

## MODELING MULTICHANNEL ADVERTISING ATTRIBUTION ACROSS COMPETITORS

Yiyi Li

Alfred Lerner College of Business and Economics, University of Delaware,  
Newark, DE 19716 U.S.A. {yiyil@udel.edu}

Ying Xie and Eric Zheng

Naveen Jindal School of Management, University of Texas at Dallas,  
Richardson, TX 75080 U.S.A. {ying.xie@utdallas.edu} {ericz@utdallas.edu}

### Appendix A

#### Robustness Checks on Different Channel Grouping, Airline Grouping, and Decay Rate

In this appendix, we present the results of several robustness checks under alternative specifications for channel grouping, airline grouping, and decay rate.

##### *Channel Grouping*

We reorganized the channels by grouping two leading referral sites—Kayak.com and Tripadvisor.com—into a distinct channel and separated them from the display channel. This new way of regrouping results in five different channels including a direct channel and four advertising channels (i.e., search, display/referral, email, and Kayak/Tripadvisor). The parameter estimates are presented in column 1 of Tables A1, A2, and A3. We find that the effect of the new channel (of these two sites) is not significant, although with a positive sign. Meanwhile, the display channel (without these two websites) still shows a significant impact on both search and purchase decisions and the magnitude of the effect has not changed significantly.

We also computed the channels' contribution to conversion probabilities using the same method described in the main text of the paper. The results are presented in Table A4. The results are consistent with our previous findings that search is the most effective channel, followed by the display/referral channel (excluding Tripadvisor.com and Kayak.com). The effect size of the display/referral channel is especially large for Orbitz.com, Travelocity.com, and Other OTAs, and is more than three times that of email. The new channel TripAdvisor/Kayak's effect size is about the same as email.

Table A1. Robustness Checks: Estimation Results for Entry Site Choice						
	5 Channels		11 Choices		0.5 Decay Rate	
	Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.
Delta.com	-0.04	<b>0.99*</b>	-0.13	<b>1.28</b>	0.20	<b>0.92</b>
AA.com	-0.25	<b>0.90</b>	<b>-0.46</b>	<b>1.15</b>	0.07	<b>0.82</b>
United.com	-0.13	0.04	0.09	0.04	0.14	0.00
Other Airlines	<b>0.38</b>	<b>1.31</b>	<b>0.53</b>	<b>1.25</b>	<b>0.58</b>	<b>1.29</b>
Expedia.com	-0.10	<b>1.56</b>	<b>-0.33</b>	<b>1.70</b>	0.01	<b>1.42</b>
Priceline.com	-0.23	<b>1.54</b>	<b>-0.40</b>	<b>1.67</b>	-0.03	<b>1.34</b>
Orbitz.com	-0.10	<b>0.77</b>	<b>-0.30</b>	<b>1.03</b>	0.06	<b>0.69</b>
Travelocity.com	-0.22	<b>1.15</b>	<b>-0.44</b>	<b>1.31</b>	0.02	<b>0.97</b>
Cheaptickets.com	0.55	0.34	1.01	<b>0.62</b>	0.66	0.08
Other OTAs			----	----	----	----
Continental.com			0.13	0.01	----	----
Lag Ad Stock: Search	<b>0.42</b>		<b>0.38</b>		<b>0.41</b>	
Lag Ad Stock: Display/Referral	<b>0.13</b>		<b>0.10</b>		<b>0.08</b>	
Lag Ad Stock: Email	<b>0.23</b>		<b>0.20</b>		<b>0.26</b>	
Lag Ad Stock: Direct	<b>0.69</b>		<b>0.66</b>		<b>0.69</b>	
Lag Ad Stock: TripAdvisor/Kayak	0.08		—		—	
Lag Visit $j$	<b>0.95</b>		<b>1.02</b>		<b>0.91</b>	
Lag Purchase $j$	-0.11		-0.04		0.12	
Cumulative Spending	<b>0.09</b>		<b>0.07</b>		<b>0.07</b>	
Ad Spending: Search Ads	-0.01		0.00		0.04	
Ad Spending: Local Display	<b>0.05</b>		<b>0.05</b>		<b>0.04</b>	
Ad Spending: National Display	-0.01		-0.01		-0.01	
Ad Spending: Local Offline	<b>0.10</b>		<b>0.08</b>		<b>0.10</b>	
Ad Spending: National Offline	0.05		0.11		0.03	
Log Likelihood	-18,016	(-25,957)	-18,630	(-27,048)	-15,632	(-23,033)
AIC	36,093		37,324		31,325	

\*The bold font indicates statistical significance at  $p < 0.05$ .

**Table A2. Robustness Checks: Estimation Results for Visit Decision on Other Websites**

	5 Channels		11 Choices		0.5 Decay Rate	
	Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.
Delta.com	<b>-1.21</b>	<b>1.47</b>	-0.72	<b>1.49</b>	-0.80	<b>1.45</b>
AA.com	<b>-1.67</b>	<b>1.42</b>	<b>-1.15</b>	<b>1.48</b>	<b>-1.25</b>	<b>1.36</b>
United.com	<b>-1.78</b>	0.01	<b>-1.37</b>	0.00	<b>-1.50</b>	0.00
Other Airlines	-0.32	<b>1.53</b>	-0.04	<b>1.38</b>	0.14	<b>1.48</b>
Expedia.com	-0.13	<b>2.45</b>	0.48	<b>2.54</b>	0.31	<b>2.32</b>
Priceline.com	-0.73	<b>2.20</b>	-0.09	<b>2.22</b>	-0.30	<b>2.14</b>
Orbitz.com	-0.91	<b>2.08</b>	-0.36	<b>2.16</b>	-0.48	<b>2.01</b>
Travelocity.com	-1.01	<b>2.03</b>	-0.45	<b>2.12</b>	-0.62	<b>2.01</b>
Cheaptickets.com	<b>-1.40</b>	<b>1.28</b>	<b>-1.27</b>	<b>1.33</b>	<b>-1.19</b>	<b>1.27</b>
Other OTAs	<b>-1.34</b>	<b>1.45</b>	-0.84	<b>1.61</b>	-0.98	<b>1.38</b>
Continental.com	—	—	<b>-1.27</b>	0.00		—
Lag Ad Stock: Search	<b>0.38</b>		<b>0.33</b>		<b>0.39</b>	
Lag Ad Stock: Display/Referral	<b>0.20</b>		<b>0.16</b>		<b>0.17</b>	
Lag Ad Stock: Email	<b>0.43</b>		<b>0.32</b>		<b>0.40</b>	
Lag Ad Stock: Direct	<b>0.56</b>		<b>0.50</b>		<b>0.58</b>	
Lag Ad Stock: TripAdvisor/Kayak	0.09		—		—	
Lag Visit $j$	<b>1.33</b>		<b>1.40</b>		<b>1.21</b>	
Lag Purchase $j$	<b>-0.28</b>		-0.19		-0.16	
Cumulative Spending	<b>0.17</b>		<b>0.13</b>		<b>0.16</b>	
Ad Spending: Search Ads	<b>-0.23</b>		<b>-0.27</b>		<b>-0.25</b>	
Ad Spending: Local Display	<b>0.04</b>		<b>0.05</b>		<b>0.03</b>	
Ad Spending: National Display	<b>0.04</b>		<b>0.04</b>		<b>0.05</b>	
Ad Spending: Local Offline	<b>0.06</b>		<b>0.04</b>		<b>0.05</b>	
Ad Spending: National Offline	-0.02		-0.07		-0.04	
-2 Log Likelihood	-51,378	(-78,138)	-53,879	(-86,006)	-46,580	(-69,336)
AIC	102,821		107,827		93,225	

<b>Table A3. Robustness Checks: Estimation Results for Purchase Decision</b>						
	<b>5 Channels</b>		<b>11 Choices</b>		<b>0.5 Decay Rate</b>	
	<b>Mean</b>	<b>Std. Dev.</b>	<b>Mean</b>	<b>Std. Dev.</b>	<b>Mean</b>	<b>Std. Dev.</b>
Delta.com	-1.11		-0.23	<b>1.56</b>	-0.98	<b>1.82</b>
AA.com	-3.59		-1.50	<b>2.62</b>	<b>-4.00</b>	<b>3.30</b>
United.com	-2.91		-1.56	<b>1.11</b>	<b>-3.05</b>	<b>2.31</b>
Other Airlines	-0.59		-0.74	<b>2.64</b>	-0.61	<b>2.88</b>
Expedia.com	0.71		1.25	<b>1.78</b>	1.12	<b>1.54</b>
Priceline.com	-0.01		0.97	<b>1.55</b>	0.34	<b>1.55</b>
Orbitz.com	-0.24		1.05	<b>1.23</b>	0.14	<b>1.19</b>
Travelocity.com	-0.63		0.43	<b>1.83</b>	-0.16	<b>1.59</b>
Cheaptickets.com	-1.60		<b>-3.17</b>	<b>0.97</b>	-1.57	<b>1.48</b>
Other OTAs	-0.72		-0.33	<b>2.13</b>	-1.44	<b>2.64</b>
Continental.com			-1.12	<b>2.48</b>	—	—
Ad Stock: Search	<b>0.94</b>		<b>0.58</b>		<b>0.66</b>	
Ad Stock: Display/Referral	<b>0.39</b>		<b>0.32</b>		<b>0.37</b>	
Ad Stock: Email	<b>0.58</b>		<b>0.39</b>		<b>0.49</b>	
Ad Stock: Direct	<b>1.13</b>		<b>0.73</b>		<b>1.01</b>	
Ad Stock: TripAdvisor/Kayak	0.47		—		—	
Entry Site	<b>1.80</b>		<b>1.62</b>		<b>1.35</b>	
Last Site	<b>1.94</b>		<b>1.77</b>		<b>1.05</b>	
Lag visit <i>j</i>	<b>0.66</b>		<b>0.64</b>		<b>0.58</b>	
Lag purchase <i>j</i>	<b>1.32</b>		<b>1.10</b>		<b>1.25</b>	
Cumulative browsing	<b>-0.53</b>		<b>-0.48</b>		<b>-0.48</b>	
Cumulative spending	<b>0.09</b>		<b>0.09</b>		<b>0.07</b>	
Ad Spending: Search Ads	<b>-0.87</b>		-0.34		<b>-0.69</b>	
Ad Spending: Local Display	0.00		0.04		-0.01	
Ad Spending: National Display	0.05		<b>0.08</b>		0.06	
Ad Spending: Local Offline	0.05		0.02		<b>0.05</b>	
Ad Spending: National Offline	-0.11		<b>-0.50</b>		-0.13	
Log Likelihood	-6,170	(-12,927)	-6,214	(-13,029)	-5,905	(-11,815)
AIC	12,433		12,521		11,900	

**Table A4. Robustness Check: Attribution under Alternative Channel Grouping**

	Search	Display/ Referral	Email	TripAdvisor/ Kayak	Direct
Delta	24.68%	1.91%	1.54%	0.59%	71.28%
AA	31.68%	1.96%	1.51%	0.31%	64.54%
United	24.07%	3.17%	0.60%	0.28%	71.88%
Other Airlines	27.89%	3.52%	1.36%	0.36%	66.86%
Expedia	28.45%	3.47%	1.59%	1.74%	64.75%
Priceline	24.38%	3.49%	1.51%	2.15%	68.46%
Orbitz	23.70%	5.22%	0.98%	2.49%	67.61%
Travelocity	25.21%	6.32%	0.84%	0.44%	67.20%
Cheaptickets	22.89%	1.62%	1.12%	0.30%	74.07%
Other OTAs	17.59%	6.10%	1.15%	1.10%	74.05%

### **Airline Grouping**

We regrouped the air booking choices by first taking Northwest out of the OA group and combining it with Delta as one option and then taking Continental out of the OA grouping and treating it as a separate option. We reestimated the model based on this alternative grouping and the model estimates are presented in the second column in Tables A1, A2, and A3. We find that “other airlines” remains the only significant brand dummy (the baseline being other OTAs). We also find that the magnitudes of the parameters for ad channels remain qualitatively unchanged. Based on these estimates, we further computed the contribution to conversion probabilities for the 11 choices, presented in Table A4. Consistent with our previous findings, search advertising has the largest contribution to conversion among all three advertising channels. Display/referral also contributes significantly to the conversion compared to the numbers suggested by the baseline model, especially for Orbitz, Travelocity, and Other OTAs.

**Table A5. Robustness Check: Attribution under Alternative Airline Grouping**

	Search	Display/ Referral	Email	TripAdvisor/ Kayak	Direct
Delta	24.68%	1.91%	1.54%	0.59%	71.28%
AA	31.68%	1.96%	1.51%	0.31%	64.54%
United	24.07%	3.17%	0.60%	0.28%	71.88%
Other Airlines	27.89%	3.52%	1.36%	0.36%	66.86%
Expedia	28.45%	3.47%	1.59%	1.74%	64.75%
Priceline	24.38%	3.49%	1.51%	2.15%	68.46%
Orbitz	23.70%	5.22%	0.98%	2.49%	67.61%
Travelocity	25.21%	6.32%	0.84%	0.44%	67.20%
Cheaptickets	22.89%	1.62%	1.12%	0.30%	74.07%
Other OTAs	17.59%	6.10%	1.15%	1.10%	74.05%

### **Decay Rate**

To further ensure the robustness of our results under different decay rates, we re-ran the model with a 0.5 monthly decay rate. The parameter estimates are reported in column 3 of Tables A1, A2, and A3. We can see that neither the significance nor the magnitude of estimates changes significantly compared to the original decay rate. Moreover, we present the Log-likelihood and AIC of the two decay rates in Table A6. The results imply that the current decay rate ( $1-0.7 = 0.3$ ) fits the data better than a higher decay rate (0.5). It stands to reason that an even higher decay rate will further harm the model fit.

**Table A6. Robustness Check: Model Fit Comparison under Different Decay Rate**

Decay rate (monthly)	Visit Decision: Entry		Visit Decision: Other		Purchase Decision	
	Log-Likelihood	AIC	Log-Likelihood	AIC	Log-Likelihood	AIC
0.5 (Robustness check)	-15,632	31,325	-46,580	93,223	-5,905	11,900
0.3 (Proposed model)	-15,426	30,911	-46,367	92,797	-5,865	11,820

## Appendix B

### Robustness Check on Window Length

In this appendix, we present our robustness check results under alternative specifications with regard to the window length of a purchase session. We first compare the average intersession visit lapse versus the intra-session visit lapse across different window lengths in Table B1. It is clear that the seven-day window yields the largest difference between these two measures, indicating that it best segregates visits. We also reestimate the model under a 3-day and a 15-day window length. The results are reported in Tables B2, B3, and B4. The magnitude of parameter estimates (especially the main variable of interest, ad effectiveness) do not change significantly (although website-specific intercepts do change a little). We therefore choose to use the seven-day window length in the main text of the paper.

**Table B1. Inter- and Intra-Session Lapse across Different Window Lengths**

Window length	InterSession lapse (A)	Intra-session lapse (B)	B-A
3	0.20	2.12	1.92
5	0.45	2.56	2.11
7	0.51	2.63	2.12
10	0.71	2.62	1.91
15	1.08	3.17	2.10
20	1.45	2.95	1.51
30	2.05	2.75	0.70

\*Including the entire year of 2010. We then use April to September as estimation sample.

**Table B2. Robustness Check: Results for Entry Site Choice under Different Window Length**

	3-day		7-day		15-day	
	Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.
Delta.com	<b>0.45*</b>	<b>0.83</b>	0.25	<b>0.83</b>	0.11	<b>1.13</b>
AA.com	0.05	<b>1.00</b>	-0.03	<b>0.79</b>	0.06	<b>0.97</b>
United.com	0.46	0.02	0.14	0.07	0.22	0.01
Other Airlines	<b>0.53</b>	<b>1.29</b>	<b>0.51</b>	<b>1.26</b>	<b>0.38</b>	<b>1.44</b>
Expedia.com	-0.17	<b>1.42</b>	-0.01	<b>1.32</b>	-0.19	<b>1.56</b>
Priceline.com	-0.12	<b>1.42</b>	0.03	<b>1.23</b>	-0.13	<b>1.37</b>
Orbitz.com	0.07	<b>0.76</b>	0.10	0.34	0.02	<b>0.60</b>
Travelocity.com	-0.02	<b>0.76</b>	-0.06	<b>0.90</b>	-0.26	<b>1.17</b>
Cheaptickets.com	1.21	0.04	0.82	0.33	0.71	0.20
Lag Ad Stock: Search	<b>0.36</b>		<b>0.34</b>		<b>0.36</b>	
Lag Ad Stock: Display/Referral	<b>0.07</b>		<b>0.07</b>		<b>0.08</b>	
Lag Ad Stock: Email	<b>0.18</b>		<b>0.20</b>		<b>0.22</b>	
Lag Ad Stock: Direct	<b>0.63</b>		<b>0.61</b>		<b>0.64</b>	
Lag Visit j	<b>0.97</b>		<b>0.89</b>		<b>0.82</b>	
Lag Purchase j	0.11		<b>0.25</b>		<b>0.38</b>	
Cumulative Spending	<b>0.06</b>		<b>0.05</b>		<b>0.04</b>	
Ad Spending: Search Ads	0.08		0.01		0.13	
Ad Spending: Local Display	0.03		0.04		<b>0.06</b>	
Ad Spending: National Display	0.00		-0.01		0.00	
Ad Spending: Local Offline	<b>0.08</b>		<b>0.09</b>		<b>0.09</b>	
Ad Spending: National Offline	0.08		0.06		0.02	

\*The bold font indicates statistical significance at  $p < 0.05$ .

**Table B3. Robustness Check: Estimation Results for Visit Decision on Other Websites under Different Window Length**

	3-day		7-day		15-day	
	Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.
Delta.com	<b>-1.68</b>	<b>1.36</b>	-0.36	0.01	-0.84	<b>1.50</b>
AA.com	<b>-2.09</b>	<b>1.34</b>	-0.66	0.02	<b>-1.45</b>	<b>1.46</b>
United.com	<b>-2.17</b>	0.01	<b>-1.42</b>	0.00	<b>-1.56</b>	0.01
Other Airlines	-1.01	<b>1.52</b>	0.34	0.05	-0.01	<b>1.50</b>
Expedia.com	-1.01	<b>2.33</b>	0.47	<b>1.50</b>	0.29	<b>2.42</b>
Priceline.com	-1.42	<b>1.98</b>	0.05	<b>1.23</b>	-0.39	<b>2.22</b>
Orbitz.com	<b>-1.53</b>	<b>1.97</b>	-0.10	<b>1.18</b>	-0.53	<b>2.13</b>
Travelocity.com	<b>-1.77</b>	<b>1.97</b>	-0.23	<b>1.19</b>	-0.75	<b>2.04</b>
Cheaptickets.com	<b>-1.89</b>	<b>1.16</b>	<b>-1.08</b>	0.00	<b>-1.07</b>	<b>1.18</b>
Other OTAs	<b>-2.24</b>	<b>1.58</b>	-0.50	0.06	-1.08	<b>1.40</b>
Lag Ad Stock: Search	<b>0.30</b>		<b>0.25</b>		<b>0.32</b>	
Lag Ad Stock: Display/Referral	<b>0.12</b>		<b>0.10</b>		<b>0.13</b>	
Lag Ad Stock: Email	<b>0.26</b>		<b>0.24</b>		<b>0.39</b>	
Lag Ad Stock: Direct	<b>0.48</b>		<b>0.39</b>		<b>0.52</b>	
Lag Visit j	<b>1.48</b>		<b>1.00</b>		<b>1.05</b>	
Lag Purchase j	-0.06		0.05		0.22	
Cumulative Spending	<b>0.13</b>		<b>0.09</b>		<b>0.11</b>	
Ad Spending: Search Ads	<b>-0.17</b>		<b>-0.18</b>		<b>-0.24</b>	
Ad Spending: Local Display	0.03		<b>0.03</b>		0.04	
Ad Spending: National Display	<b>0.05</b>		<b>0.04</b>		<b>0.08</b>	
Ad Spending: Local Offline	<b>0.04</b>		<b>0.03</b>		<b>0.04</b>	
Ad Spending: National Offline	-0.02		<b>-0.08</b>		-0.03	



**Table B4. Robustness Check: Results for Purchase Decision under Different Window Length**

	3-day		7-day		15-day	
	Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.
Delta.com	-0.70	<b>1.85</b>	-0.77	<b>1.69</b>	-0.78	<b>1.66</b>
AA.com	-2.59	<b>3.29</b>	-2.35	<b>3.04</b>	-2.20	<b>2.59</b>
United.com	-2.78	<b>2.28</b>	-2.54	<b>2.08</b>	-2.23	<b>1.80</b>
Other Airlines	-0.24	<b>3.09</b>	-0.15	<b>2.67</b>	-0.03	<b>2.44</b>
Expedia.com	1.25	<b>1.80</b>	1.15	<b>1.29</b>	1.19	<b>1.11</b>
Priceline.com	0.61	<b>1.83</b>	0.63	<b>1.30</b>	0.62	<b>1.10</b>
Orbitz.com	0.78	<b>1.48</b>	0.51	<b>1.04</b>	0.61	<b>0.83</b>
Travelocity.com	0.46	<b>1.78</b>	0.34	<b>1.42</b>	0.36	<b>1.22</b>
Cheaptickets.com	<b>-4.11</b>	<b>1.60</b>	-2.68	<b>1.19</b>	-2.57	<b>1.07</b>
Other OTAs	-0.70	<b>2.48</b>	-2.04	<b>2.87</b>	-1.22	<b>2.31</b>
Ad Stock: Search	<b>0.76</b>		<b>0.62</b>		<b>0.53</b>	
Ad Stock: Display/Referral	<b>0.55</b>		<b>0.36</b>		<b>0.25</b>	
Ad Stock: Email	<b>0.51</b>		<b>0.44</b>		0.25	
Ad Stock: Direct	<b>1.10</b>		<b>0.98</b>		<b>0.88</b>	
Entry Site	<b>1.60</b>		<b>1.28</b>		<b>1.28</b>	
Last Site	<b>1.23</b>		<b>0.99</b>		<b>0.99</b>	
Lag visit $j$	<b>0.78</b>		<b>0.66</b>		<b>0.39</b>	
Lag purchase $j$	<b>1.48</b>		<b>1.24</b>		<b>1.08</b>	
Cumulative browsing	<b>-0.51</b>		<b>-0.48</b>		<b>-0.41</b>	
Cumulative spending	<b>0.05</b>		<b>0.07</b>		<b>0.07</b>	
Ad Spending: Search Ads	-0.31		-0.43		-0.37	
Ad Spending: Local Display	-0.03		0.01		0.03	
Ad Spending: National Display	0.07		0.02		0.04	
Ad Spending: Local Offline	<b>0.06</b>		0.04		<b>0.04</b>	
Ad Spending: National Offline	<b>-0.55</b>		-0.32		-0.32	

## Appendix C

### Simulated Maximum Likelihood Estimation

We assume that the overall attractiveness of visiting or purchasing from website  $j$  (website-specific intercepts in Equations 1, 3, and 5) can vary over people but be constant over choice situations for each person (Erdem and Keane 1996). The individual website-specific intercepts  $\alpha_{1ij}$ ,  $\alpha_{2ij}$ , and  $\alpha_{3ij}$  are specified to follow normal distribution:

$$\begin{pmatrix} \alpha_{1ij} \\ \alpha_{2ij} \\ \alpha_{3ij} \end{pmatrix} \sim N \left( \begin{bmatrix} \bar{\alpha}_{1j} \\ \bar{\alpha}_{2j} \\ \bar{\alpha}_{3j} \end{bmatrix}, \Omega_j \right)$$

where  $j = 1, 2, \dots, J$ .

The probability conditional on  $\alpha_i$  is

$$L_t(\alpha) = \prod_{t=1}^{T_i} L_{it}(\alpha) = \prod_{t=1}^{T_i} P(F_{it} = j | \alpha_{1i}) \times P(V_{it} = C_{it} | F_{it}, \alpha_{2i}) \times P(B_{it} = k | F_{it}, V_{it}, \alpha_{3i})$$

The unconditional choice probability is therefore the integral of  $L_i(\theta)$  over all possible values of  $\theta_i$ :

$$P_{it} = \int L_{it}(\alpha) f(\alpha) d\alpha$$

1. Draw  $\alpha_{it}$  from its distribution.

A set of random variables  $\eta_i^r = (\eta_{j1}^r, \eta_{j2}^r, \eta_{j3}^r), j = 1, 2, \dots, J$  are drawn from i.i.d. standard normal distribution. Then we compute the website-specific intercepts for each person as follows:

$$\begin{aligned} \alpha_{1ij}^r &= \bar{\alpha}_{1j} + \sigma_{\alpha_{1j}} \eta_{j1}^r \\ \alpha_{2ij}^r &= \bar{\alpha}_{2j} + \sigma_{\alpha_{2j}} (c_{21j} \eta_{j1}^r + c_{22j} \eta_{j2}^r) \\ \alpha_{3ij}^r &= \bar{\alpha}_{3j} + \sigma_{\alpha_{3j}} (c_{31j} \eta_{j1}^r + c_{32j} \eta_{j2}^r + c_{33j} \eta_{j3}^r) \end{aligned}$$

The benefit of this specification is that the correlation between intercepts of different stages remains the same across websites, thus significantly reducing the number of parameters to be estimated. With this specification, the variance and covariance matrix can be written as follows:

$$\Omega_j = \begin{bmatrix} \sigma_{\alpha_{1j}}^2 & \sigma_{\alpha_{1j}} \sigma_{\alpha_{2j}} c_{21} & \sigma_{\alpha_{1j}} \sigma_{\alpha_{3j}} c_{31} \\ \sigma_{\alpha_{1j}} \sigma_{\alpha_{2j}} c_{21} & \sigma_{\alpha_{2j}}^2 (c_{21}^2 + c_{22}^2) & \sigma_{\alpha_{2j}} \sigma_{\alpha_{3j}} (c_{21} c_{31} + c_{22} c_{32}) \\ \sigma_{\alpha_{1j}} \sigma_{\alpha_{3j}} c_{31} & \sigma_{\alpha_{2j}} \sigma_{\alpha_{3j}} (c_{21} c_{31} + c_{22} c_{32}) & \sigma_{\alpha_{3j}}^2 (c_{31}^2 + c_{32}^2 + c_{33}^2) \end{bmatrix}$$

2.  $L_i(\alpha)$  is calculated for each period, and the product of these  $L_{it}(\theta)$ 's is taken:

$$L_i^r(\alpha) = \prod_{t=1}^{T_i} L_{it}^r(\alpha) = \prod_{t=1}^{T_i} P(F_{it} = j | \alpha_{1i}^r) \times P(V_{it} = C_{it} | F_{it}, \alpha_{2i}^r) \times P(B_{it} = k | F_{it}, V_{it}, \alpha_{3i}^r)$$

3. Repeat 1 and 2 for many time, and the results are averaged:  $\hat{P}_i = \frac{1}{R} \sum_{r=1}^R L_i^r(\alpha)$

4. Calculate the simulated log-likelihood:  $SLL = \sum_{i=1}^N \hat{P}_i$

## Reference

Erdem, T., and Keane, M. P. 1996. "Decision-Making Under Uncertainty: Capturing Dynamic Brand Choice Processes in Turbulent Consumer Goods Markets," *Marketing Science* (15:1), pp. 1-20.

# Appendix D

## Simulated Choice Probabilities

We use the change in the probability to be chosen as the entry site due to a change in information stock through the search engine for Expedia as an illustration. In step 1, we update the corresponding variable in the dataset, for example by adding 10 pages browsed a month ago through search advertising for Expedia to each individual and then recomputing the lagged individual information stock through search advertising. In step 2, we make a draw of net preferences for each website as described in step 1 in Appendix C. In step 3, we calculate the probability of being chosen as the entry website for each website using the updated data and the net preferences drawn in step 2. We repeat steps 2 and 3 as many times as necessary and, in step 4, we calculate a weighted average of these probabilities. The weight is given as

$$w_{it}^r = \frac{P(F_{it} | \alpha^r)}{\sum_r (F_{it} | \alpha^r)}$$

where  $F_{it}$  is the actual chosen entry-site for individual  $i$  in purchase session  $t$ . This is the probability of Expedia to be

chosen as the entry site due to a change in information stock through search engine. In step 5, we follow steps 2 through 4 to calculate the choice probability of Expedia to be chosen as the entry site without the change in information stock. In the last step, we calculate the differences in choice probabilities for Expedia.