Mobile App Recommendation: An Involvement-Enhanced Approach

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Appendix A: Notation

I = set of mobile apps
V = the number of mobile apps
U = set of app users
M = the number of app users
K = the number of interests
E = the number of involvement states
F = the number of browsing intensity levels
$i_{m,n}$ = the n^{th} app downloaded by user u_m
$f_{m,n}$ = browsing intensity level associated with $i_{m,n}$
$z_{m,n}$ = interest associated with $i_{m,n}$
$e_{m,n}$ = involvement state associated with $i_{m,n}$
$\boldsymbol{\theta_m}$ = K-dimensional interest distribution for user u_m
$\theta_{m,k} \in \boldsymbol{\theta_m}$ = user u_m's probability of interest k, k =1,…,K
$\varphi_{k,i} \in \boldsymbol{\varphi_k}$ = probability of downloading app i given interest k
$\boldsymbol{\varphi_k}$ =V-dimensional app distribution for interest k
$\boldsymbol{\lambda_k}$ =E-dimensional involvement distribution for interest k
$\boldsymbol{\pi_e}$ =F-dimensional browsing intensity distribution for involvement state e
$\boldsymbol{b}_m$ = most recent browsing behaviors by user u_m
$c_{m,z,i,e,f}$ = number of app i downloaded by user u_m due to interest z and with involvement state e and browsing intensity level f

Appendix B: Derivations of Equations (5) and (6)

B1: Derivation of Equation (5)

We repeat Equation (2):

$$p\left(z_{m,n}\middle|\boldsymbol{z}_{-(\boldsymbol{m},\boldsymbol{n})},\boldsymbol{i},\boldsymbol{e},\boldsymbol{f},\boldsymbol{\alpha},\boldsymbol{\beta},\boldsymbol{\tau},\boldsymbol{\varepsilon}\right)=\frac{p(\boldsymbol{z},\boldsymbol{i},\boldsymbol{e},\boldsymbol{f}|\boldsymbol{\alpha},\boldsymbol{\beta},\boldsymbol{\tau},\boldsymbol{\varepsilon})}{p\left(\boldsymbol{z}_{-(\boldsymbol{m},\boldsymbol{n})},\boldsymbol{i},\boldsymbol{e},\boldsymbol{f}|\boldsymbol{\alpha},\boldsymbol{\beta},\boldsymbol{\tau},\boldsymbol{\varepsilon}\right)}\propto p(\boldsymbol{z},\boldsymbol{i},\boldsymbol{e},\boldsymbol{f}\mid\boldsymbol{\alpha},\boldsymbol{\beta},\boldsymbol{\tau},\boldsymbol{\varepsilon}) \tag{B1}$$

We also repeat Equation (4):

$$\begin{aligned}
&p(\boldsymbol{z},\boldsymbol{i},\boldsymbol{e},\boldsymbol{f}\mid\boldsymbol{\alpha},\boldsymbol{\beta},\boldsymbol{\tau},\boldsymbol{\varepsilon})=\int\int\int\int p(\boldsymbol{z},\boldsymbol{i},\boldsymbol{e},\boldsymbol{f},\boldsymbol{\theta},\boldsymbol{\varphi},\boldsymbol{\lambda},\boldsymbol{\pi}|\,\boldsymbol{\alpha},\boldsymbol{\beta},\boldsymbol{\tau},\boldsymbol{\varepsilon})d\boldsymbol{\theta}d\boldsymbol{\varphi}d\boldsymbol{\lambda}d\boldsymbol{\pi}\\
&=\int\int\int\int p(\boldsymbol{z}|\boldsymbol{\theta})p(\boldsymbol{i}|\boldsymbol{\varphi},\boldsymbol{z})\,p(\boldsymbol{e}|\boldsymbol{\lambda},\boldsymbol{z})p(\boldsymbol{f}|\boldsymbol{\pi},\boldsymbol{e})p(\boldsymbol{\theta}|\boldsymbol{\alpha})p(\boldsymbol{\varphi}|\boldsymbol{\beta})p(\boldsymbol{\lambda}|\boldsymbol{\tau})p(\boldsymbol{\pi}|\boldsymbol{\varepsilon})d\boldsymbol{\theta}d\boldsymbol{\varphi}d\boldsymbol{\lambda}d\boldsymbol{\pi}\\
&=\int p(\boldsymbol{z}|\boldsymbol{\theta})p(\boldsymbol{\theta}|\boldsymbol{\alpha})\,d\boldsymbol{\theta}\int p(\boldsymbol{i}|\boldsymbol{\varphi},\boldsymbol{z})\;p(\boldsymbol{\varphi}|\boldsymbol{\beta})d\boldsymbol{\varphi}\\
&\quad\times\int p(\boldsymbol{e}|\boldsymbol{\lambda},\boldsymbol{z})p(\boldsymbol{\lambda}|\boldsymbol{\tau})\,d\boldsymbol{\lambda}\int p(\boldsymbol{f}|\boldsymbol{\pi},\boldsymbol{e})\,p(\boldsymbol{\pi}|\boldsymbol{\varepsilon})d\boldsymbol{\pi} \qquad \text{(B2)}
\end{aligned}$$

By integrating Equations (B1) and (B2) and dropping the term $\int p(\boldsymbol{f}|\boldsymbol{\pi},\boldsymbol{e})\,p(\boldsymbol{\pi}|\boldsymbol{\varepsilon})d\boldsymbol{\pi}$ that does not contain variable $z_{m,n}$, we have

$$\begin{aligned}
&p\left(z_{m,n}\middle|\boldsymbol{z}_{-(\boldsymbol{m},\boldsymbol{n})},\boldsymbol{i},\boldsymbol{e},\boldsymbol{f},\boldsymbol{\alpha},\boldsymbol{\beta},\boldsymbol{\tau},\boldsymbol{\varepsilon}\right)\\
&\qquad\propto\int p(\boldsymbol{z}|\boldsymbol{\theta})p(\boldsymbol{\theta}|\boldsymbol{\alpha})\,d\boldsymbol{\theta}\int p(\boldsymbol{i}|\boldsymbol{\varphi},\boldsymbol{z})\;p(\boldsymbol{\varphi}|\boldsymbol{\beta})d\boldsymbol{\varphi}\int p(\boldsymbol{e}|\boldsymbol{\lambda},\boldsymbol{z})p(\boldsymbol{\lambda}|\boldsymbol{\tau}) \qquad \text{(B3)}
\end{aligned}$$

$$\begin{aligned}
&\int p(\boldsymbol{z}|\boldsymbol{\theta})p(\boldsymbol{\theta}|\boldsymbol{\alpha})\,d\boldsymbol{\theta}\int p(\boldsymbol{i}|\boldsymbol{\varphi},\boldsymbol{z})\;p(\boldsymbol{\varphi}|\boldsymbol{\beta})d\boldsymbol{\varphi}\int p(\boldsymbol{e}|\boldsymbol{\lambda},\boldsymbol{z})p(\boldsymbol{\lambda}|\boldsymbol{\tau})\\
&=\int\prod_{m=1}^{M}p(\boldsymbol{\theta}_{\boldsymbol{m}}|\boldsymbol{\alpha})\prod_{m=1}^{M}\prod_{n=1}^{N_m}p\left(z_{m,n}\middle|\boldsymbol{\theta}_{\boldsymbol{m}}\right)d\,\boldsymbol{\theta}\\
&\qquad\times\int\prod_{k=1}^{K}p(\boldsymbol{\varphi}_{\boldsymbol{k}}|\boldsymbol{\beta})\prod_{m=1}^{M}\prod_{n=1}^{N_m}p\left(i_{m,n}\middle|\boldsymbol{\varphi}_{z_{m,n}}\right)d\boldsymbol{\varphi}\\
&\qquad\times\int\prod_{k=1}^{K}p(\boldsymbol{\lambda}_{\boldsymbol{k}}|\boldsymbol{\tau})\prod_{m=1}^{M}\prod_{n=1}^{N_m}p(e_{m,n}|\boldsymbol{\lambda}_{z_{m,n}})\,d\boldsymbol{\lambda}\\
&=\int\prod_{m=1}^{M}\frac{\Gamma\left(\sum_{k=1}^{K}\alpha_k\right)}{\prod_{k=1}^{K}\Gamma(\alpha_k)}\prod_{k=1}^{K}\theta_{m,k}^{\alpha_k-1}\prod_{m=1}^{M}\prod_{n=1}^{N_m}\theta_{m,z_{m,n}}\,d\boldsymbol{\theta}\\
&\quad\times\int\prod_{k=1}^{K}\frac{\Gamma\left(\sum_{i=1}^{V}\beta_i\right)}{\prod_{i=1}^{V}\Gamma(\beta_i)}\prod_{i=1}^{V}\varphi_{k,i}^{\beta_i-1}\prod_{m=1}^{M}\prod_{n=1}^{N_m}\varphi_{z_{m,n},i_{m,n}}\,d\boldsymbol{\varphi}\\
&\quad\times\int\prod_{k=1}^{K}\frac{\Gamma\left(\sum_{e=1}^{E}\tau_e\right)}{\prod_{e=1}^{E}\Gamma(\tau_e)}\prod_{e=1}^{E}\lambda_{k,e}^{\tau_e-1}\prod_{m=1}^{M}\prod_{n=1}^{N_m}\lambda_{z_{m,n},e_{m,n}}\,d\boldsymbol{\lambda} \qquad \text{by replacing each probabilistic term } p(.) \text{ with its corresponding density function}\\
&=\prod_{m=1}^{M}\int\frac{\Gamma\left(\sum_{k=1}^{K}\alpha_k\right)}{\prod_{k=1}^{K}\Gamma(\alpha_k)}\prod_{k=1}^{K}\theta_{m,k}^{\alpha_k-1+c_{m,k,*,*,*}}\,d\boldsymbol{\theta}_{\boldsymbol{m}}\\
&\qquad\times\prod_{k=1}^{K}\int\frac{\Gamma\left(\sum_{i=1}^{V}\beta_i\right)}{\prod_{i=1}^{V}\Gamma(\beta_i)}\prod_{i=1}^{V}\varphi_{k,i}^{\beta_i-1+c_{*,k,i,*,*}}\,d\boldsymbol{\varphi}_{\boldsymbol{k}}\\
&\qquad\times\prod_{k=1}^{K}\int\frac{\Gamma\left(\sum_{e=1}^{E}\tau_e\right)}{\prod_{e=1}^{E}\Gamma(\tau_e)}\prod_{e=1}^{E}\lambda_{k,e}^{\tau_e-1+c_{*,k,*,e,*}}\,d\boldsymbol{\lambda}_{\boldsymbol{k}} \qquad \text{by replacing the innermost products in each integral with sum of counts}\\
&=\prod_{m=1}^{M}\frac{\Gamma\left(\sum_{k=1}^{K}\alpha_k\right)}{\prod_{k=1}^{K}\Gamma(\alpha_k)}\frac{\prod_{k=1}^{K}\Gamma(\alpha_k+c_{m,k,*,*,*})}{\Gamma\left(\sum_{k=1}^{K}\alpha_k+c_{m,k,*,*,*}\right)}\int\frac{\Gamma\left(\sum_{k=1}^{K}\alpha_k+c_{m,k,*,*,*}\right)}{\prod_{k=1}^{K}\Gamma(\alpha_k+c_{m,k,*,*,*})}\prod_{k=1}^{K}\theta_{m,k}^{\alpha_k-1+c_{m,k,*,*,*}}\,d\boldsymbol{\theta}_{\boldsymbol{m}}\\
&\quad\times\prod_{k=1}^{K}\frac{\Gamma\left(\sum_{i=1}^{V}\beta_i\right)}{\prod_{i=1}^{V}\Gamma(\beta_i)}\frac{\prod_{i=1}^{V}\Gamma(\beta_i+c_{*,k,i,*,*})}{\Gamma\left(\sum_{i=1}^{V}\beta_i+c_{*,k,i,*,*}\right)}\int\frac{\Gamma\left(\sum_{i=1}^{V}\beta_i+c_{*,k,i,*,*}\right)}{\prod_{i=1}^{V}\Gamma(\beta_i+c_{*,k,i,*,*})}\prod_{i=1}^{V}\varphi_{k,i}^{\beta_i-1+c_{*,k,i,*,*}}\,d\boldsymbol{\varphi}_{\boldsymbol{k}}\\
&\quad\times\prod_{k=1}^{K}\frac{\Gamma\left(\sum_{e=1}^{E}\tau_e\right)}{\prod_{e=1}^{E}\Gamma(\tau_e)}\frac{\prod_{e=1}^{E}\Gamma(\tau_e+c_{*,k,*,e,*})}{\Gamma\left(\sum_{e=1}^{E}\tau_e+c_{*,k,*,e,*}\right)}\int\frac{\Gamma\left(\sum_{e=1}^{E}\tau_e+c_{*,k,*,e,*}\right)}{\prod_{e=1}^{E}\Gamma(\tau_e+c_{*,k,*,e,*})}\prod_{e=1}^{E}\lambda_{k,e}^{\tau_e-1+c_{*,k,*,e,*}}\,d\boldsymbol{\lambda}_{\boldsymbol{k}}\\
&=\prod_{m=1}^{M}\frac{\Gamma\left(\sum_{k=1}^{K}\alpha_k\right)}{\prod_{k=1}^{K}\Gamma(\alpha_k)}\frac{\prod_{k=1}^{K}\Gamma(\alpha_k+c_{m,k,*,*,*})}{\Gamma\left(\sum_{k=1}^{K}\alpha_k+c_{m,k,*,*,*}\right)} \qquad \text{by using the fact that all integral terms equal to 1}\\
&\qquad\times\prod_{k=1}^{K}\frac{\Gamma\left(\sum_{i=1}^{V}\beta_i\right)}{\prod_{i=1}^{V}\Gamma(\beta_i)}\frac{\prod_{i=1}^{V}\Gamma(\beta_i+c_{*,k,i,*,*})}{\Gamma\left(\sum_{i=1}^{V}\beta_i+c_{*,k,i,*,*}\right)} \qquad \text{(B4)}\\
&\qquad\times\prod_{k=1}^{K}\frac{\Gamma\left(\sum_{e=1}^{E}\tau_e\right)}{\prod_{e=1}^{E}\Gamma(\tau_e)}\frac{\prod_{e=1}^{E}\Gamma(\tau_e+c_{*,k,*,e,*})}{\Gamma\left(\sum_{e=1}^{E}\tau_e+c_{*,k,*,e,*}\right)}
\end{aligned}$$

By dropping constant terms that do not contain variable $z_{m,n}$ in Equation (B4), we have

$$\begin{aligned}
&p\left(z_{m,n}\middle|\boldsymbol{z}_{-(\boldsymbol{m},\boldsymbol{n})},\boldsymbol{i},\boldsymbol{e},\boldsymbol{f},\boldsymbol{\alpha},\boldsymbol{\beta},\boldsymbol{\tau},\boldsymbol{\varepsilon}\right)\\
&\propto\frac{\prod_{k=1}^{K}\Gamma(\alpha_k+c_{m,k,*,*,*})}{\Gamma\left(\sum_{k=1}^{K}\alpha_k+c_{m,k,*,*,*}\right)}\times\prod_{k=1}^{K}\frac{\Gamma\left(\beta_{i_{m,n}}+c_{*,k,i_{m,n},*,*}\right)}{\Gamma\left(\sum_{i=1}^{V}\beta_i+c_{*,k,i,*,*}\right)}\times\prod_{k=1}^{K}\frac{\Gamma(\tau_{e_{m,n}}+c_{*,k,*,e_{m,n},*})}{\Gamma\left(\sum_{e=1}^{E}\tau_e+c_{*,k,*,e,*}\right)} \qquad \text{(B5)}
\end{aligned}$$

Using the fact that $\Gamma(x+1)=x\Gamma(x)$, the right hand side of Equation (B5) becomes

$$\frac{\prod_{k=1}^{K}\Gamma(\alpha_k+c_{m,k,*,*,*})}{\Gamma(\sum_{k=1}^{K}\alpha_k+c_{m,k,*,*,*})}\times\prod_{k=1}^{K}\frac{\Gamma\left(\beta_{i_{m,n}}+c_{*,k,i_{m,n},*,*}\right)}{\Gamma(\sum_{i=1}^{V}\beta_i+c_{*,k,i,*,*})}\times\prod_{k=1}^{K}\frac{\Gamma(\tau_{e_{m,n}}+c_{*,k,*,e_{m,n},*})}{\Gamma(\sum_{e=1}^{E}\tau_e+c_{*,k,*,e,*})}$$
$$=\frac{\prod_{k\neq z_{m,n}}\Gamma(\alpha_k+c_{m,k,*,*,*}^{-(m,n)})}{\Gamma(1+\sum_{k=1}^{K}\alpha_k+c_{m,k,*,*,*}^{-(m,n)})}\times\Gamma(\alpha_{z_{m,n}}+c_{m,z_{m,n},*,*,*}^{-(m,n)})\times(\alpha_{z_{m,n}}+c_{m,z_{m,n},*,*,*}^{-(m,n)})$$
$$\times\prod_{k\neq z_{m,n}}\frac{\Gamma(\beta_{i_{m,n}}+c_{*,k,i_{m,n},*,*}^{-(m,n)})}{\Gamma(\sum_{i=1}^{V}\beta_i+c_{*,k,i,*,*}^{-(m,n)})}\times\frac{\Gamma(\beta_{i_{m,n}}+c_{*,z_{m,n},i_{m,n},*,*}^{-(m,n)})}{\Gamma(\sum_{i=1}^{V}\beta_i+c_{*,z_{m,n},i,*,*}^{-(m,n)})}\times\frac{\beta_{i_{m,n}}+c_{*,z_{m,n},i_{m,n},*,*}^{-(m,n)}}{\sum_{i=1}^{V}\beta_i+c_{*,z_{m,n},i,*,*}^{-(m,n)}}$$
$$\times\prod_{k\neq z_{m,n}}\frac{\Gamma(\tau_{e_{m,n}}+c_{*,k,*,e_{m,n},*}^{-(m,n)})}{\Gamma(\sum_{e=1}^{E}\tau_e+c_{*,k,*,e,*}^{-(m,n)})}\times\frac{\Gamma(\tau_{e_{m,n}}+c_{*,z_{m,n},*,e_{m,n},*}^{-(m,n)})}{\Gamma(\sum_{e=1}^{E}\tau_e+c_{*,z_{m,n},*,e,*}^{-(m,n)})}\times\frac{\tau_{e_{m,n}}+c_{*,z_{m,n},*,e_{m,n},*}^{-(m,n)}}{\sum_{e=1}^{E}\tau_e+c_{*,z_{m,n},*,e,*}^{-(m,n)}}$$
$$=\frac{\prod_{k=1}^{K}\Gamma(\alpha_k+c_{m,k,*,*,*}^{-(m,n)})}{\Gamma(1+\sum_{k=1}^{K}\alpha_k+c_{m,k,*,*,*}^{-(m,n)})}\times(\alpha_{z_{m,n}}+c_{m,z_{m,n},*,*,*}^{-(m,n)})$$
$$\times\prod_{k=1}^{K}\frac{\Gamma(\beta_{i_{m,n}}+c_{*,k,i_{m,n},*,*}^{-(m,n)})}{\Gamma(\sum_{i=1}^{V}\beta_i+c_{*,k,i,*,*}^{-(m,n)})}\times\frac{\beta_{i_{m,n}}+c_{*,z_{m,n},i_{m,n},*,*}^{-(m,n)}}{\sum_{i=1}^{V}\beta_i+c_{*,z_{m,n},i,*,*}^{-(m,n)}}$$
$$\times\prod_{k=1}^{K}\frac{\Gamma(\tau_{e_{m,n}}+c_{*,k,*,e_{m,n},*}^{-(m,n)})}{\Gamma(\sum_{e=1}^{E}\tau_e+c_{*,k,*,e,*}^{-(m,n)})}\times\frac{\tau_{e_{m,n}}+c_{*,z_{m,n},*,e_{m,n},*}^{-(m,n)}}{\sum_{e=1}^{E}\tau_e+c_{*,z_{m,n},*,e,*}^{-(m,n)}}$$

by refolding the residual $\Gamma-$function terms back into their general product
(B6)

By dropping constant terms that do not contain variable $z_{m,n}$ in Equation (B6), we obtain Equation (5).

B2: Derivation of Equation (6)

Equation (6) can be derived in a way similar to that of Equation (5). First, we repeat Equation (3):

$$p\left(e_{m,n}\middle|\boldsymbol{e}_{-(\boldsymbol{m},\boldsymbol{n})},\boldsymbol{i},\boldsymbol{z},\boldsymbol{f},\boldsymbol{\alpha},\boldsymbol{\beta},\boldsymbol{\tau},\boldsymbol{\varepsilon}\right)=\frac{p(\boldsymbol{e},\boldsymbol{i},\boldsymbol{z},\boldsymbol{f}|\boldsymbol{\alpha},\boldsymbol{\beta},\boldsymbol{\tau},\boldsymbol{\varepsilon})}{p(\boldsymbol{e}_{-(\boldsymbol{m},\boldsymbol{n})},\boldsymbol{i},\boldsymbol{z},\boldsymbol{f}|\boldsymbol{\alpha},\boldsymbol{\beta},\boldsymbol{\tau},\boldsymbol{\varepsilon})}\propto p(\boldsymbol{e},\boldsymbol{i},\boldsymbol{z},\boldsymbol{f}\mid\boldsymbol{\alpha},\boldsymbol{\beta},\boldsymbol{\tau},\boldsymbol{\varepsilon})\qquad\text{(B7)}$$

We also repeat Equation (4):

$$p(\boldsymbol{e},\boldsymbol{i},\boldsymbol{z},\boldsymbol{f}\mid\boldsymbol{\alpha},\boldsymbol{\beta},\boldsymbol{\tau},\boldsymbol{\varepsilon})=\int\int\int\int p(\boldsymbol{z},\boldsymbol{i},\boldsymbol{e},\boldsymbol{f},\boldsymbol{\theta},\boldsymbol{\varphi},\boldsymbol{\lambda},\boldsymbol{\pi}|\,\boldsymbol{\alpha},\boldsymbol{\beta},\boldsymbol{\tau},\boldsymbol{\varepsilon})d\boldsymbol{\theta}d\boldsymbol{\varphi}d\boldsymbol{\lambda}d\boldsymbol{\pi}$$
$$=\int p(\boldsymbol{z}|\boldsymbol{\theta})p(\boldsymbol{\theta}|\boldsymbol{\alpha})\,d\boldsymbol{\theta}\int p(\boldsymbol{i}|\boldsymbol{\varphi},\boldsymbol{z})\;p(\boldsymbol{\varphi}|\boldsymbol{\beta})d\boldsymbol{\varphi}$$
$$\times\int p(\boldsymbol{e}|\boldsymbol{\lambda},\boldsymbol{z})p(\boldsymbol{\lambda}|\boldsymbol{\tau})\,d\boldsymbol{\lambda}\int p(\boldsymbol{f}|\boldsymbol{\pi},\boldsymbol{e})\,p(\boldsymbol{\pi}|\boldsymbol{\varepsilon})d\boldsymbol{\pi}\qquad\text{(B8)}$$

By integrating Equations (B7) and (B8) and dropping the term $\int p(\boldsymbol{z}|\boldsymbol{\theta})p(\boldsymbol{\theta}|\boldsymbol{\alpha})\,d\boldsymbol{\theta}\int p(\boldsymbol{i}|\boldsymbol{\varphi},\boldsymbol{z})\,p(\boldsymbol{\varphi}|\boldsymbol{\beta})d\boldsymbol{\varphi}$ that does not contain the variable $e_{m,n}$, we have

$$p\left(e_{m,n}\middle|\boldsymbol{e}_{-(\boldsymbol{m},\boldsymbol{n})},\boldsymbol{i},\boldsymbol{z},\boldsymbol{f},\boldsymbol{\alpha},\boldsymbol{\beta},\boldsymbol{\tau},\boldsymbol{\varepsilon}\right)\propto\int p(\boldsymbol{e}|\boldsymbol{\lambda},\boldsymbol{z})p(\boldsymbol{\lambda}|\boldsymbol{\tau})\,d\boldsymbol{\lambda}\int p(\boldsymbol{f}|\boldsymbol{\pi},\boldsymbol{e})\,p(\boldsymbol{\pi}|\boldsymbol{\varepsilon})d\boldsymbol{\pi}\quad\int p(\boldsymbol{e}|\boldsymbol{\lambda},\boldsymbol{z})p(\boldsymbol{\lambda}|\boldsymbol{\tau})\,d\boldsymbol{\lambda}\int p(\boldsymbol{f}|\boldsymbol{\pi},\boldsymbol{e})\,p(\boldsymbol{\pi}|\boldsymbol{\varepsilon})d\boldsymbol{\pi}$$
$$=\int\prod_{k=1}^{K}p(\boldsymbol{\lambda_k}|\boldsymbol{\tau})\prod_{m=1}^{M}\prod_{n=1}^{N_m}p\left(e_{m,n}\middle|\boldsymbol{\lambda}_{z_{m,n}}\right)d\boldsymbol{\lambda}$$
$$\times\int\prod_{e=1}^{E}p(\boldsymbol{\pi}_e|\boldsymbol{\varepsilon})\prod_{m=1}^{M}\prod_{n=1}^{N_m}p(f_{m,n}|\boldsymbol{\pi}_{e_{m,n}})d\boldsymbol{\pi}$$
$$=\int\prod_{k=1}^{K}\frac{\Gamma(\sum_{e=1}^{E}\tau_e)}{\prod_{e=1}^{E}\Gamma(\tau_e)}\prod_{e=1}^{E}\lambda_{k,e}^{\tau_e-1}\prod_{m=1}^{M}\prod_{n=1}^{N_m}\lambda_{z_{m,n},e_{m,n}}\,d\boldsymbol{\lambda}$$
$$\times\int\prod_{e=1}^{E}\frac{\Gamma(\sum_{f=1}^{F}\varepsilon_f)}{\prod_{f=1}^{F}\Gamma(\varepsilon_f)}\prod_{f=1}^{F}\pi_{e,f}^{\varepsilon_f-1}\prod_{m=1}^{M}\prod_{n=1}^{N_m}\pi_{e_{m,n},f_{m,n}}\,d\boldsymbol{\pi}$$

by replacing each probabilistic term $p(.)$ with its corresponding density function

$$=\prod_{k=1}^{K}\int\frac{\Gamma(\sum_{e=1}^{E}\tau_e)}{\prod_{e=1}^{E}\Gamma(\tau_e)}\prod_{e=1}^{E}\lambda_{k,e}^{\tau_e-1+c_{*,k,*,e,*}}\,d\boldsymbol{\lambda_k}$$
$$\times\prod_{e=1}^{E}\int\frac{\Gamma(\sum_{f=1}^{F}\varepsilon_f)}{\prod_{f=1}^{F}\Gamma(\varepsilon_f)}\prod_{f=1}^{F}\pi_{e,f}^{\varepsilon_f-1+c_{*,*,*,e,f}}\,d\boldsymbol{\pi_e}$$

by replacing the innermost products in each integral with sum of counts

$$=\prod_{k=1}^{K}\frac{\Gamma(\sum_{e=1}^{E}\tau_e)}{\prod_{e=1}^{E}\Gamma(\tau_e)}\frac{\prod_{e=1}^{E}\Gamma(\tau_e+c_{*,k,*,e,*})}{\Gamma(\sum_{e=1}^{E}\tau_e+c_{*,k,*,e,*})}\int\frac{\Gamma(\sum_{e=1}^{E}\tau_e+c_{*,k,*,e,*})}{\prod_{e=1}^{E}\Gamma(\tau_e+c_{*,k,*,e,*})}\prod_{e=1}^{E}\lambda_{k,e}^{\tau_e-1+c_{*,k,*,e,*}}\,d\boldsymbol{\lambda_k}$$
$$\times\prod_{e=1}^{E}\frac{\Gamma(\sum_{f=1}^{F}\varepsilon_f)}{\prod_{f=1}^{F}\Gamma(\varepsilon_f)}\frac{\prod_{f=1}^{F}\Gamma(\varepsilon_f+c_{*,*,*,e,f})}{\Gamma(\sum_{f=1}^{F}\varepsilon_f+c_{*,*,*,e,f})}\int\frac{\Gamma(\sum_{f=1}^{F}\varepsilon_f+c_{*,*,*,e,f})}{\prod_{f=1}^{F}\Gamma(\varepsilon_f+c_{*,*,*,e,f})}\prod_{f=1}^{F}\pi_{e,f}^{\varepsilon_f-1+c_{*,*,*,e,f}}\,d\boldsymbol{\pi_e}$$
$$=\prod_{k=1}^{K}\frac{\Gamma(\sum_{e=1}^{E}\tau_e)}{\prod_{e=1}^{E}\Gamma(\tau_e)}\frac{\prod_{e=1}^{E}\Gamma(\tau_e+c_{*,k,*,e,*})}{\Gamma(\sum_{e=1}^{E}\tau_e+c_{*,k,*,e,*})}$$
$$\times\prod_{e=1}^{E}\frac{\Gamma(\sum_{f=1}^{F}\varepsilon_f)}{\prod_{f=1}^{F}\Gamma(\varepsilon_f)}\frac{\prod_{f=1}^{F}\Gamma(\varepsilon_f+c_{*,*,*,e,f})}{\Gamma\left(\sum_{f=1}^{F}\varepsilon_f+c_{*,*,*,e,f}\right)}$$

by using the fact that all integral terms equal to 1
(B9)

By dropping constant terms that do not contain the variable $e_{m,n}$ in Equation (B9), we have

$$p\left(e_{m,n} \middle| \boldsymbol{e}_{-(\boldsymbol{m},\boldsymbol{n})}, \boldsymbol{i}, \boldsymbol{z}, \boldsymbol{f}, \boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\tau}, \boldsymbol{\varepsilon}\right) \propto \frac{\prod_{e=1}^{E} \Gamma(\tau_e + c_{*,z_{m,n},*,e,*})}{\Gamma(\sum_{e=1}^{E} \tau_e + c_{*,z_{m,n},*,e,*})} \prod_{e=1}^{E} \frac{\Gamma\left(\varepsilon_{f_{m,n}} + c_{*,*,*,e,f_{m,n}}\right)}{\Gamma\left(\sum_{f=1}^{F} \varepsilon_f + c_{*,*,*,e,f}\right)}$$

(B10)

Using the fact that $\Gamma(x+1) = x\Gamma(x)$, the right-hand side of Equation (B10) becomes,

$$\frac{\prod_{e=1}^{E} \Gamma(\tau_e + c_{*,z_{m,n},*,e,*})}{\Gamma(\sum_{e=1}^{E} \tau_e + c_{*,z_{m,n},*,e,*})} \times \prod_{e=1}^{E} \frac{\Gamma\left(\varepsilon_{f_{m,n}} + c_{*,*,*,e,f_{m,n}}\right)}{\Gamma\left(\sum_{f=1}^{F} \varepsilon_f + c_{*,*,*,e,f}\right)}$$

$$= \frac{\prod_{e \neq e_{m,n}} \Gamma(\tau_e + c_{*,z_{m,n},*,e,*}^{-(m,n)})}{\Gamma(1 + \sum_{e=1}^{E} \tau_e + c_{*,z_{m,n},*,e,*}^{-(m,n)})} \times \Gamma(\tau_{e_{m,n}} + c_{*,z_{m,n},*,e_{m,n},*}^{-(m,n)}) \times (\tau_{e_{m,n}} + c_{*,z_{m,n},*,e_{m,n},*}^{-(m,n)})$$

$$\times \prod_{e \neq e_{m,n}} \frac{\Gamma\left(\varepsilon_{f_{m,n}} + c_{*,*,*,e,f_{m,n}}^{-(m,n)}\right)}{\Gamma\left(\sum_{f=1}^{F} \varepsilon_{f_{m,n}} + c_{*,*,*,e,f}^{-(m,n)}\right)} \times \frac{\Gamma\left(\varepsilon_{f_{m,n}} + c_{*,*,*,e_{m,n},f_{m,n}}^{-(m,n)}\right)}{\Gamma\left(\sum_{f=1}^{F} \varepsilon_{f_{m,n}} + c_{*,*,*,e_{m,n},f}^{-(m,n)}\right)} \times \frac{\varepsilon_{f_{m,n}} + c_{*,*,*,e_{m,n},f_{m,n}}^{-(m,n)}}{\sum_{f=1}^{F} \varepsilon_{f_{m,n}} + c_{*,*,*,e_{m,n},f}^{-(m,n)}}$$

$$= \frac{\prod_{e=1}^{E} \Gamma(\tau_e + c_{*,z_{m,n},*,e,*}^{-(m,n)})}{\Gamma(1 + \sum_{e=1}^{E} \tau_e + c_{*,z_{m,n},*,e,*}^{-(m,n)})} \times (\tau_{e_{m,n}} + c_{*,z_{m,n},*,e_{m,n},*}^{-(m,n)})$$

$$\times \prod_{e=1}^{E} \frac{\Gamma\left(\varepsilon_{f_{m,n}} + c_{*,*,*,e,f_{m,n}}^{-(m,n)}\right)}{\Gamma\left(\sum_{f=1}^{F} \varepsilon_{f_{m,n}} + c_{*,*,*,e,f}^{-(m,n)}\right)} \times \frac{\varepsilon_{f_{m,n}} + c_{*,*,*,e_{m,n},f_{m,n}}^{-(m,n)}}{\sum_{f=1}^{F} \varepsilon_{f_{m,n}} + c_{*,*,*,e_{m,n},f}^{-(m,n)}}$$

by refolding the residual Γ-function terms back into their general products

(B11)

By dropping constant terms that do not contain variable $e_{m,n}$ in Equation (B11), we obtain Equation (6).

Appendix C: Derivations of Equations (7) to (10)

Let $p(\boldsymbol{\theta}_m|\boldsymbol{i},\boldsymbol{f},\boldsymbol{z},\boldsymbol{e},\boldsymbol{\alpha})$be the posterior distribution of $\boldsymbol{\theta}_m$, $m = 1, \ldots, M$, given observed app downloads $\boldsymbol{i}$**,** browsing intensity levels $\boldsymbol{f}$, learned hidden variables $\boldsymbol{z}$ and $\boldsymbol{e}$, and hyper-parameter $\boldsymbol{\alpha}$. We have

$$p(\boldsymbol{\theta}_m|\boldsymbol{i},\boldsymbol{f},\boldsymbol{z},\boldsymbol{e},\boldsymbol{\alpha}) = \frac{p(\boldsymbol{\theta}_m|\boldsymbol{\alpha})\prod_{m=1}^{Nm} p(z_{m,n}|\boldsymbol{\theta}_m)}{\int p(\boldsymbol{\theta}_m|\boldsymbol{\alpha})\prod_{m=1}^{Nm} p(z_{m,n}|\boldsymbol{\theta}_m) d\boldsymbol{\theta}_m}$$

$$= \frac{\frac{\Gamma\left(\sum_{k=1}^{K}\alpha_k\right)}{\prod_{k=1}^{K}\Gamma(\alpha_k)}\prod_{k=1}^{K}\theta_{m,k}^{\alpha_k-1}\prod_{n=1}^{Nm}\theta_{m,z_{m,n}}}{\int\frac{\Gamma\left(\sum_{k=1}^{K}\alpha_k\right)}{\prod_{k=1}^{K}\Gamma(\alpha_k)}\prod_{k=1}^{K}\theta_{m,k}^{\alpha_k-1}\prod_{n=1}^{Nm}\theta_{m,z_{m,n}}d\theta_m}$$

by replacing each probabilistic term $p(.)$ with its corresponding density function

$$= \frac{\frac{\Gamma\left(\sum_{k=1}^{K}\alpha_k\right)\Gamma\left(\sum_{k=1}^{K}\alpha_k+c_{m,k,*,*,*}\right)}{\prod_{k=1}^{K}\Gamma(\alpha_k)\prod_{k=1}^{K}\Gamma(\alpha_k+c_{m,k,*,*,*})}\prod_{k=1}^{K}\theta_{m,k}^{\alpha_k+c_{m,k,*,*,*}-1}}{\frac{\Gamma\left(\sum_{k=1}^{K}\alpha_k\right)}{\prod_{k=1}^{K}\Gamma(\alpha_k)}\int\frac{\Gamma\left(\sum_{k=1}^{K}\alpha_k+c_{m,k,*,*,*}\right)}{\prod_{k=1}^{K}\Gamma(\alpha_k+c_{m,k,*,*,*})}\prod_{k=1}^{K}\theta_{m,k}^{\alpha_k+c_{m,k,*,*,*}-1}d\theta_m}$$

by replacing the innermost product with sum of counts and multiplying denominator and nominator by a same term

$$= \frac{\Gamma\left(\sum_{k=1}^{K}\alpha_k+c_{m,k,*,*,*}\right)}{\prod_{k=1}^{K}\Gamma(\alpha_k+c_{m,k,*,*,*})}\prod_{k=1}^{K}\theta_{m,k}^{\alpha_k+c_{m,k,*,*,*}-1}$$

by using the fact that the integral term in the denominator equal to 1

According to the equation above, given $\boldsymbol{i},\boldsymbol{f},\boldsymbol{z},\boldsymbol{e},$ and $\boldsymbol{\alpha}$, $\boldsymbol{\theta}_m$ follows a *Dirichlet* distribution with *K*-vector hyper-parameter $\boldsymbol{\alpha_k} + \boldsymbol{c_{m,k}}_{,*,*,*} = (\alpha_1 + c_{m,1,*,*,*}, \ldots, \alpha_K + c_{m,K,*,*,*})$. Given a *K*-dimensional variable $\boldsymbol{X} = (X_1, X_2, \ldots, X_K)$, which follows a *Dirichlet* distribution with *K*-vector hyper-parameter $\boldsymbol{\alpha} = (\alpha_1, \alpha_2, \ldots, \alpha_K)$, we know the fact that $E(X_i) = \frac{\alpha_i}{\sum_i \alpha_i}$. Applying this fact to $\boldsymbol{\theta}_m$, we obtain Equation (7). Equations (8) to (10) can be obtained in a way similar to that of Equation (7).

Appendix D: Derivations of Equation (15) and $Perp(\boldsymbol{b})$

D1: Details of Obtaining Equation (15)

By comparing $p(x_{m,j}|\boldsymbol{x}_{-(m,j)},\boldsymbol{b},\boldsymbol{z},\boldsymbol{i},\boldsymbol{\alpha},\boldsymbol{\beta})$ with $p(z_{m,n}|\boldsymbol{z}_{-(\boldsymbol{m},\boldsymbol{n})},\boldsymbol{e},\boldsymbol{i},\boldsymbol{f},\boldsymbol{\alpha},\boldsymbol{\beta},\boldsymbol{\tau},\boldsymbol{\varepsilon})$ in Equation (5), we notice that both $x_{m,j}$ and $z_{m,n}$ are interests. However, overall interest $z_{m,n}$ is different from current interest $x_{m,j}$ in that the former is conditioned on browsing intensity $\boldsymbol{f}$ but the latter is not. Specifically, we obtain Equation (15) by (1) dropping the third term in Equation (5), which is associated with browsing intensity $\boldsymbol{f}$; (2) replacing parameters in the first two terms of Equation (5) with their corresponding parameters for $x_{m,j}$; (3) replacing $c_{*,z_{m,n},i_{m,n},*,*}^{-(m,n)}$ in Equation (5) with $c_{*,x_{m,j},i_{m,j},*,*} + d_{*,x_{m,j},i_{m,j}}^{-(m,j)}$ because $x_{m,j}$ in $p(x_{m,j}|\boldsymbol{x}_{-(m,j)},\boldsymbol{b},\boldsymbol{z},\boldsymbol{i},\boldsymbol{\alpha},\boldsymbol{\beta})$ is conditioned on both download behaviors $\boldsymbol{i}$ and most recent browsing behaviors $\boldsymbol{b}$.

D2: Calculation of $Perp(\boldsymbol{b})$

We have

$$Perp(\boldsymbol{b}) = exp(-\frac{\sum_{m=1}^{M}\sum_{j=1}^{J_m} logP(i_{m,j})}{\sum_{m=1}^{M} J_m}) \qquad \text{(D1)}$$

where

$$logP(i_{m,j}) = log(\sum_{k=1}^{K}\gamma_{m,k}\delta_{k,i_{m,j}}) \qquad \text{(D2)}$$

$$\gamma_{m,k} = \frac{\alpha_k + d_{m,k,*}}{\sum_{k=1}^{K}\alpha_k + d_{m,k,*}} \qquad \text{(D3)}$$

and

$$\delta_{k,i} = \frac{\beta_i + c_{*,k,i,*,*} + d_{*,k,i}}{\sum_{i=1}^{V}\beta_i + c_{*,k,i,*,*} + d_{*,k,i}} \qquad \text{(D4)}$$

In Equation (D1), $logP(i_{m,j})$ denotes the log-likelihood of browsing app $i_{m,j}$, which is calculated using Equation (D2). In Equation (D2), $\gamma_{m,k}$ denotes the probability of interest *k* discovered from most recent browsing behaviors $\boldsymbol{b}_m$, and $\delta_{k,i}$ is the probability of browsing app *i* given interest *k*. Equations (D3) and (D4) for calculating $\gamma_{m,k}$ and $\delta_{k,i}$ can be obtained by analogy to Equations (7) and (8), with two changes: (1) $c_{m,k,*,*,*}$ in Equation (7) changes to $d_{m,k,*}$ in Equation (D3); (2) $c_{*,k,i,*,*}$ in Equation (8) changes to $c_{*,k,i,*,*} + d_{*,k,i}$ in Equation (D4), because $x_{m,j}$ in $p(x_{m,j}|\boldsymbol{x}_{-(m,j)},\boldsymbol{b},\boldsymbol{z},\boldsymbol{i},\boldsymbol{\alpha},\boldsymbol{\beta})$ is conditioned on both download behaviors $\boldsymbol{i}$ and most recent browsing behaviors $\boldsymbol{b}$**.**

Appendix E: Performance Comparison between IMAR and IMAR-Gaussian

We develop a variant of our proposed method, namely IMAR-Gaussian. The only difference between IMAR-Gaussian (Figure E1) and IMAR (Figure 4) is that IMAR-Gaussian models involvement state as a Gaussian distribution over browsing intensities g with mean μ_e and standard deviation σ_e whereas IMAR models involvement state as a multinomial distribution over browsing intensity levels.

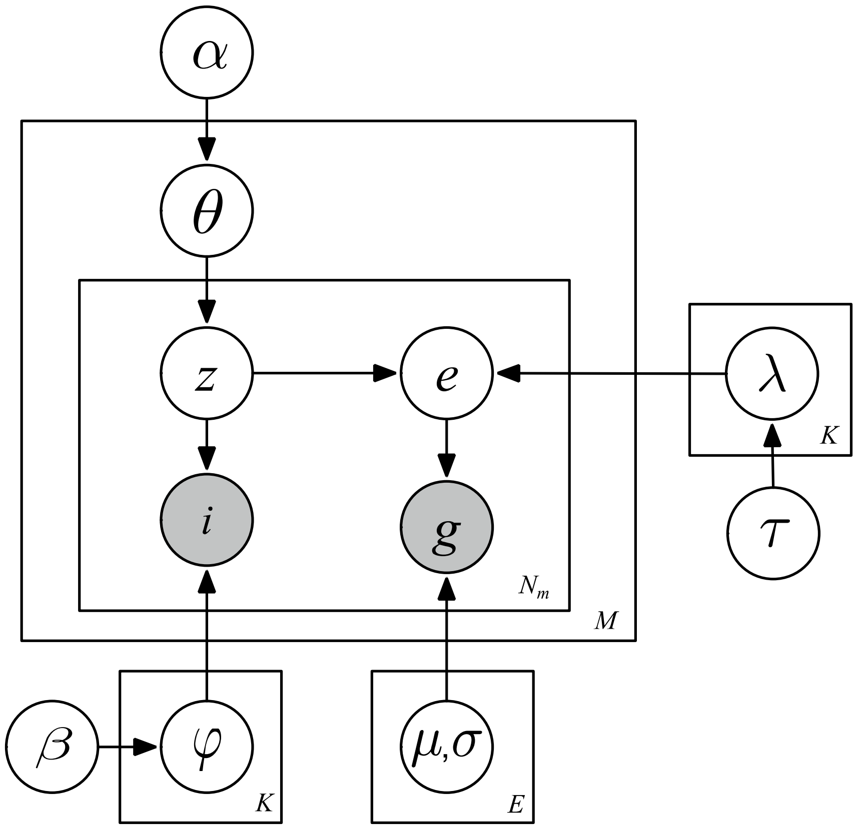

Figure E1. The Graphical Model for IMAR-Gaussian

The model parameters in IMAR-Gaussian are inferred with a combined Gibbs Sampling and EM algorithm. To evaluate its performance, we test IMAR-Gaussian on the same dataset used in this study. As shown in Tables E1 and E2, our method consistently outperforms IMAR-Gaussian in both recall and DCG, as the length N of the recommendation list increases from 3 to 15. One possible explanation of the underperformance of IMAR-Gaussian could be that multinomial distribution allows for a more flexible structure for data modeling than Gaussian distribution. For example, the distribution of an involvement state over browsing intensities could be skewed. Gaussian distribution, a symmetric distribution, is not a good option for modeling that distribution, whereas multinomial distribution can model skewed distributions well, despite of its discrete characteristic. In addition, we would like to explain why IMAR treats involvement state as a categorical variable. In IMAR, differentiating various involvement states is sufficient for model learning and ordinal information among involvement states is not required for model learning. For example, differentiating between "high involvement" and "low involvement" is sufficient while the ordinal information that one is "higher" than the other is not necessary for model learning. Therefore, in the model learning phase, IMAR treats involvement state as a categorical variables and model it as a multinomial distribution over browsing intensity levels. In the recommendation phase, IMAR identifies low or high involvement state according to its distribution over browsing intensity levels (e.g., the low-involvement state concentrates more on low browsing intensity levels than the high-involvement state).

Table E1. Recommendation Performances of IMAR and IMAR-Gaussian: Recall

Method	Recall (N=3)	Recall (N=5)	Recall (N=10)	Recall (N=15)
IMAR (Our Method)	0.0419	0.0620	0.1030	0.1360
IMAR-Gaussian	0.0387	0.0587	0.0989	0.131
IMAR over IMAR-Gaussian	8.27%	5.62%	4.15%	3.82%

Table E2. Recommendation Performances of IMAR and IMAR-Gaussian: DGG

Method	DCG (N=3)	DCG (N=5)	DCG (N=10)	DCG (N=15)
IMAR (Our Method)	0.0312	0.0393	0.0526	0.0613
IMAR-Gaussian	0.0287	0.0369	0.0498	0.0583
IMAR over IMAR-Gaussian	8.71%	6.50%	5.62%	5.15%

Appendix F: Sample Interests Discovered by Our Method

In this appendix, we report sample interests discovered by our method. In our method, an interest is represented as a probability distribution over apps. Table F1 lists six interests discovered by our method, along with apps with top download probabilities in each interest. In this table, we manually label each interest according to the top apps in the interest. For example, the top five apps in interest "Racing Games" are *Truck Simulator City*, *Need for Speed Most Wanted*, *Crazy Taxi: Urban Surge*, *High-speed Road Race*, and *Hill Climbing Racing*, with download probabilities of 0.022, 0.021, 0.017, 0.016, and 0.015, respectively.

Our method also discovers the distribution of involvement states for each interest, shown in Table F2. For example, the probabilities that interests "Racing Games," "Mom & Kids," and "Learning English" being at the high-involvement state are 0.999, 0.851, and 0.756 respectively. The top downloaded apps in interests "Racing Games" and "Mom & Kids" are of high hedonic value and emotional appeal and thus can elicit high involvement from users (Nicolau 2013; Zaichkowsky 1985). The interest "Learning English" has a high probability at the high-involvement state because users are highly motivated to improve their English and thus carefully compare alternative apps and select the most appropriate one to download for learning English.

Comparatively, the probabilities that interests "Hot Apps," "Videos," and "Navigation Services" being at the high-involvement state are 0.00004, 0.0003, and 0.0006 respectively. These interests are more likely at the low-involvement state because (1) top downloaded apps of these interests are more utilitarian than hedonic and thus are unlikely to arouse users' involvement; (2) top downloaded apps of these interests are similar to each other without much differences in attributes. Thus, there is no need for users to carefully compare alternatives before a download. The sample interests discussed in this appendix show that our method can effectively discover interests and their involvement distributions.

Table F1. Sample Interests Discovered by IMAR

Interest: Racing Games		**Interest: Hot Apps**	
Truck Simulator City	0.022	Wechat	0.119
Need for Speed Most Wanted	0.021	QQ	0.109
Crazy Taxi: Urban Surge	0.017	KuGou Music	0.062
High-speed Road Race	0.016	Paypal	0.061
Hill Climbing Racing	0.015	Mobile Taobao	0.051
Interest: Mom & Kids		**Interest: Videos**	
Kids Hospital	0.034	Youku	0.138
Kids Kindergarten	0.029	IQIYI Video	0.132
Kids Kitchen	0.027	Sohu Video	0.13
Kids Cleaning	0.021	Tudou Video	0.099
Kids Love Eating	0.020	Mango Video	0.094
Interest: Learning English		**Interest: Navigation Services**	
Fluent Oral English	0.059	AutoMap	0.217
Hundred Words Killer	0.057	AutoNavi	0.144
Hj Happy Words	0.043	Baidu Map	0.097
Zhimi Word Tutor	0.041	Google Map	0.072
Palm English	0.039	Tencent Map	0.048

Table F2. Involvement Distributions of the Six Sample Interests

Interest	Probability at the High-Involvement State	Probability at the Low-Involvement State
Racing Games	0.999	0.001
Mom & Kids	0.851	0.149
Learning English	0.756	0.244
Hot Apps	0.00004	0.99996
Videos	0.0003	0.9997
Navigation Services	0.0006	0.9994

References

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