NN, NB, BN, and BB are contained in both Case (A) and Case (B); outcomes NY, BY, GN, GB, YY, and GY are contained in Case (A); outcomes NG, BG, YN, YB, GG, and YG are contained in Case (B).

Based on the results in step 1, we know the demand patterns and waiting times are related across different outcomes by horizontal flip. Therefore, we can compare the equilibrium fixed fees F_C and F_D for the following groups of outcomes.

Outcomes NN, NB, BN, and BB

For outcomes NN, NB, BN, and BB, we have $x_{C\{NN\}} = x_{D\{NN\}} = x_{C\{NB\}} = x_{D\{NB\}} = x_{C\{BN\}} = x_{D\{BN\}} = x_{C\{BB\}} = x_{D\{BB\}} = \frac{1}{2}$ and $z_{Y\{NN\}} = z_{G\{NN\}} = z_{Y\{NB\}} = z_{G\{NB\}} = z_{Y\{BN\}} = z_{G\{BN\}} = z_{Y\{BB\}} = z_{G\{BB\}} < \frac{1}{2}$. In addition, we know that $w_{CG\{NN\}} = w_{CY\{NN\}} = w_{CG\{NB\}} = w_{CY\{NB\}} = w_{CG\{BN\}} = w_{CY\{BN\}} = w_{CG\{BB\}} = w_{CY\{BB\}}$ and $w_{DY\{NN\}} = w_{DG\{NN\}} = w_{DY\{NB\}} = w_{DG\{NB\}} = w_{DY\{BN\}} = w_{DG\{BN\}} = w_{DY\{BB\}} = w_{DG\{BB\}}$. Therefore, we know that $F_{C\{NN\}} = F_{C\{NB\}} = F_{C\{BN\}} = F_{C\{BB\}}$ and $F_{D\{NN\}} = F_{D\{NB\}} = F_{D\{BN\}} = F_{D\{BB\}}$.

Outcomes NY, NG, BY, and BG

For outcomes NY, NG, BY, and BG, we have $x_{C\{NY\}} = x_{C\{NG\}} = x_{C\{BY\}} = x_{C\{BG\}} = \frac{1}{2}$, $x_{D\{NY\}} = 1 - x_{D\{NG\}} = 1 - x_{D\{BG\}} = x_{D\{BY\}}$, $z_{Y\{NY\}} = z_{G\{NG\}} = z_{Y\{BY\}} = z_{G\{BG\}}$, and $z_{G\{NY\}} = z_{Y\{NG\}} = z_{G\{BY\}} = z_{Y\{BG\}}$. In addition, we know that $w_{CG\{NY\}} = w_{CY\{NG\}} = w_{CG\{BY\}} = w_{CY\{BG\}}$ and $w_{DY\{NY\}} = w_{DG\{NG\}} = w_{DY\{BY\}} = w_{DG\{BG\}}$. Therefore, we know that $F_{C\{NY\}} = F_{C\{NG\}} = F_{C\{BY\}} = F_{C\{BG\}}$ and $F_{D\{NY\}} = F_{D\{NG\}} = F_{D\{BY\}} = F_{D\{BG\}}$.

Outcomes GN, YN, GB, and YB

For outcomes GN, YN, GB, and YB, we have $x_{D\{GN\}} = x_{D\{YN\}} = x_{D\{GB\}} = x_{D\{YB\}} = \frac{1}{2}$, $x_{C\{GN\}} = 1 - x_{C\{YN\}} = 1 - x_{C\{YB\}} = x_{C\{GB\}}$, $z_{Y\{GN\}} = z_{G\{YN\}} = z_{Y\{GB\}} = z_{G\{YB\}}$, and $z_{G\{GN\}} = z_{Y\{YN\}} = z_{G\{GB\}} = z_{Y\{YB\}}$. In addition, we know that $w_{CG\{GN\}} = w_{CY\{YN\}} = w_{CG\{GB\}} = w_{CY\{YB\}}$ and $w_{DY\{GN\}} = w_{DG\{YN\}} = w_{DY\{GB\}} = w_{DG\{YB\}}$. Therefore, we know that $F_{C\{GN\}} = F_{C\{YN\}} = F_{C\{GB\}} = F_{C\{YB\}}$ and $F_{D\{GN\}} = F_{D\{YN\}} = F_{D\{GB\}} = F_{D\{YB\}}$.

Outcomes GY and YG

For outcomes GY and YG, we have $x_{C\{GY\}} = 1 - x_{C\{YG\}}$, $x_{D\{GY\}} = 1 - x_{D\{YG\}}$, $z_{Y\{GY\}} = z_{G\{YG\}}$, and $z_{G\{GY\}} = z_{Y\{YG\}}$. In addition, we know that $w_{CG\{GY\}} = w_{CY\{YG\}}$ and $w_{DY\{GY\}} = w_{DG\{YG\}}$. Therefore, we know that $F_{C\{GY\}} = F_{C\{YG\}}$ and $F_{D\{GY\}} = F_{D\{YG\}}$.

Step 3: Eliminate Dominated and Infeasible Outcomes in Asymmetric Equilibrium

Next we compare groups of outcomes and eliminate the dominated outcomes from further analysis of asymmetric equilibrium.

Outcomes NN, NB, and BN are dominated

In outcome NN, the ISPs' profit functions are $\pi_{C\{NN\}} = (N_{CY\{NN\}} + N_{CG\{NN\}})F_{C\{NN\}}$ and $\pi_{D\{NN\}} = (N_{DY\{NN\}} + N_{DG\{NN\}})F_{D\{NN\}}$. In outcome NB, the ISPs' profit functions are $\pi_{C\{NB\}} = (N_{CY\{NB\}} + N_{CG\{NB\}})F_{C\{NB\}}$ and $\pi_{D\{NB\}} = (N_{DY\{NB\}} + N_{DG\{NB\}})(F_{D\{NB\}} + \lambda p_{D\{NB\}})$. In outcome BN, the ISPs' profit functions are $\pi_{C\{BN\}} = (N_{CY\{BN\}} + N_{CG\{BN\}})(F_{C\{BN\}} + \lambda p_{C\{BN\}})$ and $\pi_{D\{BN\}} = (N_{DY\{BN\}} + N_{DG\{BN\}})F_{D\{BN\}}$. In outcome BB, the ISPs' profit functions are $\pi_{C\{BB\}} = (N_{CY\{BB\}} + N_{CG\{BB\}})(F_{C\{BB\}} + \lambda p_{C\{BB\}})$ and $\pi_{D\{BB\}} = (N_{DY\{BB\}} + N_{DG\{BB\}})(F_{D\{BB\}} + \lambda p_{D\{BB\}})$. Based on the results in step 1, we know that $N_{CY\{NN\}} = N_{CG\{NN\}} = N_{CY\{NB\}} = N_{CG\{NB\}} = N_{CY\{BN\}} = N_{CG\{BN\}} = N_{CY\{BB\}} = N_{CG\{BB\}}$ and $N_{DY\{NN\}} = N_{DG\{NN\}} = N_{DY\{NB\}} = N_{DG\{NB\}} = N_{DY\{BN\}} = N_{DG\{BN\}} = N_{DY\{BB\}} = N_{DG\{BB\}}$. Based on the results in step 2, we know that $F_{C\{NN\}} = F_{C\{NB\}} = F_{C\{BN\}} = F_{C\{BB\}}$ and $F_{D\{NN\}} = F_{D\{NB\}} = F_{D\{BN\}} = F_{D\{BB\}}$.

Comparing pairs of these outcomes yields $\pi_{D\{NN\}} < \pi_{D\{NB\}}$, $\pi_{D\{BN\}} < \pi_{D\{BB\}}$, and $\pi_{C\{NB\}} < \pi_{C\{BB\}}$. Therefore, outcomes NN, BN, and NB are dominated and can be eliminated from further analysis of asymmetric equilibrium.

Outcome NY is dominated by outcome NG

The feasible region of $p_{C\{NY\}}$ and $p_{D\{NY\}}$ is determined by the six incentive compatibility constraints: $\pi_{Y\{NY\}} - \pi_{Y\{YY\}} \geq 0$, $\pi_{Y\{NY\}} - \pi_{Y\{NN\}} \geq 0$, $\pi_{Y\{NY\}} - \pi_{Y\{YN\}} \geq 0$, $\pi_{G\{NY\}} - \pi_{G\{GY\}} \geq 0$, $\pi_{G\{NY\}} - \pi_{G\{NB\}} \geq 0$, and $\pi_{G\{NY\}} - \pi_{G\{GB\}} \geq 0$. These constraints respectively imply $(N_{CY\{NY\}} + N_{DY\{NY\}} - N_{CY\{YY\}} - N_{DY\{YY\}})r_Y + N_{CY\{YY\}}p_{C\{NY\}} + (N_{DY\{YY\}} - N_{DY\{NY\}})p_{D\{NY\}} \geq 0$, $\left(N_{CY\{NY\}} + N_{DY\{NY\}} - \frac{1}{2}\right)r_Y - N_{DY\{NY\}}p_{D\{NY\}} \geq 0$, $(N_{CY\{NY\}} + N_{DY\{NY\}} - N_{CY\{YN\}} + N_{DY\{YN\}})r_Y + N_{CY\{YN\}}p_{C\{NY\}} - N_{DY\{NY\}}p_{D\{NY\}} \geq 0$, $\left(N_{CG\{NY\}} + N_{DG\{NY\}} - \frac{1}{2}\right)r_G + N_{CG\{GY\}}p_{C\{NY\}} \geq 0$, $\left(N_{CG\{NY\}} + N_{DG\{NY\}} - \frac{1}{2}\right)r_G + N_{DG\{NB\}}p_{D\{NY\}} \geq 0$, and $(N_{CG\{NY\}} + N_{DG\{NY\}} - N_{CY\{GB\}} - N_{DY\{GB\}})r_G + N_{CG\{GB\}}p_{C\{NY\}} + N_{DG\{GB\}}p_{D\{NY\}} \geq 0$.

The feasible region of $p_{C\{NG\}}$ and $p_{D\{NG\}}$ is determined by the six incentive compatibility constraints: $\pi_{Y\{NG\}} - \pi_{Y\{YG\}} \geq 0$, $\pi_{Y\{NG\}} - \pi_{Y\{NB\}} \geq 0$, $\pi_{Y\{NG\}} - \pi_{Y\{YB\}} \geq 0$, $\pi_{G\{NG\}} - \pi_{G\{GG\}} \geq 0$, $\pi_{G\{NG\}} - \pi_{G\{NN\}} \geq 0$, and $\pi_{G\{NG\}} - \pi_{G\{GN\}} \geq 0$. These constraints respectively imply $\left(N_{CY\{NG\}} + N_{DY\{NG\}} - \frac{1}{2}\right)r_Y + N_{CY\{YG\}}p_{C\{NG\}} \geq 0$, $\left(N_{CY\{NG\}} + N_{DY\{NG\}} - \frac{1}{2}\right)r_Y + N_{DY\{NB\}}p_{D\{NG\}} \geq 0$, $(N_{CY\{NG\}} + N_{DY\{NG\}} - N_{CY\{YB\}} - N_{DY\{YB\}})r_Y + N_{CY\{YB\}}p_{C\{NG\}} + N_{DY\{YB\}}p_{D\{NG\}} \geq 0$, $(N_{CG\{NG\}} + N_{DG\{NG\}} - N_{CG\{GG\}} - N_{DG\{GG\}})r_G + N_{CG\{GG\}}p_{C\{NG\}} +$

$\left(N_{DG\{GG\}}-N_{DG\{NG\}}\right)p_{D\{NG\}}\geq 0$, $\left(N_{CG\{NG\}}+N_{DG\{NG\}}-\frac{1}{2}\right)r_G-N_{DG\{NG\}}p_{D\{NG\}}\geq 0$, and $\left(N_{CG\{NG\}}+N_{DG\{NG\}}-N_{CG\{GN\}}-N_{DG\{GN\}}\right)r_G+N_{CG\{GN\}}p_{C\{NG\}}-N_{DG\{NG\}}p_{D\{NG\}}\geq 0$.

The feasible region of $p_{C\{NG\}}$ and $p_{D\{NG\}}$ contains the feasible region of $p_{C\{NY\}}$ and $p_{D\{NY\}}$ since $r_G\geq r_Y$ and the demand patterns across outcomes are related by horizontal flip. Based on the results in step 2, we know that $F_{C\{NY\}}=F_{C\{NG\}}$ and $F_{D\{NY\}}=F_{D\{NG\}}$. In outcome NY, the ISPs' profit functions are $\pi_{C\{NY\}}=\left(N_{CY\{NY\}}+N_{CG\{NY\}}\right)F_{C\{NY\}}$ and $\pi_{D\{NY\}}=\left(N_{DY\{NY\}}+N_{DG\{NY\}}\right)F_{D\{NY\}}+\lambda N_{DY\{NY\}}p_{D\{NY\}}$. In outcome NG, the ISPs' profit functions are $\pi_{C\{NG\}}=\left(N_{CY\{NG\}}+N_{CG\{NG\}}\right)F_{C\{NG\}}$ and $\pi_{D\{NG\}}=\left(N_{DY\{NG\}}+N_{DG\{NG\}}\right)F_{D\{NG\}}+\lambda N_{DG\{NG\}}p_{D\{NG\}}$. Therefore, outcome NY is dominated by outcome NG since $\pi_{D\{NY\}}\leq\pi_{D\{NG\}}$.

Outcomes BY is dominated by outcome BG

The feasible region of $p_{C\{BY\}}$ and $p_{D\{BY\}}$ is determined by the six incentive compatibility constraints: $\pi_{Y\{BY\}}-\pi_{Y\{GY\}}\geq 0$, $\pi_{Y\{BY\}}-\pi_{Y\{BN\}}\geq 0$, $\pi_{Y\{BY\}}-\pi_{Y\{GN\}}\geq 0$, $\pi_{G\{BY\}}-\pi_{G\{YY\}}\geq 0$, $\pi_{G\{BY\}}-\pi_{G\{BB\}}\geq 0$, and $\pi_{G\{BY\}}-\pi_{G\{YB\}}\geq 0$. These constraints respectively imply $\left(N_{CY\{BY\}}+N_{DY\{BY\}}-\frac{1}{2}\right)r_Y-N_{CY\{BY\}}p_{C\{BY\}}+\left(N_{DY\{GY\}}-N_{DY\{BY\}}\right)p_{D\{BY\}}\geq 0$, $\left(N_{CY\{BY\}}+N_{DY\{BY\}}-\frac{1}{2}\right)r_Y+\left(N_{CY\{BN\}}-N_{CY\{BY\}}\right)p_{C\{BY\}}-N_{DY\{BY\}}p_{D\{BY\}}\geq 0$, $\left(N_{CY\{BY\}}+N_{DY\{BY\}}-N_{CY\{GN\}}-N_{DY\{GN\}}\right)r_Y-N_{CY\{BY\}}p_{C\{BY\}}-N_{DY\{BY\}}p_{D\{BY\}}\geq 0$, $\left(N_{CG\{BY\}}+N_{DG\{BY\}}-N_{CG\{YY\}}-N_{DG\{YY\}}\right)r_G-N_{CG\{YB\}}p_{C\{BY\}}\geq 0$, $\left(N_{CG\{BY\}}+N_{DG\{BY\}}-\frac{1}{2}\right)r_G+\left(N_{CG\{BB\}}-N_{CG\{BY\}}\right)p_{C\{BY\}}+N_{DG\{BB\}}p_{D\{BY\}}\geq 0$, and $\left(N_{CG\{BY\}}+N_{DG\{BY\}}-N_{CG\{YB\}}-N_{DG\{YB\}}\right)r_G-N_{CG\{BY\}}p_{C\{BY\}}+N_{DG\{YB\}}p_{D\{BY\}}\geq 0$.

The feasible region of $p_{C\{BG\}}$ and $p_{D\{BG\}}$ is determined by the six incentive compatibility constraints: $\pi_{Y\{BG\}}-\pi_{Y\{GG\}}\geq 0$, $\pi_{Y\{BG\}}-\pi_{Y\{BB\}}\geq 0$, $\pi_{Y\{BG\}}-\pi_{Y\{GB\}}\geq 0$, $\pi_{G\{BG\}}-\pi_{G\{YG\}}\geq 0$, $\pi_{G\{BG\}}-\pi_{G\{BN\}}\geq 0$, and $\pi_{G\{BG\}}-\pi_{G\{YN\}}\geq 0$. These constraints respectively imply $\left(N_{CY\{BG\}}+N_{DY\{BG\}}-N_{CY\{GG\}}-N_{DY\{GG\}}\right)r_Y-N_{CY\{BG\}}p_{C\{BG\}}\geq 0$, $\left(N_{CY\{BG\}}+N_{DY\{BG\}}-\frac{1}{2}\right)r_Y+\left(N_{CY\{BB\}}-N_{CY\{BG\}}\right)p_{C\{BG\}}+N_{DY\{BB\}}p_{D\{BG\}}\geq 0$, $\left(N_{CY\{BG\}}+N_{DY\{BG\}}-N_{CY\{GB\}}-N_{DY\{GB\}}\right)r_Y-N_{CY\{BG\}}p_{C\{BG\}}+N_{DY\{GB\}}p_{D\{BG\}}\geq 0$, $\left(N_{CG\{BG\}}+N_{DG\{BG\}}-N_{CG\{YG\}}-N_{DG\{YG\}}\right)r_G-N_{CG\{BG\}}p_{C\{BG\}}+\left(N_{DG\{YG\}}-N_{DG\{BG\}}\right)p_{D\{BG\}}\geq 0$, $\left(N_{CG\{BG\}}+N_{DG\{BG\}}-\frac{1}{2}\right)r_G+\left(N_{CG\{BN\}}-N_{CG\{BG\}}\right)p_{C\{BG\}}-N_{DG\{BG\}}p_{D\{BG\}}\geq 0$, and $\left(N_{CG\{BG\}}+N_{DG\{BG\}}-N_{CG\{YN\}}-N_{DG\{YN\}}\right)r_G-N_{CG\{BG\}}p_{C\{BG\}}-N_{DG\{BG\}}p_{D\{BG\}}\geq 0$.

The feasible region of $p_{C\{BG\}}$ and $p_{D\{BG\}}$ contains the feasible region of $p_{C\{BY\}}$ and $p_{D\{BY\}}$ since $r_G\geq r_Y$ and the demand patterns across outcomes are related by horizontal flip. Based on the results in step 2, we know that $F_{C\{BY\}}=F_{C\{BG\}}$ and $F_{D\{BY\}}=F_{D\{BG\}}$. In outcome BY, the ISPs' profit functions are $\pi_{C\{BY\}}=\left(N_{CY\{BY\}}+N_{CG\{BY\}}\right)\left(F_{C\{BY\}}+\lambda p_{C\{BY\}}\right)$ and $\pi_{D\{BY\}}=\left(N_{DY\{BY\}}+N_{DG\{BY\}}\right)F_{D\{BY\}}+\lambda N_{DY\{BY\}}p_{D\{BY\}}$. In outcome BG, the ISPs' profit functions are $\pi_{C\{BG\}}=\left(N_{CY\{BG\}}+N_{CG\{BG\}}\right)\left(F_{C\{BG\}}+\lambda p_{C\{BG\}}\right)$ and $\pi_{D\{BG\}}=\left(N_{DY\{BG\}}+N_{DG\{BG\}}\right)F_{D\{BG\}}+\lambda N_{DG\{BG\}}p_{D\{BG\}}$. Therefore, outcome BY is dominated by outcome BG since $\pi_{D\{BY\}}\leq\pi_{D\{BG\}}$.

Outcomes NG is dominated by outcome BG

In outcome NG, the ISPs' profit functions are $\pi_{C\{NG\}}=\left(N_{CY\{NG\}}+N_{CG\{NG\}}\right)F_{C\{NG\}}$ and $\pi_{D\{NG\}}=\left(N_{DY\{NG\}}+N_{DG\{NG\}}\right)F_{D\{NG\}}+\lambda N_{DG\{NG\}}p_{D\{NG\}}$. In outcome BG, the ISPs' profit functions are $\pi_{C\{BG\}}=\left(N_{CY\{BG\}}+N_{CG\{BG\}}\right)\left(F_{C\{BG\}}+\lambda p_{C\{BG\}}\right)$ and $\pi_{D\{BG\}}=\left(N_{DY\{BG\}}+N_{DG\{BG\}}\right)F_{D\{BG\}}+\lambda N_{DG\{BG\}}p_{D\{BG\}}$. Based on the results in step 1, we know that $N_{CY\{NG\}}=N_{CY\{BG\}}$ and $N_{CG\{NG\}}=N_{CG\{BG\}}$. Based on the results in step 2, we know that $F_{C\{NG\}}=F_{C\{BG\}}$. Therefore, outcome NG is dominated by outcome BG since $\pi_{C\{NG\}}\leq\pi_{C\{BG\}}$.

Outcomes YN is dominated by outcome GN

The feasible region of $p_{C\{YN\}}$ and $p_{D\{YN\}}$ is determined by the six incentive compatibility constraints: $\pi_{Y\{YN\}}-\pi_{Y\{NN\}}\geq 0$, $\pi_{Y\{YN\}}-\pi_{Y\{YY\}}\geq 0$, $\pi_{Y\{YN\}}-\pi_{Y\{NY\}}\geq 0$, $\pi_{G\{YN\}}-\pi_{G\{BN\}}\geq 0$, $\pi_{G\{YN\}}-\pi_{G\{YG\}}\geq 0$, and $\pi_{G\{YN\}}-\pi_{G\{BG\}}\geq 0$. These constraints respectively imply $\left(N_{CY\{YN\}}+N_{DY\{YN\}}-\frac{1}{2}\right)r_Y-N_{CY\{YN\}}p_{C\{YN\}}\geq 0$, $\left(N_{CY\{YN\}}+N_{DY\{YN\}}-N_{CY\{YY\}}-N_{DY\{YY\}}\right)r_Y+\left(N_{CY\{YY\}}-N_{CY\{YN\}}\right)p_{C\{YN\}}+N_{DY\{YY\}}p_{D\{YN\}}\geq 0$, $\left(N_{CY\{YN\}}+N_{DY\{YN\}}-N_{CY\{NY\}}-N_{DY\{NY\}}\right)r_Y-N_{CY\{YN\}}p_{C\{YN\}}+N_{DY\{NY\}}p_{D\{YN\}}\geq 0$, $\left(N_{CG\{YN\}}+N_{DG\{YN\}}-\frac{1}{2}\right)r_G+N_{CG\{BN\}}p_{C\{YN\}}\geq 0$, $\left(N_{CG\{YN\}}+N_{DG\{YN\}}-N_{CG\{YG\}}-N_{DG\{YG\}}\right)r_G+N_{DG\{YG\}}p_{D\{YN\}}\geq 0$, and $\left(N_{CG\{YN\}}+N_{DG\{YN\}}-N_{CG\{BG\}}-N_{DG\{BG\}}\right)r_G+N_{CG\{BG\}}p_{C\{YN\}}+N_{DG\{BG\}}p_{D\{YN\}}\geq 0$.

The feasible region of $p_{C\{GN\}}$ and $p_{D\{GN\}}$ is determined by the six incentive compatibility constraints: $\pi_{Y\{GN\}}-\pi_{Y\{BN\}}\geq 0$, $\pi_{Y\{GN\}}-\pi_{Y\{GY\}}\geq 0$, $\pi_{Y\{GN\}}-\pi_{Y\{BY\}}\geq 0$, $\pi_{G\{GN\}}-\pi_{G\{NN\}}\geq 0$, $\pi_{G\{GN\}}-\pi_{G\{GG\}}\geq 0$, and $\pi_{G\{GN\}}-\pi_{G\{NG\}}\geq 0$. These constraints respectively imply $\left(N_{CY\{GN\}}+N_{DY\{GN\}}-\frac{1}{2}\right)r_Y+N_{CY\{BN\}}p_{C\{GN\}}\geq 0$, $\left(N_{CY\{GN\}}+N_{DY\{GN\}}-N_{CY\{GY\}}-N_{DY\{GY\}}\right)r_Y+N_{DY\{GY\}}p_{D\{GN\}}\geq 0$,

$(N_{CY\{GN\}} + N_{DY\{GN\}} - N_{CY\{BY\}} - N_{DY\{BY\}})r_Y + N_{CY\{BY\}}p_{C\{GN\}} + N_{DY\{BY\}}p_{D\{GN\}} \geq 0$, $\left(N_{CG\{GN\}} + N_{DG\{GN\}} - \frac{1}{2}\right)r_G - N_{CG\{GN\}}p_{C\{GN\}} \geq 0$, $(N_{CG\{GN\}} + N_{DG\{GN\}} - N_{CG\{GG\}} - N_{DG\{GG\}})r_G + (N_{CG\{GG\}} - N_{CG\{GN\}})p_{C\{GN\}} + N_{DG\{GG\}}p_{D\{GN\}} \geq 0$, and $(N_{CG\{GN\}} + N_{DG\{GN\}} - N_{CG\{NG\}} - N_{DG\{NG\}})r_G - N_{CG\{GN\}}p_{C\{GN\}} + N_{DG\{NG\}}p_{D\{GN\}} \geq 0$.

The feasible region of $p_{C\{GN\}}$ and $p_{D\{GN\}}$ contains the feasible region of $p_{C\{YN\}}$ and $p_{D\{YN\}}$ since $r_G \geq r_Y$ and the demand patterns across outcomes are related by horizontal flip. Based on the results in step 2, we know that $F_{C\{GN\}} = F_{C\{YN\}}$ and $F_{D\{GN\}} = F_{D\{YN\}}$. In outcome GN, the ISPs' profit functions are $\pi_{C\{GN\}} = (N_{CY\{GN\}} + N_{CG\{GN\}})F_{C\{GN\}} + \lambda N_{CG\{GN\}}p_{C\{GN\}}$ and $\pi_{D\{GN\}} = (N_{DY\{GN\}} + N_{DG\{GN\}})F_{D\{GN\}}$. In outcome YN, the ISPs' profit functions are $\pi_{C\{YN\}} = (N_{CY\{YN\}} + N_{CG\{YN\}})F_{C\{YN\}} + \lambda N_{CY\{YN\}}p_{C\{YN\}}$ and $\pi_{D\{YN\}} = (N_{DY\{YN\}} + N_{DG\{YN\}})F_{D\{YN\}}$. Therefore, outcome YN is dominated by outcome GN since $\pi_{C\{YN\}} \leq \pi_{C\{GN\}}$.

Outcomes YB is dominated by outcome GB

The feasible region of $p_{C\{YB\}}$ and $p_{D\{YB\}}$ is determined by the six incentive compatibility constraints: $\pi_{Y\{YB\}} - \pi_{Y\{NB\}} \geq 0$, $\pi_{Y\{YB\}} - \pi_{Y\{YG\}} \geq 0$, $\pi_{Y\{YB\}} - \pi_{Y\{NG\}} \geq 0$, $\pi_{G\{YB\}} - \pi_{G\{BB\}} \geq 0$, $\pi_{G\{YB\}} - \pi_{G\{YY\}} \geq 0$, and $\pi_{G\{YB\}} - \pi_{G\{BY\}} \geq 0$. These constraints respectively imply $\left(N_{CY\{YB\}} + N_{DY\{YB\}} - \frac{1}{2}\right)r_Y - N_{CY\{YB\}}p_{C\{YB\}} + (N_{DY\{NB\}} - N_{DY\{YB\}})p_{D\{YB\}} \geq 0$, $\left(N_{CY\{YB\}} + N_{DY\{YB\}} - \frac{1}{2}\right)r_Y + (N_{CY\{YG\}} - N_{CY\{YB\}})p_{C\{YB\}} - N_{DY\{YB\}}p_{D\{YB\}} \geq 0$, $(N_{CY\{YB\}} + N_{DY\{YB\}} - N_{CY\{NG\}} - N_{DY\{NG\}})r_Y - N_{CY\{YB\}}p_{C\{YB\}} - N_{DY\{YB\}}p_{D\{YB\}} \geq 0$, $\left(N_{CG\{YB\}} + N_{DG\{YB\}} - \frac{1}{2}\right)r_G + N_{CG\{BB\}}p_{C\{YB\}} + (N_{DG\{BB\}} - N_{DG\{YB\}})p_{D\{YB\}} \geq 0$, $(N_{CG\{YB\}} + N_{DG\{YB\}} - N_{CG\{YY\}} - N_{DG\{YY\}})r_G - N_{DG\{YB\}}p_{D\{YB\}} \geq 0$, and $(N_{CG\{YB\}} + N_{DG\{YB\}} - N_{CG\{BY\}} - N_{DG\{BY\}})r_G + N_{CG\{BY\}}p_{C\{YB\}} - N_{DG\{YB\}}p_{D\{YB\}} \geq 0$.

The feasible region of $p_{C\{GB\}}$ and $p_{D\{GB\}}$ is determined by the six incentive compatibility constraints: $\pi_{Y\{GB\}} - \pi_{Y\{BB\}} \geq 0$, $\pi_{Y\{GB\}} - \pi_{Y\{GG\}} \geq 0$, $\pi_{Y\{GB\}} - \pi_{Y\{BG\}} \geq 0$, $\pi_{G\{GB\}} - \pi_{G\{NB\}} \geq 0$, $\pi_{G\{GB\}} - \pi_{G\{GY\}} \geq 0$, and $\pi_{G\{GB\}} - \pi_{G\{NY\}} \geq 0$. These constraints respectively imply $\left(N_{CY\{GB\}} + N_{DY\{GB\}} - \frac{1}{2}\right)r_Y + N_{CY\{BB\}}p_{C\{GB\}} + (N_{DY\{BB\}} - N_{DY\{GB\}})p_{D\{GB\}} \geq 0$, $(N_{CY\{GB\}} + N_{DY\{GB\}} - N_{CY\{GG\}} - N_{DY\{GG\}})r_Y - N_{DY\{GB\}}p_{D\{GB\}} \geq 0$, $(N_{CY\{GB\}} + N_{DY\{GB\}} - N_{CY\{BG\}} - N_{DY\{BG\}})r_Y + N_{CY\{BG\}}p_{C\{GB\}} - N_{DY\{GB\}}p_{D\{GB\}} \geq 0$, $\left(N_{CG\{GB\}} + N_{DG\{GB\}} - \frac{1}{2}\right)r_G - N_{CG\{GB\}}p_{C\{GB\}} + (N_{DG\{NB\}} - N_{DG\{GB\}})p_{D\{GB\}} \geq 0$, $\left(N_{CG\{GB\}} + N_{DG\{GB\}} - \frac{1}{2}\right)r_G + (N_{CG\{GY\}} - N_{CG\{GB\}})p_{C\{GB\}} - N_{DG\{GB\}}p_{D\{GB\}} \geq 0$, and $(N_{CG\{GB\}} + N_{DG\{GB\}} - N_{CG\{NY\}} - N_{DG\{NY\}})r_G - N_{CG\{GB\}}p_{C\{GB\}} - N_{DG\{GB\}}p_{D\{GB\}} \geq 0$.

The feasible region of $p_{C\{GB\}}$ and $p_{D\{GB\}}$ contains the feasible region of $p_{C\{YB\}}$ and $p_{D\{YB\}}$ since $r_G \geq r_Y$ and the demand patterns across outcomes are related by horizontal flip. Based on the results in step 2, we know that $F_{C\{GB\}} = F_{C\{YB\}}$ and $F_{D\{GB\}} = F_{D\{YB\}}$. In outcome YB, the ISPs' profit functions are $\pi_{C\{YB\}} = (N_{CY\{YB\}} + N_{CG\{YB\}})F_{C\{YB\}} + \lambda N_{CY\{YB\}}p_{C\{YB\}}$ and $\pi_{D\{YB\}} = (N_{DY\{YB\}} + N_{DG\{YB\}})(F_{D\{YB\}} + \lambda p_{C\{YB\}})$. In outcome GB, the ISPs' profit functions are $\pi_{C\{GB\}} = (N_{CY\{GB\}} + N_{CG\{GB\}})F_{C\{GB\}} + \lambda N_{CG\{GB\}}p_{C\{GB\}}$ and $\pi_{D\{GB\}} = (N_{DY\{GB\}} + N_{DG\{GB\}})(F_{D\{GB\}} + \lambda p_{C\{GB\}})$. Therefore, outcome YB is dominated by outcome GB since $\pi_{C\{YB\}} \leq \pi_{C\{GB\}}$.

Outcomes GN is dominated by outcome GB

In outcome GN, the ISPs' profit functions are $\pi_{C\{GN\}} = (N_{CY\{GN\}} + N_{CG\{GN\}})F_{C\{GN\}} + \lambda N_{CG\{GN\}}p_{C\{GN\}}$ and $\pi_{D\{GN\}} = (N_{DY\{GN\}} + N_{DG\{GN\}})F_{D\{GN\}}$. In outcome GB, the ISPs' profit functions are $\pi_{C\{GB\}} = (N_{CY\{GB\}} + N_{CG\{GB\}})F_{C\{GB\}} + \lambda N_{CG\{GB\}}p_{C\{GB\}}$ and $\pi_{D\{GB\}} = (N_{DY\{GB\}} + N_{DG\{GB\}})(F_{D\{GB\}} + \lambda p_{C\{GB\}})$. Based on the results in step 1, we know that $N_{DY\{GN\}} = N_{DY\{GB\}}$ and $N_{DG\{GN\}} = N_{DG\{GB\}}$. Based on the results in step 2, we know that $F_{D\{GN\}} = F_{D\{GB\}}$. Therefore, outcome GN is dominated by outcome GB since $\pi_{D\{GN\}} \leq \pi_{D\{GB\}}$.

Outcomes GY and YG are infeasible

Here we focus on showing that there is no feasible p for outcome YG, as the analysis for outcome GY is similar. For outcomes YG to be feasible, all the CPs' incentive compatibility constraints need to be satisfied: (1) $\pi_{Y\{YG\}} - \pi_{Y\{NG\}} \geq 0$; (2) $\pi_{Y\{YG\}} - \pi_{Y\{YB\}} \geq 0$; (3) $\pi_{Y\{YG\}} - \pi_{Y\{NB\}} \geq 0$; (4) $\pi_{G\{YG\}} - \pi_{G\{BG\}} \geq 0$; (5) $\pi_{G\{YG\}} - \pi_{G\{YN\}} \geq 0$; (6) $\pi_{G\{YG\}} - \pi_{G\{BN\}} \geq 0$.

Inequality (3) is $(N_{CY\{YG\}} + N_{DY\{YG\}} - N_{CY\{NB\}} - N_{DY\{NB\}})r_Y + N_{DY\{NB\}}p_D - N_{CY\{YG\}}p_C \geq 0$. Since $N_{CY\{NB\}} + N_{DY\{NB\}} = \frac{1}{2}$, inequality (3) can be reduced to $p_D \geq \left(\frac{N_{CY\{YG\}}}{N_{DY\{NB\}}}\right)p_C + \left(\frac{\frac{1}{2} - N_{CY\{YG\}} - N_{DY\{YG\}}}{N_{DY\{NB\}}}\right)r_Y$.

Inequality (6) is $\left(N_{CG\{YG\}} + N_{DG\{YG\}} - N_{CG\{BN\}} - N_{DG\{BN\}}\right)r_G + N_{CG\{BN\}}p_C - N_{DG\{YG\}}p_D \geq 0$. Since $N_{CG\{BN\}} + N_{DG\{BN\}} = \frac{1}{2}$, inequality (6) can be reduced to $p_D \leq \left(\frac{N_{CG\{BN\}}}{N_{DG\{YG\}}}\right)p_C + \left(\frac{\frac{1}{2} - N_{CG\{YG\}} - N_{DG\{YG\}}}{N_{DG\{YG\}}}\right)r_G$.

Based on the result in step 1, we have $\frac{1}{2} - N_{CY\{YG\}} - N_{DY\{YG\}} = N_{CG\{YG\}} + N_{DG\{YG\}} - \frac{1}{2} > 0$. This gives $\left(\frac{\frac{1}{2} - N_{CY\{YG\}} - N_{DY\{YG\}}}{N_{DY\{NB\}}}\right)r_Y > 0$ and $\left(\frac{\frac{1}{2} - N_{CG\{YG\}} - N_{DG\{YG\}}}{N_{DG\{YG\}}}\right)r_G < 0$. Next we show that $\frac{N_{CY\{YG\}}}{N_{DY\{NB\}}} > \frac{N_{CG\{BN\}}}{N_{DG\{YG\}}}$. We first note that $N_{DY\{NB\}} = N_{DG\{NN\}}$ and $N_{CG\{BN\}} = N_{CY\{NN\}}$. Thus, $\frac{N_{CY\{YG\}}}{N_{DY\{NB\}}} > \frac{N_{CG\{BN\}}}{N_{DG\{YG\}}} \Leftrightarrow \frac{N_{CY\{YG\}}}{N_{DG\{NN\}}} > \frac{N_{CY\{NN\}}}{N_{DG\{YG\}}} \Leftrightarrow \frac{N_{CY\{YG\}}}{N_{CY\{NN\}}} > \frac{N_{DG\{NN\}}}{N_{DG\{YG\}}}$.

Since $N_{CY\{YG\}} > N_{CY\{NN\}}$ and $N_{DG\{YG\}} > N_{DG\{NN\}}$, we have $\frac{N_{CY\{YG\}}}{N_{CY\{NN\}}} > 1 > \frac{N_{DG\{NN\}}}{N_{DG\{YG\}}}$. Thus, we also have $\frac{N_{CY\{YG\}}}{N_{DY\{NB\}}} > \frac{N_{CG\{BN\}}}{N_{DG\{YG\}}}$. Then (3) and (6) implies that p_C and p_D are both negative. Therefore, outcome YG is infeasible.

Similarly, we can show that there is no feasible p for outcome GY. Therefore, both outcomes YG and GY are infeasible.

Outcomes YY is infeasible

For outcomes YY to be feasible, all the CPs' incentive compatibility constraints need to be satisfied: (1) $\pi_{Y\{YY\}} - \pi_{Y\{NG\}} \geq 0$; (2) $\pi_{Y\{YY\}} - \pi_{Y\{GN\}} \geq 0$; (3) $\pi_{Y\{YY\}} - \pi_{Y\{NN\}} \geq 0$; (4) $\pi_{G\{YY\}} - \pi_{G\{BY\}} \geq 0$; (5) $\pi_{G\{YY\}} - \pi_{G\{YB\}} \geq 0$; (6) $\pi_{G\{YY\}} - \pi_{G\{BB\}} \geq 0$.

Inequality (3) is $\left(N_{CY\{YY\}} + N_{DY\{YY\}} - N_{CY\{NN\}} - N_{DY\{NN\}}\right)r_Y - N_{CY\{YY\}}p_C - N_{DY\{YY\}}p_D \geq 0$. Since $N_{CY\{NN\}} + N_{DY\{NN\}} = \frac{1}{2}$, inequality (3) can be reduced to $\left(N_{CY\{YY\}} + N_{DY\{YY\}} - \frac{1}{2}\right)r_Y \geq N_{CY\{YY\}}p_C + N_{DY\{YY\}}p_D$.

Inequality (6) is $\left(N_{CG\{YY\}} + N_{DG\{YY\}} - N_{CG\{BB\}} - N_{DG\{BB\}}\right)r_G + N_{CG\{BB\}}p_C + N_{DG\{BB\}}p_D \geq 0$. Since $N_{CG\{BB\}} + N_{DG\{BB\}} = \frac{1}{2}$, inequality (6) can be reduced to $N_{CG\{BB\}}p_C + N_{DG\{BB\}}p_D \geq \left(\frac{1}{2} - N_{CG\{YY\}} - N_{DG\{YY\}}\right)r_G$.

Based on the result in step 1, we have $N_{CY\{NN\}} = N_{CG\{BB\}}$, $N_{DY\{NN\}} = N_{DG\{BB\}}$ and $\left(\frac{1}{2} - N_{CG\{YY\}} - N_{DG\{YY\}}\right) = \left(N_{CY\{YY\}} + N_{DY\{YY\}} - \frac{1}{2}\right)$. We also know that $r_G \geq r_Y$. Thus, $N_{CY\{NN\}}p_C + N_{DY\{NN\}}p_D = N_{CG\{BB\}}p_C + N_{DG\{BB\}}p_D \geq N_{CY\{YY\}}p_C + N_{DY\{YY\}}p_D$, which implies $\left(N_{CY\{YY\}} - N_{CY\{NN\}}\right)p_C + \left(N_{DY\{YY\}} - N_{DY\{NN\}}\right)p_D \leq 0$.

Since Y pays for priority delivery on both C and D, we know that $N_{CY\{YY\}} > N_{CY\{NN\}}$ and $N_{DY\{YY\}} > N_{DY\{NN\}}$, i.e., $\left(N_{CY\{YY\}} - N_{CY\{NN\}}\right) > 0$ and $\left(N_{DY\{YY\}} - N_{DY\{NN\}}\right) > 0$. Thus, (3) and (6) imply that either p_C or p_D is negative. Therefore, outcome YY is infeasible.

Therefore, after eliminating all the dominated and infeasible outcomes, we conclude that outcomes GG, GB, BG, and BB as the only four possible asymmetric equilibria.

Appendix F

Proof of Lemma 3

From Lemma 2, we know that outcomes GG, GB, BG, and BB as the only four possible equilibria. Here we conduct symmetric equilibrium analysis ($F_C = F_D = F$ and $p_C = p_D = p$) and derive the ISPs' equilibrium pricing strategies and the corresponding equilibrium outcomes in the packet discrimination regime in the following two steps.

Step 1: Solve for the Equilibrium Fixed Fee F and Preferential Delivery Fee p for the Candidate Outcomes

In step 1, we solve for the equilibrium fixed fee F and preferential delivery fee p for the candidate outcomes one by one. Among the four candidate equilibria, outcome BG and outcome GB are symmetric. Thus, we focus on outcomes BB, BG, and GG in this analysis.

Outcome BB

The preferential delivery fee p for outcome BB is determined by the following two CPs' incentive compatibility constraints: $\pi_{Y\{BB\}} \geq \pi_{Y\{BG\}}$ yields $p_{\{BB\}} \leq \frac{(1/2 - N_{DY\{BG\}} - N_{CY\{BG\}})r_Y}{1/2 - N_{CY\{BG\}}}$; $\pi_{Y\{BB\}} \geq \pi_{Y\{GG\}}$ yields $p_{\{BB\}} \leq \frac{(1/2 - N_{CY\{GG\}} - N_{DY\{GG\}})r_Y}{1/2}$. Therefore, $p^*_{\{BB\}} = H_{\{BB\}} r_Y$, where $H_{\{BB\}} = \min\left\{\frac{1/2 - N_{DY\{BG\}} - N_{CY\{BG\}}}{1/2 - N_{CY\{BG\}}}, \frac{1/2 - N_{CY\{GG\}} - N_{DY\{GG\}}}{1/2}\right\}$. In addition, we know from the results in Lemma 2 that $F^*_{\{BB\}} = V - \frac{t}{2} - \frac{k}{2} - \frac{d\lambda}{\mu - \lambda/2}$.

Outcome BG

The preferential delivery fee p for outcome BG is determined by the following three CPs' incentive compatibility constraints: $\pi_{Y\{BG\}} \geq \pi_{Y\{GG\}}$ yields $p_{\{BG\}} \leq \frac{(N_{CY\{BG\}} + N_{DY\{BG\}} - N_{CY\{GG\}} - N_{DY\{GG\}})r_Y}{N_{CY\{BG\}}}$; $\pi_{Y\{BG\}} \geq \pi_{Y\{BB\}}$ yields $p_{\{BG\}} \geq \frac{(1/2 - N_{CY\{BG\}} - N_{DY\{BG\}})r_Y}{1/2 - N_{CY\{BG\}}}$; $\pi_{G\{BG\}} \geq \pi_{G\{BN\}}$ yields $p_{\{BG\}} \leq \frac{(N_{CG\{BG\}} + N_{DG\{BG\}} - 1/2)r_G}{N_{CG\{BG\}} + N_{DG\{BG\}} - 1/4}$. Thus, there exists a feasible $p_{\{BG\}}$ if and only if $\frac{(1/2 - N_{CY\{BG\}} - N_{DY\{BG\}})r_Y}{1/2 - N_{CY\{BG\}}} \leq \min\left\{\frac{(N_{CY\{BG\}} + N_{DY\{BG\}} - N_{CY\{GG\}} - N_{DY\{GG\}})r_Y}{N_{CY\{BG\}}}, \frac{(N_{CG\{BG\}} + N_{DG\{BG\}} - 1/2)r_G}{N_{CG\{BG\}} + N_{DG\{BG\}} - 1/4}\right\}$, which can be reduced to $r_G \geq \frac{(N_{CG\{BG\}} + N_{DG\{BG\}} - 1/4)r_Y}{1/2 - N_{CY\{BG\}}}$ and $\frac{1/2 - N_{CY\{BG\}} - N_{DY\{BG\}}}{1/2 - N_{CY\{BG\}}} \leq \frac{N_{CY\{BG\}} + N_{DY\{BG\}} - N_{CY\{GG\}} - N_{DY\{GG\}}}{N_{CY\{BG\}}}$. When these feasible conditions hold, we obtain $p^*_{\{BG\}} = \min\{H_{Y\{BG\}} r_Y, H_{G\{BG\}} r_G\}$, where $H_{Y\{BG\}} = \frac{N_{CY\{BG\}} + N_{DY\{BG\}} - N_{CY\{GG\}} - N_{DY\{GG\}}}{N_{CY\{BG\}}}$ and $H_{G\{BG\}} = \frac{N_{CG\{BG\}} + N_{DG\{BG\}} - 1/2}{N_{CG\{BG\}} + N_{DG\{BG\}} - 1/4}$.

We know that in a symmetric equilibrium, $\pi_{C\{BG\}} = \pi_{D\{BG\}}$, i.e., $(N_{DY\{BG\}} + N_{DG\{BG\}})F_{\{BG\}} + \lambda p_{\{BG\}} N_{DG\{BG\}} = (N_{CY\{BG\}} + N_{CG\{BG\}})(F_{\{BG\}} + \lambda p_{\{BG\}})$. Thus, $F^*_{\{BG\}} = \frac{(N_{CY\{BG\}} + N_{CG\{BG\}} - N_{DG\{BG\}})\lambda p^*_{\{BG\}}}{N_{DY\{BG\}} + N_{DG\{BG\}} - N_{CY\{BG\}} - N_{CG\{BG\}}}$. Note that since $N_{CY\{BG\}} + N_{CG\{BG\}} > N_{DG\{BG\}}$ and $F^*_{\{BG\}} \geq 0$, we have $N_{DY\{BG\}} + N_{DG\{BG\}} > N_{CY\{BG\}} + N_{CG\{BG\}}$.

Outcome GG

The preferential delivery fee p for outcome GG is determined by the following three CPs' incentive compatibility constraints: $\pi_{Y\{GG\}} \geq \pi_{Y\{BG\}}$ yields $p_{\{GG\}} \geq \frac{(N_{CY\{BG\}} + N_{DY\{BG\}} - N_{CY\{GG\}} - N_{DY\{GG\}})r_Y}{N_{CY\{BG\}}}$; $\pi_{Y\{GG\}} \geq \pi_{Y\{BB\}}$ yields $p_{\{GG\}} \geq \frac{(1/2 - N_{CY\{GG\}} - N_{DY\{GG\}})r_Y}{1/2}$; $\pi_{G\{GG\}} \geq \pi_{G\{NN\}}$ yields $p_{\{GG\}} \leq \frac{(N_{CG\{GG\}} + N_{DG\{GG\}} - 1/2)r_G}{N_{CG\{GG\}} + N_{DG\{GG\}}}$; $\pi_{G\{GG\}} \geq \pi_{G\{NG\}}$ yields $p_{\{GG\}} \leq \frac{(N_{CG\{GG\}} + N_{DG\{GG\}} - N_{CG\{BG\}} - N_{DG\{BG\}})r_G}{N_{CG\{GG\}} + N_{DG\{GG\}} - N_{DG\{BG\}}}$.
Let $L_{\{GG\}} = \max\left\{\frac{N_{CY\{BG\}} + N_{DY\{BG\}} - N_{CY\{GG\}} - N_{DY\{GG\}}}{N_{CY\{BG\}}}, \frac{1/2 - N_{CY\{GG\}} - N_{DY\{GG\}}}{1/2}\right\}$ and $H_{\{GG\}} = \min\left\{\frac{N_{CG\{GG\}} + N_{DG\{GG\}} - 1/2}{N_{CG\{GG\}} + N_{DG\{GG\}}}, \frac{N_{CG\{GG\}} + N_{DG\{GG\}} - N_{CG\{BG\}} - N_{DG\{BG\}}}{N_{CG\{GG\}} + N_{DG\{GG\}} - N_{DG\{BG\}}}\right\}$. Thus, there exists a feasible $p_{\{GG\}}$ if and only if $L_{\{GG\}} r_Y \leq H_{\{GG\}} r_G$. Here we note that $L_{\{GG\}} \geq \frac{1/2 - N_{CY\{GG\}} - N_{DY\{GG\}}}{1/2}$ and $H_{\{GG\}} \leq \frac{N_{CG\{GG\}} + N_{DG\{GG\}} - 1/2}{N_{CG\{GG\}} + N_{DG\{GG\}}}$. So we have $\frac{L_{\{GG\}}}{H_{\{GG\}}} \geq \frac{\frac{1/2 - N_{CY\{GG\}} - N_{DY\{GG\}}}{1/2}}{\frac{N_{CG\{GG\}} + N_{DG\{GG\}} - 1/2}{N_{CG\{GG\}} + N_{DG\{GG\}}}} = \frac{N_{CG\{GG\}} + N_{DG\{GG\}}}{1/2} > 1$. When these feasible conditions hold, we obtain $p^*_{\{GG\}} = H_{\{GG\}} r_G$. In addition, we know from the results in step 2 in the proof of Lemma 2 that $F^*_{\{GG\}} = V - t(1 - x_{\{GG\}}) - \frac{k}{2} - \frac{d\lambda}{\mu - (1 - x_{D\{GG\}})\lambda/2}$.

We note here that the solution of price p in outcomes BB, BG, and GG form three non-overlapping intervals. Specifically, we have $p_{\{BB\}} \leq H_{\{BB\}} r_Y \leq \frac{(1/2 - N_{CY\{BG\}} - N_{DY\{BG\}})r_Y}{1/2 - N_{CY\{BG\}}} \leq p_{\{BG\}} \leq \min\{H_{Y\{BG\}} r_Y, H_{G\{BG\}} r_G\} \leq L_{\{GG\}} r_Y \leq p_{\{GG\}} \leq H_{\{GG\}} r_G$. The non-overlapping solution reflects the fact that incentive criteria for content providers in outcomes BB, BG, and GG are mutually exclusive. Observe also that the endpoints of the non-overlapping intervals are given by constant multiples of the revenue rates r_Y and r_G.

Step 2: Compare the Candidate Outcomes and Derive Equilibrium Outcomes

In step 2, we compare the ISPs' profits in outcomes BB, BG, and GG to determine the equilibrium outcomes. Since ISPs C and D have the same profit level in a given outcome, we simplify the notations to $\pi_{C\{BB\}} = \pi_{D\{BB\}} = \pi_{\{BB\}}$, $\pi_{C\{BG\}} = \pi_{D\{BG\}} = \pi_{\{BG\}}$, and $\pi_{C\{GG\}} = \pi_{D\{GG\}} = \pi_{\{GG\}}$. Outcome GG is the equilibrium provided all the following inequalities are satisfied: $L_{\{GG\}} r_Y \leq H_{\{GG\}} r_G$, $\pi_{\{GG\}} \geq \pi_{\{BG\}}$, and $\pi_{\{GG\}} \geq \pi_{\{BB\}}$. These reduces to the following inequalities: $r_G \geq \frac{L_{\{GG\}} r_Y}{H_{\{GG\}}} \equiv \beta_1 r_Y$, $r_G \geq \frac{(N_{CY\{BG\}} + N_{CG\{BG\}})p_{\{BG\}}}{H_{\{GG\}} N_{CG\{GG\}}} + \frac{(N_{CY\{BG\}} + N_{CG\{BG\}})F_{\{BG\}} - F_{\{GG\}}/2}{\lambda H_{\{GG\}} N_{CG\{GG\}}} \equiv \alpha_1$, and $r_G \geq \frac{H_{\{BB\}} r_Y}{2H_{\{GG\}} N_{CG\{GG\}}} + \frac{F_{\{BB\}} - F_{\{GG\}}}{2\lambda H_{\{GG\}} N_{CG\{GG\}}} \equiv \beta_2 r_Y + \alpha_2$. Outcome BG (or outcome GB) is the equilibrium provided all the following inequalities are satisfied: $r_G \geq \frac{(N_{CG\{BG\}} + N_{DG\{BG\}} - 1/4)r_Y}{1/2 - N_{CY\{BG\}}} \equiv \beta_3 r_Y$, $\pi_{\{BG\}} > \pi_{\{GG\}}$, and $\pi_{\{BG\}} \geq \pi_{\{BB\}}$. These reduces to the following inequalities:

$r_G \geq \beta_3 r_Y$, $r_G < \alpha_1$, and $r_Y \leq \frac{2(N_{CY\{BG\}}+N_{CG\{BG\}})p_{\{BG\}}}{H_{\{BB\}}} - \frac{F_{\{BB\}}-2(N_{CY\{BG\}}+N_{CG\{BG\}})F_{\{BG\}}}{\lambda H_{\{BB\}}} \equiv \alpha_3$. When the above market conditions are not satisfied, outcome BB is the equilibrium. Summarizing the above analysis yields Lemma 3.

Appendix G

Proof of Proposition 1

Since the net neutrality regime is essentially equivalent to outcome NN, where neither CP pays for preferential delivery even though they have the option to do so. Based on the results from Lemma 2, we know that in the net neutrality regime, $\pi_C^{NN} = \pi_D^{NN} = \pi_{\{NN\}}^* = \frac{F_{\{NN\}}^*}{2}$. In addition, there are four possible equilibria in the packet discrimination regime, i.e., $\pi_C^{PD} = \pi_D^{PD} = \pi_{\{GG\}}^* = \frac{F_{\{GG\}}^*+\lambda p_{\{GG\}}^*(1-x_{\{GG\}})}{2}$, or $\pi_C^{PD} = \pi_D^{PD} = \pi_{\{BG\}}^* = N_{C\{BG\}}(F_{\{BG\}}^* + \lambda p_{\{BG\}}^*) = \pi_{\{GB\}}^* = N_{D\{GB\}}(F_{\{GB\}}^* + \lambda p_{\{GB\}}^*)$, or $\pi_C^{PD} = \pi_D^{PD} = \pi_{\{BB\}}^* = \frac{F_{\{BB\}}^*+\lambda p_{\{BB\}}^*}{2}$. From the results in step 3 in the proof of Lemma 2, we know $\pi_{\{BB\}}^* \geq \pi_{\{NN\}}^*$. Therefore, we get $\pi_C^{PD} = \pi_D^{PD} \geq \pi_{\{BB\}}^* \geq \pi_{\{NN\}}^* = \pi_C^{NN} = \pi_D^{NN}$.

Appendix H

Proof of Proposition 2

In the net neutrality regime, we know that $\pi_G^{NN} = \pi_{G\{NN\}}^* = \frac{\lambda r_G}{2}$. In the packet discrimination regime, there are three possible equilibria – outcomes GG, BG, and BB. The corresponding profit for content provider G is: $\pi_{G\{GG\}}^* = \lambda(N_{CG\{GG\}} + N_{DG\{GG\}})(r_G - p_{\{GG\}}^*)$, $\pi_{G\{BG\}}^* = \lambda(N_{CG\{BG\}} + N_{DG\{BG\}})(r_G - p_{\{BG\}}^*)$, and $\pi_{G\{BB\}}^* = \frac{\lambda(r_G - p_{\{BB\}}^*)}{2}$. Next we focus on comparing $\pi_{G\{GG\}}^*$ and $\pi_{G\{NN\}}^*$.

Recall that $p_{\{GG\}}^* = H_{\{GG\}} r_G$, where $H_{\{GG\}} = \min\left\{\frac{N_{CG\{GG\}}+N_{DG\{GG\}}-1/2}{N_{CG\{GG\}}+N_{DG\{GG\}}}, \frac{N_{CG\{GG\}}+N_{DG\{GG\}}-N_{CG\{BG\}}-N_{DG\{BG\}}}{N_{CG\{GG\}}+N_{DG\{GG\}}-N_{DG\{BG\}}}\right\}$. If $p_{\{GG\}} = \frac{(N_{CG\{GG\}}+N_{DG\{GG\}}-1/2)r_G}{N_{CG\{GG\}}+N_{DG\{GG\}}}$, then $\pi_{G\{GG\}}^* = \lambda(N_{CG\{GG\}} + N_{DG\{GG\}})\left(r_G - \frac{(N_{CG\{GG\}}+N_{DG\{GG\}}-1/2)r_G}{N_{CG\{GG\}}+N_{DG\{GG\}}}\right) = \frac{\lambda r_G}{2} = \pi_{G\{NN\}}^*$. If $p_{\{GG\}} = \frac{(N_{CG\{GG\}}+N_{DG\{GG\}}-N_{CG\{BG\}}-N_{DG\{BG\}})r_G}{N_{CG\{GG\}}+N_{DG\{GG\}}-N_{DG\{BG\}}}$, then $\pi_{G\{GG\}}^* = \lambda(N_{CG\{GG\}} + N_{DG\{GG\}})\left(r_G - \frac{(N_{CG\{GG\}}+N_{DG\{GG\}}-N_{CG\{BG\}}-N_{DG\{BG\}})r_G}{N_{CG\{GG\}}+N_{DG\{GG\}}-N_{DG\{BG\}}}\right) = \frac{N_{CG\{BG\}}(N_{CG\{GG\}}+N_{DG\{GG\}})\lambda r_G}{N_{CG\{GG\}}+N_{DG\{GG\}}-N_{DG\{BG\}}} \geq \frac{\lambda r_G}{2} = \pi_{G\{NN\}}^*$. Thus, CP G's profit in outcome GG is higher than that in outcome NN if and only if $\frac{N_{CG\{GG\}}+N_{DG\{GG\}}-N_{CG\{BG\}}-N_{DG\{BG\}}}{N_{CG\{GG\}}+N_{DG\{GG\}}-N_{DG\{BG\}}} < \frac{N_{CG\{GG\}}+N_{DG\{GG\}}-1/2}{N_{CG\{GG\}}+N_{DG\{GG\}}}$, which can be simplified to $(N_{CG\{GG\}} + N_{DG\{GG\}})N_{CG\{BG\}} > \frac{N_{CG\{GG\}}+N_{DG\{GG\}}-N_{DG\{BG\}}}{2}$.

From the proof of Lemma 1, we know $N_{CG\{GG\}} + N_{DG\{GG\}} = 1 - x_{\{GG\}}$, $N_{CG\{BG\}} = \frac{z_{G\{BG\}}}{2}$, and $\frac{1}{2} - x_{D\{BG\}} = \frac{k}{t}(z_{Y\{BG\}} - z_{G\{BG\}})$. This gives: $N_{DG\{BG\}} = (1 - x_{D\{BG\}})(1 - z_{G\{BG\}}) - \frac{1}{2}(z_{Y\{BG\}} - z_{G\{BG\}})\left(\frac{1}{2} - x_{D\{BG\}}\right) = (1 - x_{D\{BG\}})(1 - z_{G\{BG\}}) - \frac{k}{2t}(z_{Y\{BG\}} - z_{G\{BG\}})^2$.

Substituting these equations into the $(N_{CG\{GG\}} + N_{DG\{GG\}})N_{CG\{BG\}} > \frac{N_{CG\{GG\}}+N_{DG\{GG\}}-N_{DG\{BG\}}}{2}$ yields $(1 - x_{\{GG\}})\left(\frac{z_{G\{BG\}}}{2}\right) > \frac{1}{2}\left((1 - x_{\{GG\}}) - (1 - x_{D\{BG\}})(1 - z_{G\{BG\}}) + \frac{k}{2t}(z_{Y\{BG\}} - z_{G\{BG\}})^2\right)$. Rearranging this inequality gives $\frac{t}{k} > \frac{(z_{Y\{BG\}}-z_{G\{BG\}})^2}{2(x_{\{GG\}}-x_{D\{BG\}})(1-z_{G\{BG\}})}$. Therefore, if the ratio of $\frac{t}{k}$ is higher than a threshold, $\pi_{G\{GG\}}^* > \pi_{G\{NN\}}^*$.

In general, comparisons of $\pi_{G\{GG\}}^*$, $\pi_{G\{BB\}}^*$, and $\pi_{G\{NN\}}^*$ show that CP G's profit may be lower, unchanged or higher in the packet discrimination regime than that in the net neutrality regime. Specifically, it is lower under equilibrium BB, but is unchanged or higher under equilibrium GG, i.e., $\pi_{G\{GG\}}^* \geq \pi_{G\{NN\}}^* \geq \pi_{G\{BB\}}^*$.

Appendix I

Proof of Proposition 3

In the net neutrality regime, we know that $\pi_Y^{NN} = \pi_{Y\{NN\}}^* = \frac{\lambda r_Y}{2}$. In the packet discrimination regime, there are three possible equilibria – outcomes GG, BG, and BB. The corresponding profit for content provider Y is: $\pi_{Y\{GG\}}^* = \lambda x_{\{GG\}} r_Y$, $\pi_{Y\{BG\}}^* = \lambda\left(N_{Y\{BG\}} r_Y - N_{CY\{BG\}} p_{\{BG\}}^*\right)$, and $\pi_{Y\{BB\}}^* = \frac{\lambda\left(r_Y - p_{\{BB\}}^*\right)}{2}$. We compare Y's profit in the three possible equilibria in the packet discrimination regime to its profit in the net neutrality regime one by one. We first note that $\pi_{Y\{NN\}}^* \geq \pi_{Y\{BB\}}^*$. Furthermore, since $x_{\{GG\}} \leq \frac{1}{2}$, we get that $\pi_{Y\{NN\}}^* \geq \pi_{Y\{GG\}}^*$. Lastly, since $p_{\{BB\}}^* \leq p_{\{BG\}}^*$, $\pi_{Y\{BB\}}^* \geq \left(N_{CY\{BG\}} + N_{DY\{BG\}}\right)\lambda r_Y - N_{CY\{BG\}}\lambda p_{\{BB\}}^* \geq \left(N_{CY\{BG\}} + N_{DY\{BG\}}\right)\lambda r_Y - N_{CY\{BG\}}\lambda p_{\{BG\}}^* = \pi_{Y\{BG\}}^*$.

Summarizing the above, we conclude that Y's profit is higher in the net neutrality regime than that in all three possible equilibria in the packet discrimination regime. Therefore, $\pi_Y^{NN} \geq \pi_Y^{PD}$.

Appendix J

Proof of Proposition 4

Substituting the equilibrium prices into the social welfare formula $SW_{ij} = \pi_{Cij} + \pi_{Dij} + \pi_{Yij} + \pi_{Gij} + \int_0^1 \int_0^1 U_{ij}(x,z)dxdz$, we get that, in the net neutrality regime, $SW^{NN} = SW_{\{NN\}} = V - \frac{t+k}{4} - \frac{d\lambda}{\mu - \lambda/2} + \frac{\lambda(r_Y + r_G)}{2}$. In the packet discrimination regime, there are three possible equilibria – outcomes GG, BG, and BB. The corresponding social welfare is: $SW_{\{GG\}} = V - t\left(\frac{1}{2} - x_{\{GG\}}^2\right) - \frac{k}{4} - \frac{d\lambda}{\mu - (1 - x_{\{GG\}})\lambda/2} + \lambda x_{\{GG\}} r_Y + \lambda\left(1 - x_{\{GG\}}\right) r_G$, $SW_{\{BG\}} = F_{\{BG\}} + \frac{\lambda(r_Y + r_G)}{2} + \frac{\lambda(r_G - r_Y)}{2}\left(2 - z_{Y\{BG\}} - z_{G\{BG\}}\right)\left(\frac{1}{2} - x_{D\{BG\}}\right) + \lambda N_{Y\{BG\}} r_Y + \lambda N_{G\{BG\}} r_G + t\left(x_{D\{BG\}} - \frac{1}{2}\right) + k\left(z_{G\{BG\}} - \frac{1}{2}\right) + \frac{k}{2}\left(x_{D\{BG\}} + \frac{1}{2}\right)\left(z_{Y\{BG\}} - z_{G\{BG\}}\right) + \frac{t}{2}\left(z_{Y\{BG\}} + z_{G\{BG\}}\right)\left(\frac{1}{2} - x_{D\{BG\}}\right) + \frac{2t^2}{3k}\left(\frac{1}{8} - x_{D\{BG\}}^3\right) + \frac{2k^2}{3t}\left(z_{Y\{BG\}}^3 - z_{G\{BG\}}^3\right) - \frac{t}{2k}\left(t + 2kz_{G\{BG\}}\right)\left(\frac{1}{4} - x_{D\{BG\}}^2\right) - \frac{k}{2t}\left(t + 2kz_{G\{BG\}}\right)\left(z_{Y\{BG\}}^2 - z_{G\{BG\}}^2\right)$, and $SW_{\{BB\}} = V - \frac{t+k}{4} - \frac{d\lambda}{\mu - \lambda/2} + \frac{\lambda(r_Y + r_G)}{2}$.

We first note that $SW_{\{BB\}} = SW_{\{NN\}}$. Furthermore, since $x_{\{GG\}} \leq \frac{1}{2}$, we get that $SW_{\{GG\}} \geq SW_{\{NN\}}$. Lastly, we compare $SW_{\{BG\}}$ and $SW_{\{NN\}}$. Let $\Delta SW = SW_{\{BG\}} - SW_{\{NN\}}$. We can show that $\frac{\partial \Delta SW}{\partial \mu} \geq 0$ and $\Delta SW = 0$ at $\mu = \lambda$. Therefore, $SW_{\{BG\}} \geq SW_{\{NN\}}$.

Summarizing the above, we conclude that social welfare is weakly higher in all three possible equilibria in the packet discrimination regime than that in the net neutrality regime. Therefore, $SW^{PD} \geq SW^{NN}$.

Appendix K

Numerical Analysis of the Asymmetric Equilibrium

In this appendix, we numerically explore the asymmetric equilibrium. There are eight parameters (V, t, k, d, λ, μ, r_Y, and r_G) in our model. Note that not all the parameters need to be changed independent of the other parameters. For example, with respect to the parameters μ and λ, what is important is not their absolute values but the utilization rate of the service queue, i.e., λ/μ, and hence we set $\lambda = 0.5$ and varied the value of μ to achieve a wide range of utilization rate. Specifically, $\mu \in (0.5,5]$ in our numerical analysis, which resulted in a range for the utilization rate of $[0.1,1)$. In addition, parameters V, t, k, and d can theoretically vary within an infinite range and they all affect the consumer's utility. Thus, one of these parameters can be kept fixed relative to the others and here we normalized $d = 1$. We then conducted the numerical analysis on a wide range of the other three parameters $V \in [1,5]$, $t \in [0.5,3]$, and $k \in [0.5,3]$. Finally, recent empirical evidence[1] shows that the revenue rates (measured by the average revenue per user, i.e., ARPU) vary widely[2]. Therefore, we chose a reasonable range of revenue rates $r_Y \in (0,5]$ and $r_G \in (0,5]$. In summary, the total number of exploration points for the entire parameter space was 1,593,750, which generated 38.8 GB of data. We implemented this asymmetric equilibrium analysis in Mathematica 10 and ran the solution procedure on clusters hosted by the High Performance Computing facility at a university. The total running time for all the simulations was around 180 hours.

Figure K1 shows the result of the symmetric equilibria for parameters $V = 3$, $t = 2$, $k = 1$, $d = 1$, $\lambda = 0.5$, and $\mu = 1$. Results for other parameter values are qualitatively the same. These numerical results validate the analytical results (all the lemmas and propositions for the symmetric equilibrium) that we present in the paper.

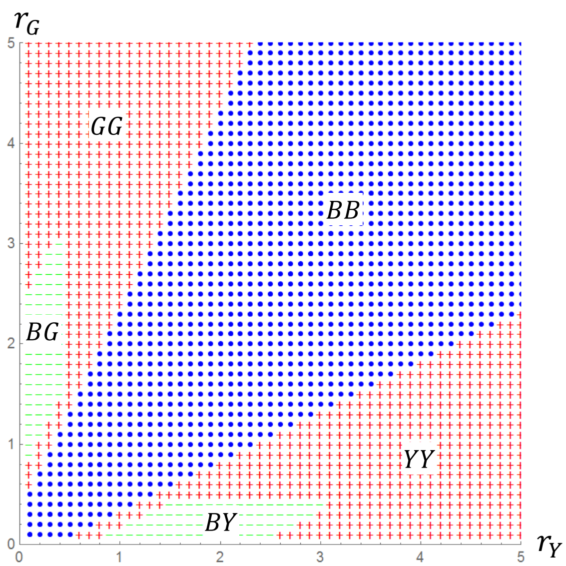


Figure K1. Types of Symmetric Equilibria

Notes:

- The separating lines between the regions for different equilibria may shift based on different parameter values.
- For symmetric ISPs (with the same capacity levels), outcome BG – where both CPs pay ISP C and only G pays ISP D – is equivalent to outcome GB. Thus, a set of parameter values that result in outcome BG can also (equivalently) result in outcome GB. Similarly, outcome BY is equivalent to outcome YB.

Next, we consider the asymmetric equilibria results with symmetric ISPs. As we show in Figure K2, the results show a somewhat more complex set of possible equilibrium outcomes. There are some regions which correspond to a single type of dominating outcomes (for example, the regions in green corresponding to the dominating equilibrium outcome BG, or similarly BY if $r_Y \geq r_G$, or the region in blue corresponding to the outcome BB), and there are others that correspond to regions where there are two possible types of equilibrium outcomes (e.g., the region in red corresponding to either outcome GG or outcome BG).

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[1] http://www.forbes.com/sites/tristanlouis/2013/08/31/how-much-is-a-user-worth/

[2] The ARPU for four popular websites are \$1.63 (Facebook), \$1.53 (LinkedIn), \$1.81 (Yahoo), and \$10.09 (Google).

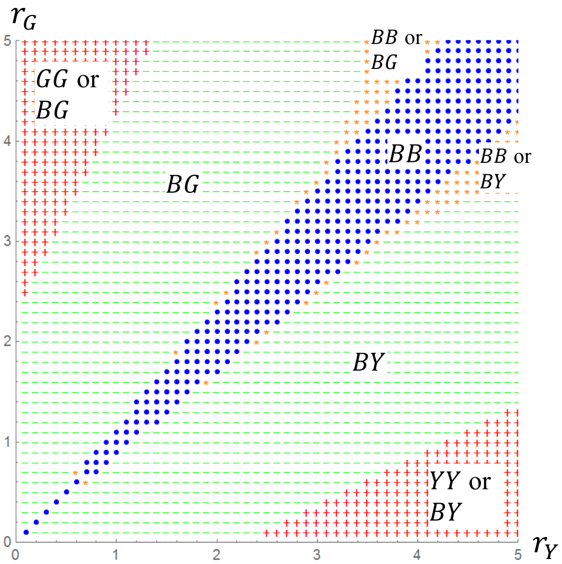
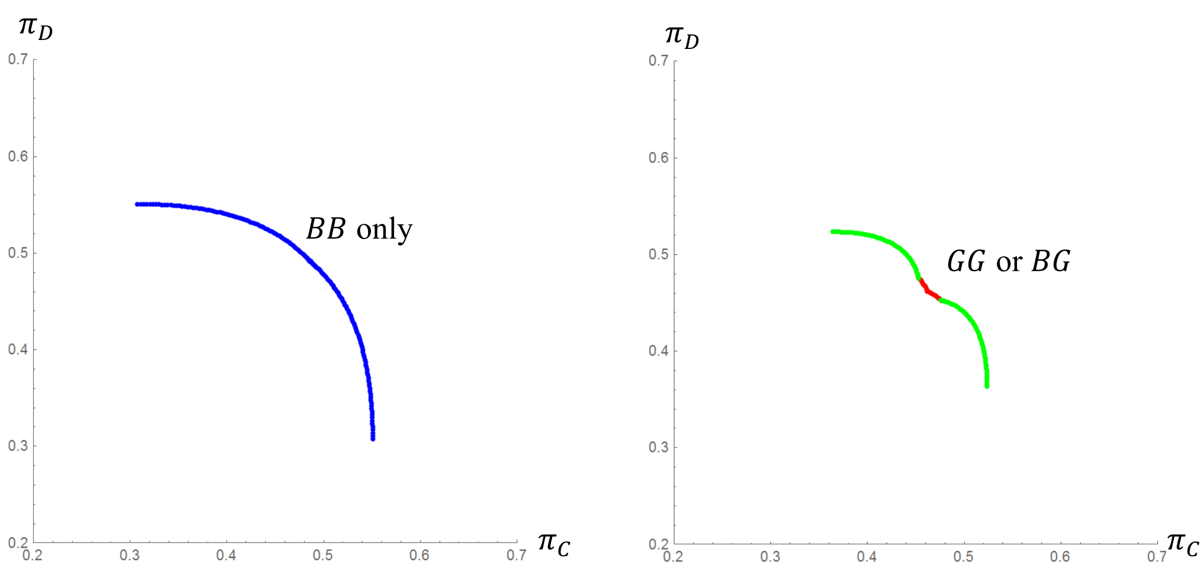


Figure K2. Types of Pareto Frontiers for Asymmetric Equilibria
Notes:

- The separating lines between the regions for different equilibria may shift based on different parameter values.
- For symmetric ISPs (with the same capacity levels), outcome BG is equivalent to outcome GB. Similarly, outcome BY is equivalent to outcome YB.

To understand why we may have multiple possible types of asymmetric equilibrium outcomes for a certain combination of r_G and r_Y, it is instructive to look at the Pareto frontier of the asymmetric equilibria. Figure K3 shows two examples of the Pareto frontier results with different values of r_G and r_Y.

In Figure K3, every point on the curve corresponds to the profit of ISP C (on the x-axis) and the profit of ISP D (on the y-axis), such that if C and D choose the corresponding (F_C, p_C) and (F_D, p_D) that results in these profits, such a strategy choice is not dominated by any other strategy in the strategy space of C and D forming a Pareto frontier. Thus, for a certain combination of r_Y and r_G, there may be multiple asymmetric strategy choices contained in the Pareto frontier. Consider the example on the left in Figure K3 (with $r_Y = 3$ and $r_G = 3$), all such strategy choices result in the equilibrium outcome BB, i.e., both Y and G pay both ISPs. However, in the example on the right in Figure K3 (with $r_Y = 0.5$ and $r_G = 4$), for some strategy choices of C and D, the equilibrium outcome is GG, but for other strategy choices, the equilibrium outcome is BG.



$r_Y = 3$ and $r_G = 3$ | $r_Y = 0.5$ and $r_G = 4$

Figure K3. Examples of Pareto Frontier of Asymmetric Equilibria

Note that under these asymmetric strategy choices, the profits of C and D are different, which means that one of them is engaging in predatory pricing, with the intent of getting higher profits. In real life, such an action will likely result in a retaliatory action from the other ISP, which is harmful to both firms in the long run. As Farrell (1987)[3] showed, it is very easy for *symmetric* firms who can engage in asymmetric equilibria to signal their intent at a very little cost to the other firm (in the words of Farrell, by engaging in "cheap talk") and thereby arrive at the mutually beneficial symmetric equilibrium.

Furthermore, our numerical results show that the main findings for the symmetric equilibrium case still hold for the asymmetric equilibrium case. We have already shown analytically in Lemma 3 that ISP competition does not substitute for net neutrality regulation even considering the asymmetric equilibria. In addition, our numerical analysis for the asymmetric equilibria confirms that the dominant CP still sometimes benefits in the absence of net neutrality. While we cannot develop similar generalized "conditions" with numerical analysis, we find however that when the ratio of t/k is high and the ratio of r_G/r_Y is high (in other words, when the conditions of Proposition 2 hold), CP G is better off under packet discrimination.

The case is different however if the ISPs are *asymmetric* with respect to their capacities. In such situations, the asymmetric equilibrium is not just a theoretical exercise but can actually occur. We numerically explore the details of the asymmetric equilibria for asymmetric ISPs with different capacities in Appendix L.

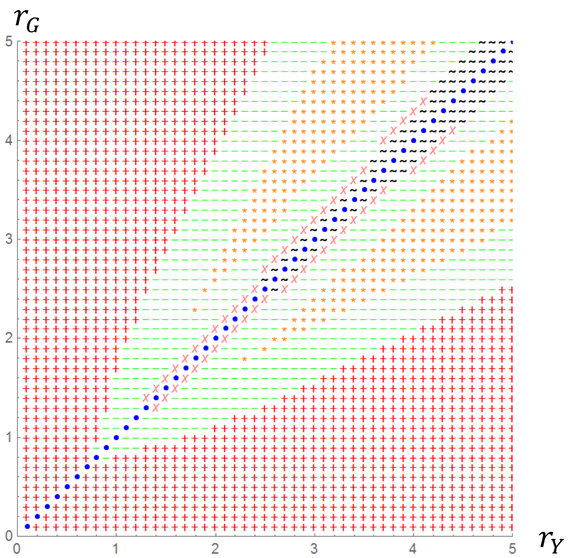
Appendix L

Numerical Analysis of the Asymmetric ISPs

In this appendix, we numerically explore the asymmetric equilibria for asymmetric ISPs with different capacities. Without loss of generality, we assume $\mu_C \geq \mu_D$.

As compared to the symmetric equilibrium or the asymmetric equilibrium with symmetric ISPs, the equilibrium outcomes with asymmetric ISPs is more complicated with more possible types of strategy choices (as shown in Figure L1). For example, for a certain combination of r_G and r_Y, there can be three or even four outcomes that are part of the Pareto frontier, i.e. C and D can choose three or four different types of pricing strategies.

[3] Farrell, J. 1987. "Cheap Talk, Coordination, and Entry," *RAND Journal of Economics* (18:1), pp. 34-39.



Red "+": GG, GB, or BG (YY, YB, or BY)
Green "—": GG, GB, BG, or BB (YY, YB, BY, or BB)
Orange "*": GB, BG, or BB (YB, BY, or BB)
Blue "·": BB only
Pink "X": GG, GB, or BB (YY, YB, or BB)
Black "~": GB or BB (YB or BB)

Figure L1. Types of Pareto Frontiers for Asymmetric Equilibria with Asymmetric ISPs

Notes:

- Figure L1 is generated based on parameter values $\mu_C = 5$, $\mu_D = 1$, $V = 3$, $t = 1$, $k = 1$, $d = 1$, and $\lambda = 0.5$.
- The separating lines between the regions for different equilibria may shift based on different parameter values.
- Unlike the equilibria with symmetric ISPs, with asymmetric ISPs, the equilibrium outcomes BG and GB are *not* equivalent. Similarly, outcomes BY and YB are not equivalent for the asymmetric ISP case.

Figure L1 shows that when r_G and r_Y are somewhat comparable, the equilibrium outcome is BB. Also, when r_G is much greater than r_Y, the equilibrium outcome is either just GG or it also includes the outcome BG or GB as part of the Pareto frontier. For intermediate values of r_G and r_Y, the strategy choices for the two ISPs and the CPs get more varied.

Figure L2 shows two examples of the Pareto frontier with two sets of r_G and r_Y values ($r_Y = 3$, $r_G = 3$ and $r_Y = 1$, $r_G = 3$). The Pareto frontier is no longer symmetric as for symmetric ISPs (Figure K3) because ISP C has a higher capacity than ISP D.

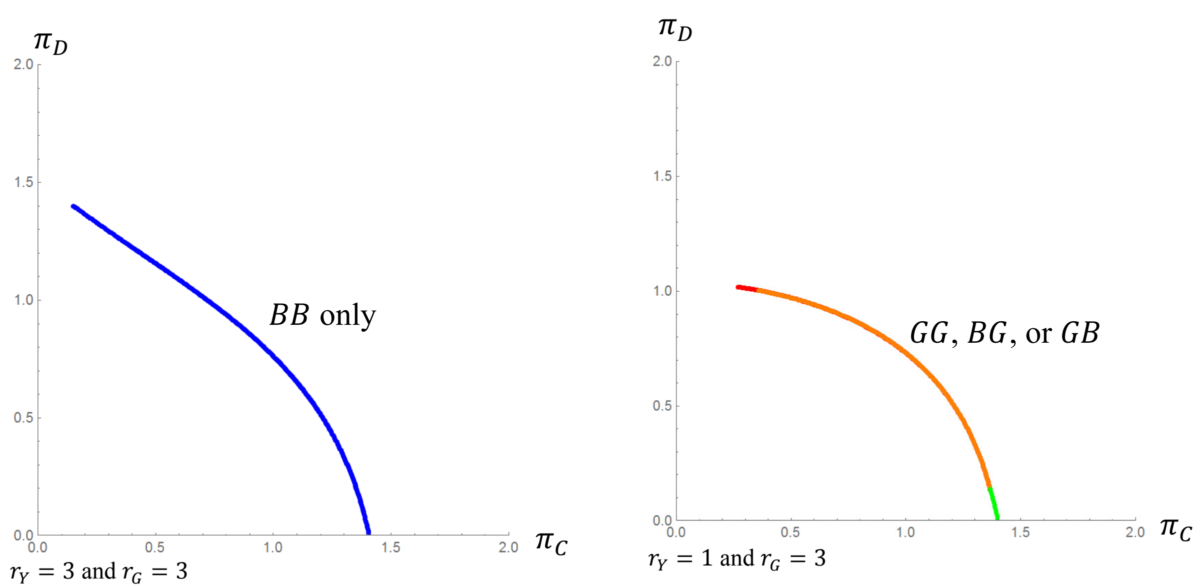


Figure L2. Examples of Pareto Frontier of Asymmetric Equilibria with Asymmetric ISPs

Furthermore, just as shown in Appendix K, our numerical analysis for asymmetric ISPs show that the main findings for the symmetric ISP case still hold for the asymmetric ISP case. For example, our numerical analysis for asymmetric ISPs confirms that ISP competition does not substitute for net neutrality regulation even for ISPs with different capacity levels as in the symmetric ISP case (Lemma 3). In addition, the dominant CP still sometimes benefits in the absence of net neutrality.