Shared or Dedicated Infrastructures? On the Impact of Reprovisioning Ability

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Appendix A

Proofs of Lemmas and Proposition 1

Proof of Results for the Impact of Decreases in Sharing Costs: It can be derived that $\frac{\partial(\Pi_s-\Pi_d)}{\partial a_s}=\frac{1}{2}\left(-2f\mathrm{X}_1-aX_2^{max}-arX_2^{max}-\frac{(1-a)^2(V_2-a_s)(1+r)X_2^{max}}{aa_s+(1-a)V_2+(1-a)(V_2-V_1)r}-\frac{(1-a)^2(V_2-a_s)(1+r)(V_2+(1-a)(V_2-V_1)r)X_2^{max}}{(aa_s+(1-a)V_2+(1-a)(V_2-V_1)r)^2}\right)$, and $\frac{\partial(\Pi_s-\Pi_d)}{\partial c_s}=-1$. It can be easily checked that both are negative.

Proof of Lemma 1: It can be derived that $\frac{\partial K_{d2}^*}{\partial\frac{V_2}{a_{d2}}}=\frac{(1-\alpha)X_2^{max}}{\left((1-\alpha)\frac{V_2}{a_{d2}}+\alpha\right)^2}$, $\frac{\partial K_s^*}{\partial\frac{V_1}{a_s}}=\frac{(1-\alpha)^2\left(\frac{V_2}{a_s}-1\right)r(1+r)X_2^{max}}{\left(\alpha+(1-\alpha)\frac{V_2}{a_s}+(1-\alpha)\left(\frac{V_2}{a_s}-\frac{V_1}{a_s}\right)r\right)^2}$, $\frac{\partial K_s^*}{\partial\frac{V_2}{a_s}}=\frac{(1-\alpha)(1+r)\left(1-(1-\alpha)\left(\frac{V_1}{a_s}-1\right)r\right)X_2^{max}}{\left(\alpha+(1-\alpha)\frac{V_2}{a_s}+(1-\alpha)\left(\frac{V_2}{a_s}-\frac{V_1}{a_s}\right)r\right)^2}$, $\frac{\partial K_{d2}^*}{\partial\alpha}=\frac{-\left(\frac{V_2}{a_{d2}}-1\right)X_2^{max}}{\left((1-\alpha)\frac{V_2}{a_{d2}}+\alpha\right)^2}$, $\frac{\partial K_s^*}{\partial\alpha}=\frac{-(1+r)\left(\frac{V_2}{a_{d2}}-1\right)X_2^{max}}{\left(\alpha+(1-\alpha)\frac{V_2}{a_s}+(1-\alpha)\left(\frac{V_2}{a_s}-\frac{V_1}{a_s}\right)r\right)^2}$, $\frac{\partial K_s^*}{\partial r}=\frac{(1-\alpha)\left(\frac{V_2}{a_s}-1\right)\left(\alpha+(1-\alpha)\frac{V_1}{a_s}\right)X_2^{max}}{\left(\alpha+(1-\alpha)\frac{V_2}{a_s}+(1-\alpha)\left(\frac{V_2}{a_s}-\frac{V_1}{a_s}\right)r\right)^2}$ and $\frac{\partial K_s^*}{\partial f}=X_1$. Given $r\left(\frac{V_1}{a_s}-1\right)<1$, it can be easily checked that $\frac{\partial K_{d2}^*}{\partial\alpha}<0$, $\frac{\partial K_s^*}{\partial\alpha}<0$, $\frac{\partial K_s^*}{\partial f}>0$, and in addition, that $\frac{\partial K_{d2}^*}{\partial\frac{V_2}{a_{d2}}}$, $\frac{\partial K_s^*}{\partial\frac{V_1}{a_s}}$, $\frac{\partial K_s^*}{\partial\frac{V_2}{a_s}}$ and $\frac{\partial K_s^*}{\partial r}$ are all greater than or equal to zero, and that they are zero only if $\alpha=1$.

Proof of Lemma 2: It can be derived that $\frac{\partial^2 K_s^*}{\partial\alpha\partial r}=\frac{-a_s(V_2-a_s)(V_1(2+r)(1-a)-V_2(1+r)(1-a)+aa_s)X_2^{max}}{(aa_s+(1-a)V_2+(1-a)(V_2-V_1)r)^3}$. It can be easily checked that it is greater than zero if $V_1<\frac{(1+r)V_2}{(2+r)}-\frac{\alpha a_s}{(1-\alpha)(2+r)}$.

Proof related to Proposition 1 (uniqueness property of $\mathbf{h(\alpha)}$): $h'(\alpha)$ satisfies the uniqueness property (i.e., changes its sign at most once) for $\alpha\in[0,1]$ if the equation $h'(\alpha)=0$ has at most one solution for $\alpha\in[0,1]$. To examine the behavior of $h'(\alpha)=0$, we derive:

$$h'(\alpha)=\frac{a_s\left(\frac{V_2}{a_s}-1\right)+a_s\left(\frac{V_1}{a_s}-1\right)r\left(1+(1-\alpha)\left(\frac{V_2}{a_s}-\frac{V_1}{a_s}\right)\mathrm{r}\left(\left(1+\alpha+(1-\alpha)\frac{V_2}{a_s}\right)+(1-\alpha)\left(\frac{V_2}{a_s}-\frac{V_1}{a_s}\right)r\right)\right)}{\left(\alpha+(1-\alpha)\frac{V_2}{a_s}+(1-\alpha)\left(\frac{V_2}{a_s}-\frac{V_1}{a_s}\right)r\right)^2}-\frac{a_{d2}\left(\frac{V_2}{a_{d2}}-1\right)}{\left((1-\alpha)\frac{V_2}{a_{d2}}+\alpha\right)^2}$$

$$h''(\alpha)=\frac{2a_s\left(\frac{V_2}{a_s}-1\right)^2(1+r)}{\left(\alpha+(1-\alpha)\frac{V_2}{a_s}+(1-\alpha)\left(\frac{V_2}{a_s}-\frac{V_1}{a_s}\right)r\right)^3}-\frac{2a_{d2}\left(\frac{V_2}{a_{d2}}-1\right)^2}{\left(\alpha+(1-\alpha)\frac{V_2}{a_{d2}}\right)^3}.$$

Here, $h''(\alpha) = 0$ can be rewritten as:

$$\alpha + (1 - \alpha)\frac{V_2}{a_s} + (1 - \alpha)\left(\frac{V_2}{a_s} - \frac{V_1}{a_s}\right)r = \left(\frac{a_s\left(\frac{V_2}{a_s}-1\right)^2(1+r)}{a_{d2}\left(\frac{V_2}{a_{d2}}-1\right)^2}\right)^{\frac{1}{3}}\left(\alpha + (1 - \alpha)\frac{V_2}{a_{d2}}\right).$$

It can be easily seen that $h''(\alpha) = 0$ has at most one solution for $\alpha \in [0,1]$. If it doesn't have a solution for $\alpha \in [0,1]$, $h'(0) = 0$ has at most one solution for $\alpha \in [0,1]$. If the solution exists, it is

$$\alpha = \frac{\frac{V_2}{a_{d2}}\Phi - \Omega\Psi}{\left(\frac{V_2}{a_{d2}}-1\right)\Phi - (\Omega-1)\Psi},$$

where $\Phi = \left(a_s\left(\frac{V_2}{a_s} - 1\right)^2(1+r)\right)^{\frac{1}{3}}$, $\Psi = \left(a_{d2}\left(\frac{V_2}{a_{d2}} - 1\right)^2\right)^{\frac{1}{3}}$, and $\Omega = \frac{V_2}{a_s} + \left(\frac{V_2}{a_s} - \frac{V_1}{a_s}\right)r$. Let $\bar{\alpha} = \frac{\frac{V_2}{a_{d2}}\Phi - \Omega\Psi}{\left(\frac{V_2}{a_{d2}}-1\right)\Phi - (\Omega-1)\Psi}$. $\bar{\alpha} \in [0,1]$ if either of the following conditions holds:

(1) $\frac{a_{d2}\left(\frac{V_2}{a_{d2}}-1\right)^2\Omega^3}{(1+r)\left(\frac{V_2}{a_{d2}}\right)^3\left(\frac{V_2}{a_s}-1\right)^2} < a_s < \frac{a_{d2}\left(\frac{V_2}{a_{d2}}-1\right)^2}{(1+r)\left(\frac{V_2}{a_s}-1\right)^2}$ and $\frac{V_2}{a_{d2}} > \Omega$,

(2) $\frac{a_{d2}\left(\frac{V_2}{a_{d2}}-1\right)^2}{(1+r)\left(\frac{V_2}{a_s}-1\right)^2} < a_s < \frac{a_{d2}\left(\frac{V_2}{a_{d2}}-1\right)^2\Omega^3}{(1+r)\left(\frac{V_2}{a_{d2}}\right)^3\left(\frac{V_2}{a_s}-1\right)^2}$ and $\frac{V_2}{a_{d2}} < \Omega$.

Under Condition (1), we can confirm that $h''(\alpha) > 0$ for $0 < \alpha < \bar{\alpha}$, $h''(\alpha) < 0$ for $\bar{\alpha} < \alpha < 1$, and $h'(0) > 0$. Therefore, $h'(0) = 0$ has at most one solution for $\alpha \in [0,1]$.

Under Condition (2), we can confirm that $h''(\alpha) < 0$ for $0 < \alpha < \bar{\alpha}$, $h''(\alpha) > 0$ for $\bar{\alpha} < \alpha < 1$. In this case, $h'(0) = 0$ has at most one solution for $\alpha \in [0,1]$ except when $h'(0) > 0$, $h'(1) > 0$ and $h'(\bar{\alpha}) < 0$, for which $h'(0) = 0$ has two solutions. This region appears if

$$Max\left[\frac{a_{d2}\left(\frac{V_2}{a_{d2}}-1\right)\Omega^2}{\left(\frac{V_2}{a_{d2}}\right)^2\left(\frac{V_2}{a_s}-1+\left(\frac{V_1}{a_s}-1\right)r\left(1+\left(\frac{V_2}{a_s}-\frac{V_1}{a_s}\right)r(1+\Omega)\right)\right)}, \frac{a_{d2}\left(\frac{V_2}{a_{d2}}-1\right)}{\frac{V_2}{a_s}-1+\left(\frac{V_1}{a_s}-1\right)r}\right] < a_s < \frac{a_{d2}\left(\frac{V_2}{a_{d2}}-1\right)(\Omega-1)^2 - 3\Psi\Phi\left((\Omega-1)\Psi - \left(\frac{p_{d2}}{a_{d2}}-1\right)\Phi\right)}{\left(\left(\frac{V_2}{a_{d2}}-1\right)^2\left(\frac{V_2}{a_s}-1+\left(\frac{V_1}{a_s}-1\right)r\right)+\left(\frac{V_1}{a_s}-1\right)\left(\frac{V_2}{a_s}-\frac{V_1}{a_s}\right)r^2\left(1-2\frac{V_2}{a_{d2}}+\Omega\right)\right)}$$

In summary, $h'(\alpha)$ satisfies the uniqueness property except when the following two conditions are both satisfied:

(C1) $\frac{V_2}{a_{d2}} < \Omega$

(C2) $\mathrm{Max}\left[\frac{a_{d2}\left(\frac{V_2}{a_{d2}}-1\right)\Omega^2}{\left(\frac{V_2}{a_{d2}}\right)^2\left(\frac{V_2}{a_s}-1+\left(\frac{V_1}{a_s}-1\right)r\left(1+\left(\frac{V_2}{a_s}-\frac{V_1}{a_s}\right)r(1+\Omega)\right)\right)}, \frac{a_{d2}\left(\frac{V_2}{a_{d2}}-1\right)}{\frac{V_2}{a_s}-1+\left(\frac{V_1}{a_s}-1\right)r}, \frac{a_{d2}\left(\frac{V_2}{a_{d2}}-1\right)^2}{(1+r)\left(\frac{V_2}{a_s}-1\right)^2}\right] < a_s <$

$$\mathrm{Min}\left[\frac{a_{d2}\left(\frac{V_2}{a_{d2}}-1\right)^2\Omega^3}{(1+r)\left(\frac{V_2}{a_{d2}}\right)^3\left(\frac{V_2}{a_s}-1\right)^2}, \frac{a_{d2}\left(\frac{V_2}{a_{d2}}-1\right)(\Omega-1)^2 - 3\Psi\Phi\left((\Omega-1)\Psi - \left(\frac{V_2}{a_{d2}}-1\right)\Phi\right)}{\left(\left(\frac{V_2}{a_{d2}}-1\right)^2\left(\frac{V_2}{a_s}-1+\left(\frac{V_1}{a_s}-1\right)r\right)+\left(\frac{V_1}{a_s}-1\right)\left(\frac{V_2}{a_s}-\frac{V_1}{a_s}\right)r^2\left(1-2\frac{V_2}{a_{d2}}+\Omega\right)\right)}\right]$$

When (C1) and (C2) are satisfied, $h'(0) = 0$ has two solutions for $\alpha \in [0,1]$, $h'(0) > 0$, and $h'(1) > 0$. In this case, a shared infrastructure benefits more from increases in α at low and high values of α and a dedicated infrastructure at intermediate values of $\boldsymbol{\alpha}$. Thus our main result that improving α does not always benefit the shared infrastructure still holds. A numerical example in Figure A1 shows how the choice of infrastructure is affected by α in this scenario.

Proof of Proposition 2: According to Corollary 1, a shared infrastructure benefits from better reprovisioning at high α if

$$V_2 - a_{d2} < (V_2 - a_s) + r(V_1 - a_s).$$

This condition is less likely to hold when r increases. Therefore, at high α, as r increases, a shared infrastructure is always more likely to benefit more from better reprovisioning.

According to Corollary 2, a shared infrastructure benefits more from better reprovisioning at low α if

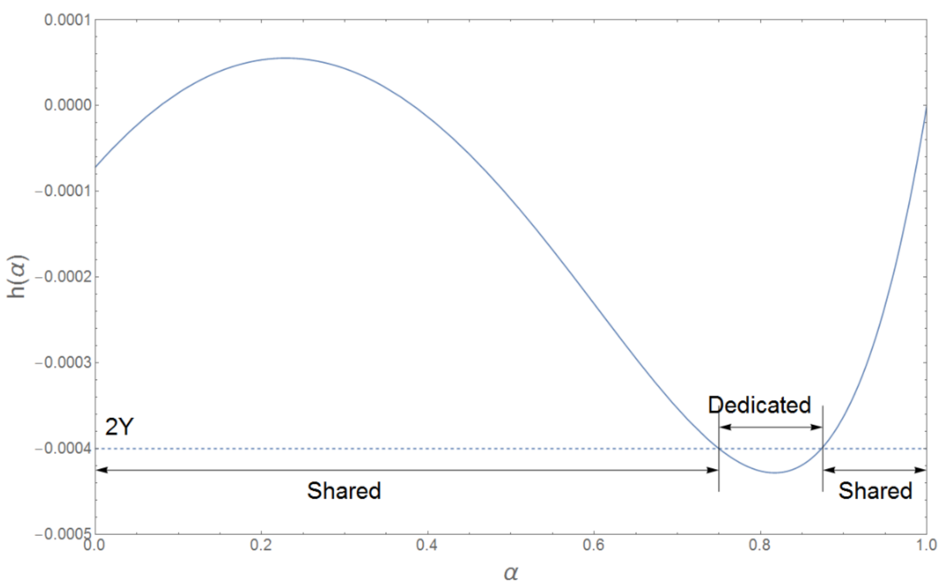
$$\frac{\frac{V_2}{a_{d2}}}{\sqrt{V_2-a_{d2}}} > \frac{\frac{V_2}{a_s}+r\left(\frac{V_2}{a_s}-\frac{V_1}{a_s}\right)}{\sqrt{V_2-a_s+r(V_1-a_s)\left(1+r\left(\frac{V_2}{a_s}-\frac{V_1}{a_s}\right)\left(1+\frac{V_2}{a_s}+r\left(\frac{V_2}{a_s}-\frac{V_1}{a_s}\right)\right)\right)}}.$$

This condition is equivalent to

$$\frac{V_2-a_{d2}}{\left(\frac{V_2}{a_{d2}}\right)^2} < \frac{V_2-a_s+r(V_1-a_s)\left(1+r\left(\frac{V_2}{a_s}-\frac{V_1}{a_s}\right)\left(1+\frac{V_2}{a_s}+r\left(\frac{V_2}{a_s}-\frac{V_1}{a_s}\right)\right)\right)}{\left(\frac{V_2}{a_s}+r\left(\frac{V_2}{a_s}-\frac{V_1}{a_s}\right)\right)^2}.$$

The left hand side of the above inequality is independent of r, and the first derivative of the right hand side on r equals $a_s\left(\frac{V_1}{a_s}-1-\frac{2\frac{V_1}{a_s}\left(\frac{V_2}{a_s}-1\right)^2}{\left(\frac{V_2}{a_s}-\frac{V_1}{a_s}r+\frac{V_2}{a_s}r\right)^3}+\frac{\left(\frac{V_1}{a_s}-1\right)\left(\frac{V_2}{a_s}-1\right)^2}{\left(\frac{V_2}{a_s}-\frac{V_1}{a_s}r+\frac{V_2}{a_s}r\right)^2}\right)$, which is greater than zero if V_1 is greater than the solution to the following equation that falls in the range of $(a_s,\ V_2)$: [1]

$$\frac{V_1}{a_s}\left(\frac{V_2}{a_s}-\frac{V_1}{a_s}r+\frac{V_2}{a_s}r\right)^3-\left(\frac{V_2}{a_s}-\frac{V_1}{a_s}r+\frac{V_2}{a_s}r\right)^3-\left(\frac{V_1}{a_s}-1\right)\left(\frac{V_2}{a_s}-1\right)^2\left(\frac{V_2}{a_s}-\frac{V_1}{a_s}r+\frac{V_2}{a_s}r\right)-2\frac{V_1}{a_s}\left(\frac{V_2}{a_s}-1\right)^2=0.$$



(Parameters: $V_2 = 1.956$, $V_1 = 1.467$, $a_s = 0.978$, $a_{s1} = 1$, , $a_{d2} = 0.945$, $r = 0.2$, $f = 0.1$, $a_{d1} = 1$, $X_1 = 1$, $X_2^{\max} = 1$, $c_s - c_{d1} - c_{d2} = 0.13$)

Figure A1. Impact of α on Infrastructure Choice when $h(\alpha)$ Does Not Satisfy the Uniqueness Property

[1]There is a closed-form solution but it is too complex to show.

Appendix B

Robustness to Model Changes

In this appendix, we demonstrate that the results are robust to several changes in the model.

Economies of Scale and Alternative Demand Distributions

In this section, we show that the behaviors and outcomes that the model helps elucidate are still present when economies of scale are included or when using a non-uniform demand distribution. The investigation is carried out by numerically computing optimal provisioning decisions for shared and dedicated infrastructures under these new conditions. It reveals that changes in the reprovisioning factor α still affect which infrastructure choice yields a higher profit. Furthermore, scenarios where multiple such changes arise as α varies in the range [0, 1] remain present as well.

The inclusion of economies of scale is a natural extension, as they represent a common benefit associated with shared solutions. It is, therefore, of interest to verify that the presence of such a benefit (for shared solutions) does not eliminate the impact that the coefficient α can have on determining the solution of choice. Similarly, validating that changes in demand distribution do not significantly affect the outcome is another standard test of the robustness of the results.

In Figure B1(a), we use $a_{d2}K_{d2}^{0.8}$ and $a_{d1}X_1^{0.8}$ to capture economies of scale in capacity costs for Services 1 and 2 respectively in the dedicated infrastructure, $a_sK_s^{0.8}$ for the flexible capacity in the shared infrastructure, and $a_{s1}X_1^{0.8}$ for the existing capacity for Service 1 in the shared infrastructure. The example shows an instance of infrastructure choice where dedicated infrastructures are preferred at both high and low α, while a shared infrastructure is preferred at intermediate values of α. In Figure B1(b), Service 2’s demand distribution follows a beta distribution with parameters (1.5, 1), which is negatively skewed. In this scenario, a shared infrastructure is preferred at both high and low α, while dedicated infrastructures are preferred at intermediate values of α. Figure B1(c) displays a similar example with the demand distribution of Service 2 now following an Erlang distribution with parameters (2, 5), which is positively skewed. In this scenario, dedicated infrastructures are preferred at both high and low values of α, while a shared infrastructure is preferred at intermediate values of α.

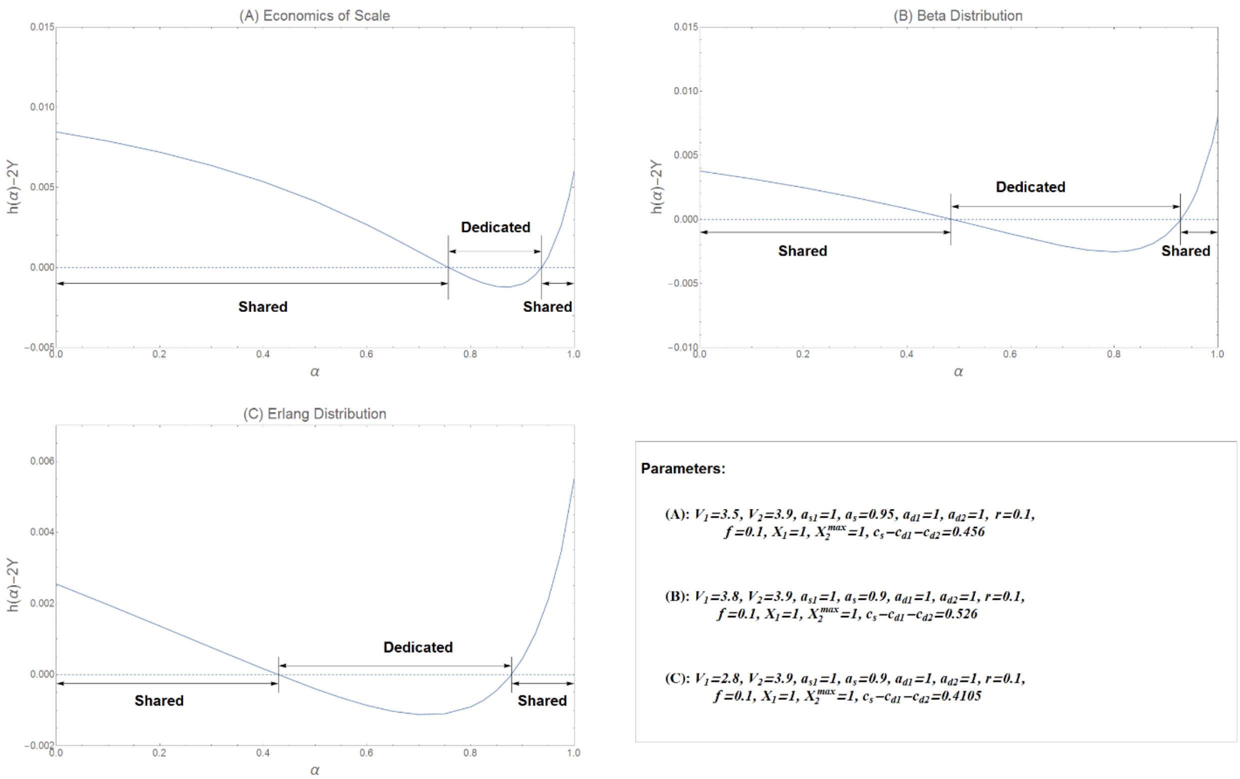
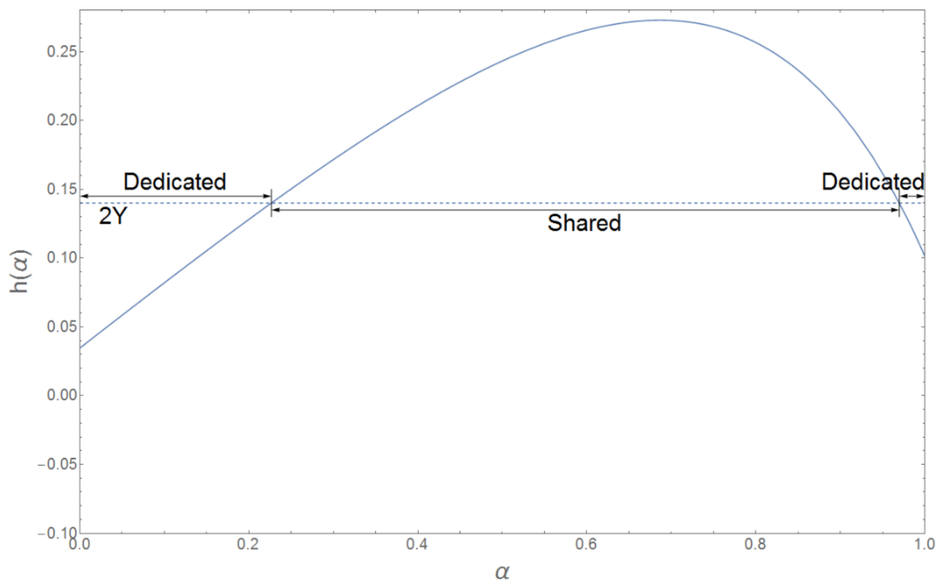


Figure B1. Impact of α on Infrastructure Choice When Economics of Scale or Different Form of Demand Distribution Is Assumed

Different α_s and α_d for Infrastructure Options

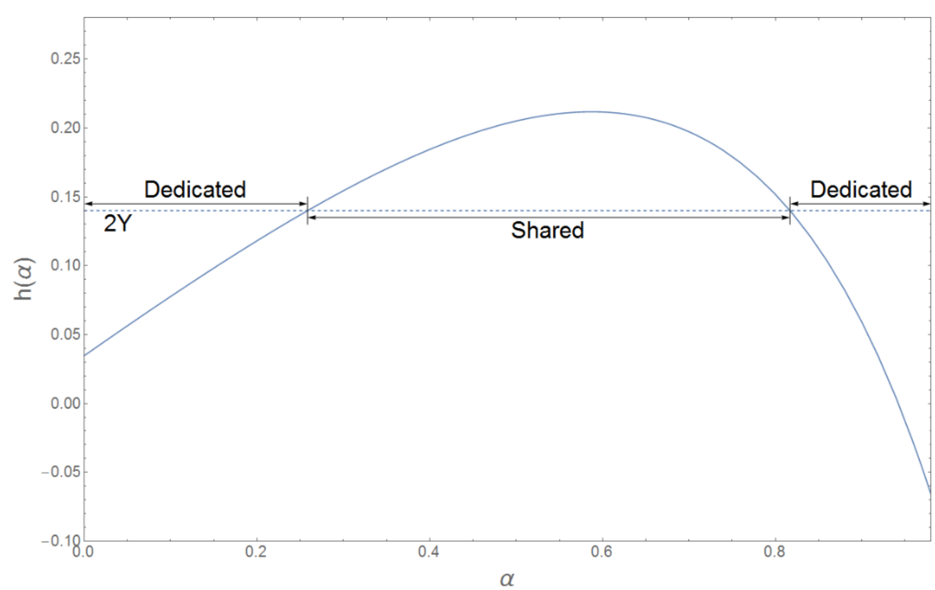
In this section, we numerically demonstrate the robustness of our findings in scenarios with $\alpha_d \neq \alpha_s$. Figures B2 and B3 provide the plots for the cases when α_d is 2% lower or higher than α_s, respectively. We see that qualitatively these plots are similar to those of Figure 4(b) for the case when $\alpha_d = \alpha_s = \alpha$ in the main model. As before, the dedicated infrastructure is preferred at both low and high α, whereas a shared infrastructure is preferred only for intermediate values of α.

Of course, when α_d and α_s are significantly different, that is, when the reprovisioning capability of one infrastructure significantly dominates the other, one infrastructure will be the optimal choice for all values of α. We can also have this result when the cost of one infrastructure is significantly lower than the other as well. But the key result that the reprovisioning ability plays an important role in impacting the correct choice of infrastructure and that the shared network is not necessarily preferred when α is large still holds even if α is different for the two infrastructure options.



(Parameters: $V_2 = 4.6, V_1 = 4.2, a_s = 3, a_{s1} = 1, a_{d2} = 2, a_{d1} = 1, X_1 = 1, X_2^{\max} = 2, c_s - c_{d1} - c_{d2} = -0.65, r = 0.1, f = 0.1, \alpha_s = \alpha,\ \alpha_d = 0.98\alpha$)

Figure B2. Plots for the Case When the Reprovisioning Ability α_d for the Dedicated Option is 2% Lower than α_s



(Parameters: $V_2 = 4.6, V_1 = 4.2, a_s = 3, a_{s1} = 1, a_{d2} = 2, a_{d1} = 1, X_1 = 1, X_2^{\max} = 2, c_s - c_{d1} - c_{d2} = -0.65, r = 0.1, f = 0.1, \alpha_s = \alpha,\ \alpha_d = 1.02\alpha$)

Figure B3. Plots for the Case When the Reprovisioning Ability α_d for the Dedicated Option is 2% Higher than α_s